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MULTIPLE STATE PROBLEM REDUCTION AND DECISION MAKING CRITERIA HYBRIDIZATION

Background. Due to that decision making is always involving a great deal of approaches and heuristics, and poor statistics and time course can generate series of decision making problems, the problem of regarding multiple states and criteria is considered.

Objective. The goal is to develop an approach for reducing the multiple state decision making problem along with regarding multiple criteria by their hybridization to solve disambiguously a single decision making problem.

Methods. An algorithm of reducing a finite series of decision making problems to a single problem is suggested. Also a statement is formulated to hybridize decision making criteria allowing to get a single optimal alternatives' set.

Results. Practically, this set contains just a single alternative. And, owing to the law of large numbers (of multiple criteria), the greater number of criteria is involved into the hybridization, the more reliable decision by the formulated statement is.

Conclusions. The represented multiple state problem reduction and decision making criteria hybridization both provide a researcher with the one decision making problem whose number of optimal solutions must be less than that by any other approaches. Besides, it allows to rank alternatives at higher reliability and validity. Furthermore, reliable weights (priorities) for scalarizing multicriteria problems are produced.

Keywords: decision making problem; multiple state problem; reduction; hybridization of criteria.

Introduction

Decision making is always involving a great deal of approaches and heuristics. They concern both estimation procedures [1, 2] and criteria to optimize decisions [3, 4]. Selection of a single approach or criterion along with the point evaluation is a non-trivial problem needing supplementary knowledge and statistical observations. Otherwise, without prior statistics, a selected method over the ordinarily point-evaluated decision matrix is going to fail or just be ineffective [1, 2, 5, 6].

The similar difficulty exists when multicriteria problems are solved. Without statistical data, scalarization appears the only way to pay attention to every plausible method and criterion. For this, minimax-based approaches are widely applied [7, 8]. Besides, sets and their cardinalities of both alternatives and states may vary as time goes by [1, 2, 6, 9, 10]. Therefore, to solve properly decision making problems (DMPs) under uncontrollable uncertainties, any non-excluded aspects and methods should be regarded.

Problem statement

Inasmuch as a finite series of DMPs is an aftermath of poor statistics and time course influence, an approach to reduce this series into a solvable DMP is needed. Variety of decision making criteria should be admitted as well. The goal lies in reducing the multiple state DMP (MSDMP) along with regarding multiple criteria to solve a single DMP. This goal is going to be reached after fulfilling the following steps:

1. Formalization of MSDMP.
2. Reduction of a finite series of DMPs generating an MSDMP in order to get an optimal alternatives' set (OAS) at disambiguation.
3. Decision making criteria hybridization for a single DMP.
4. Discussion of the reduction and hybridization.

Reduction of a finite series of DMPs

Henceforward, let all decision evaluations be kind of risks. Any risk is evaluated non-negatively. Suppose that, in the k-th condition (metastate), there is a finite set of alternatives (decisions) $X_k = \{x_i^{\langle k \rangle}\}_{i=1}^{M_k}$ by $M_k \in \mathbb{N} \backslash \{1\}$ and a finite set of states $S_k = \{s_j^{\langle k \rangle}\}_{j=1}^{N_k}$ by $N_k \in \mathbb{N} \backslash \{1\}$, where $k = \overline{1, K}$ by $K \in \mathbb{N} \backslash \{1\}$. Consequently, the decision matrix $\mathbf{R}_k = [r_{ij}^{\langle k \rangle}]_{M_k \times N_k}$ corresponds to the k-th metastate, where the entry $r_{ij}^{\langle k \rangle}$ is a risk after the decision $x_i^{\langle k \rangle}$ which fell into the state $s_j^{\langle k \rangle}$. Thus an MSDMP is modeled as decision matrices $\{\mathbf{R}_k\}_{k=1}^{K}$

along with sets $\{X_k\}_{k=1}^{K}$ and $\{S_k\}_{k=1}^{K}$. Note that it is not necessary that

$$\bigcap_{k=1}^{K} X_k = \varnothing \quad (1)$$

or

$$\bigcap_{k=1}^{K} S_k = \varnothing \quad (2)$$

because those K DMPs are related anyhow.

Occasionally, $M_k \times N_k$ DMP associated with the matrix $\mathbf{R}_k$ may be assigned to a probability p_k by

$$p_k > 0, \ \sum_{k=1}^{K} p_k = 1. \quad (3)$$

Denote by X_k^* the OAS by a decision making criterion applied to $\mathbf{R}_k$ DMP, $X_k^* \subset X_k$. Obviously, if subsets $\{X_k^*\}_{k=1}^{K}$ had common elements by

$$\bigcap_{k=1}^{K} X_k^* \neq \varnothing \quad (4)$$

then probabilities $\{p_k\}_{k=1}^{K}$ would not be needed, and MSDMP would be solved to an OAS

$$X^{**} = \bigcap_{k=1}^{K} X_k^*. \quad (5)$$

But this is rare case even when every of those K DMPs is solved by the same decision making criterion. However, the condition (4) is not excluded.

If

$$\bigcap_{k=1}^{K} X_k^* = \varnothing \quad (6)$$

then availability of probabilities $\{p_k\}_{k=1}^{K}$ does not solve this MSDMP at once. This is because we get into a probabilistic domain requiring strong statistical series. Particularly, if conditions and metastates of MSDMP recur periodically for at least a few hundred times then OAS X_k^* should be practiced with the probability p_k. But if they recur just a few times or singly at all, then probabilities $\{p_k\}_{k=1}^{K}$ are counted unavailable anyway.

Consequently, by the occasion (6) and a short-term statistical trend, the union of solutions of those K DMPs should be considered. This makes sense, however, only if

$$\bigcap_{k=1}^{K} X_k \neq \varnothing. \quad (7)$$

So if

$$\bigcup_{k=1}^{K} X_k^* \subset \bigcap_{k=1}^{K} X_k \quad (8)$$

then a new single DMP may be derived whose set of alternatives is

$$X = \bigcup_{k=1}^{K} X_k^*. \quad (9)$$

The set of states for this DMP is

$$S = \bigcup_{k=1}^{K} S_k. \quad (10)$$

Denoting $M = |X|$ and $N = |S|$, the single $M \times N$ DMP is finally formalized upon the decision matrix $\mathbf{R} = [r_{i_0 j_0}]_{M \times N}$ is evaluated, whose entry $r_{i_0 j_0}$ is a risk after the decision

$$x_{i_0} \in X = \{x_{i_0}\}_{i_0=1}^{M}$$

which fell into the state

$$s_{j_0} \in S = \{s_{j_0}\}_{j_0=1}^{N}.$$

If (8) fails (Fig. 1) for a short-term statistical trend then the most probable OAS should be practiced one after another, according to the descend-

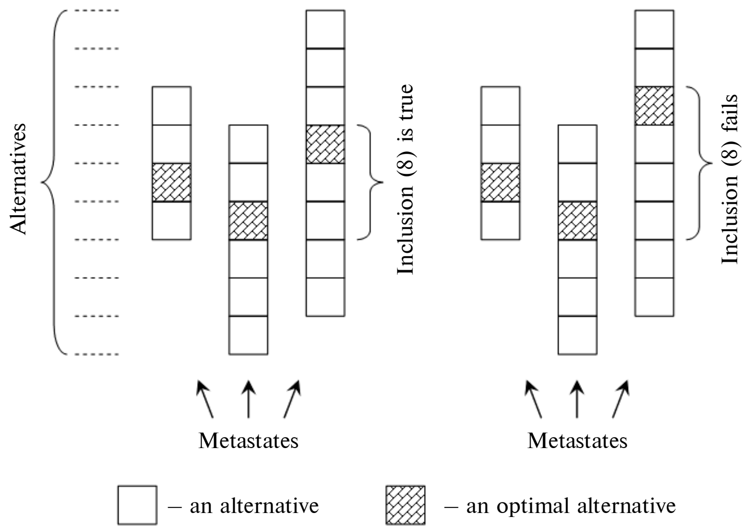

Fig. 1. A sketch for cases when the inclusion (8) is true and when it fails

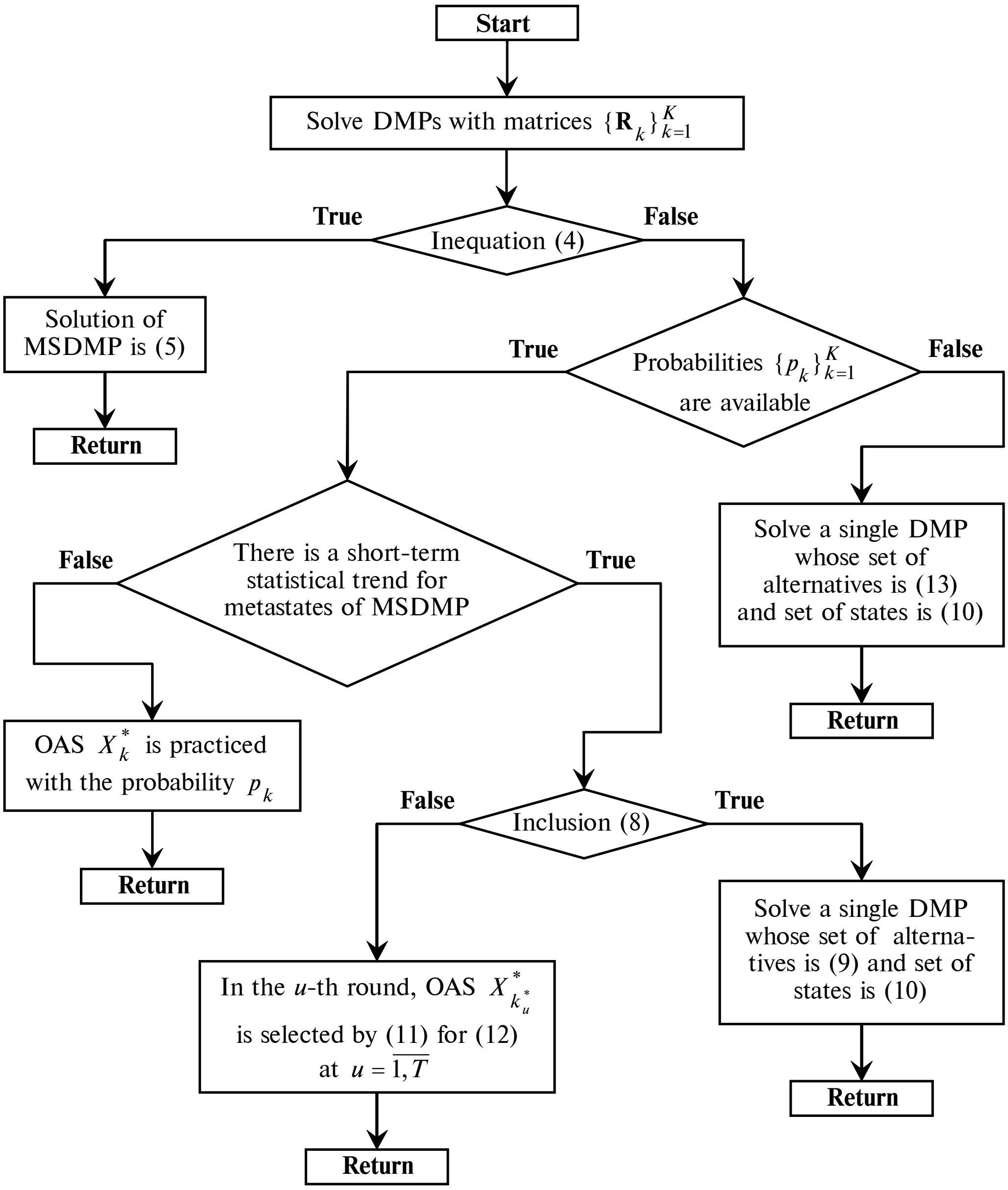

Fig. 2. An algorithm of reducing a finite series of K DMPs to a single DMP

ing probabilities. Statistically improbable metastates are disregarded. So if conditions and metastates of MSDMP recur for at least $T \in \mathbb{N}$ times (rounds) then, in the u-th round, OAS $X^*_{k^*_u}$ is selected by

$$k_1^* \in \arg\max_{k=\overline{1,K}} \{\{p_k\}_{k=1}^{K}\} \text{ and}$$

$$k_u^* \in \arg\max_{k=\overline{1,K-u+1}} \{\{\{p_k\}_{k=1}^{K}\} \backslash \{\{p_z^*\}_{z=1}^{u-1}\}\} \quad (11)$$

for

$$p_1^* = \max\{\{p_k\}_{k=1}^{K}\} \text{ and}$$

$$p_u^* = \max\{\{\{p_k\}_{k=1}^{K}\} \backslash \{\{p_z^*\}_{z=1}^{u-1}\}\} \quad (12)$$

at $u=\overline{1,T}$. Such a selection is relevant for $T<K$ or about that.

The worst occasion is when (6) is true and probabilities $\{p_k\}_{k=1}^{K}$ are unavailable. Then a new single DMP is derived whose set of alternatives is

$$X = \bigcup_{k=1}^{K} X_k \quad (13)$$

by the set of states (10). This single $M \times N$ DMP is finally formalized upon the decision matrix $\mathbf{R} = [r_{i_0 j_0}]_{M \times N}$ is evaluated.

An algorithmic representation of the described reduction of K DMPs is in Fig. 2. Practicing an OAS X_k^* with the probability p_k refers to [11]. A variate Θ which is uniformly distributed on half-interval $[0;1)$ is raffled. Its value is θ. And if

$$\theta \in \left[\sum_{k=1}^{z-1} p_k ; \sum_{k=1}^{z} p_k \right) \text{ for } z \in \{\overline{1, K}\} \tag{14}$$

then, in the current round, OAS X_z^* is chosen.

For reducing, the set of OAS $\{X_k^*\}_{k=1}^K$ is required. The algorithm in Figure 2 does not specify what criterion is applied to solve either DMPs with matrices $\{\mathbf{R}_k\}_{k=1}^K$ or the single DMP with $\mathbf{R}$. Selection of criteria is a separate task.

Decision making criteria hybridization

A large number of decision making criteria can be applied to solve an DMP [3, 4, 10, 12, 13]. A consequence of that, generally speaking, are different OASs whose intersection often occurs empty. Hence a single criterion which might include merits of all plausible criteria should better be used. The single criterion or approach will produce just an OAS disambiguating in the final decision selection.

Various criteria operate with differently measured values. This is why the risk decision nonnegative matrix $\mathbf{R} = [r_{ij}]_{M \times N}$ must be normalized into matrix $\tilde{\mathbf{R}} = [\tilde{r}_{ij}]_{M \times N}$ whose entry $\tilde{r}_{ij} \in [0;1]$ is a standardized risk after the decision $x_i \in X = \{x_i\}_{i=1}^M$ which fell into the state $s_j \in S = \{s_j\}_{j=1}^N$. And this is the known standardization rule:

$$\tilde{r}_{ij} = \frac{r_{ij} - \min\limits_{q=\overline{1,M}} \min\limits_{t=\overline{1,N}} r_{qt}}{\max\limits_{q=\overline{1,M}} \max\limits_{t=\overline{1,N}} r_{qt} - \min\limits_{q=\overline{1,M}} \min\limits_{t=\overline{1,N}} r_{qt}}. \tag{15}$$

The Savage criterion normalized regret matrix (SCNRM) $\tilde{\mathbf{F}}$ is deduced from the matrix $\tilde{\mathbf{R}}$. When the Germeyer criterion is on, it uses the stochastic matrix

$$\mathbf{P} = [p_{ij}]_{M \times N} \text{ by } \sum_{j=1}^{N} p_{ij} = 1 \tag{16}$$

whose entry p_{ij} is probability of situation $\{x_i, y_j\}$. The Germeyer criterion takes the decision matrix $\mathbf{R_P} = [r_{ij}^{\langle \mathbf{P} \rangle}]_{M \times N}$ by $r_{ij}^{\langle \mathbf{P} \rangle} = r_{ij} \cdot p_{ij}$. Thus matrix $\mathbf{R_P} = [r_{ij}^{\langle \mathbf{P} \rangle}]_{M \times N}$ is normalized into matrix $\tilde{\mathbf{R}}_\mathbf{P} = [\tilde{r}_{ij}^{\langle \mathbf{P} \rangle}]_{M \times N}$ where

$$\tilde{r}_{ij}^{\langle \mathbf{P} \rangle} = \frac{r_{ij}^{\langle \mathbf{P} \rangle} - \min\limits_{q=\overline{1,M}} \min\limits_{t=\overline{1,N}} r_{qt}^{\langle \mathbf{P} \rangle}}{\max\limits_{q=\overline{1,M}} \max\limits_{t=\overline{1,N}} r_{qt}^{\langle \mathbf{P} \rangle} - \min\limits_{q=\overline{1,M}} \min\limits_{t=\overline{1,N}} r_{qt}^{\langle \mathbf{P} \rangle}} \tag{17}$$

identically to (15), giving $\tilde{r}_{ij}^{\langle \mathbf{P} \rangle} \in [0;1]$.

The standardization rule (15) is not suitable for the product criterion because all M products $\prod_{j=1}^{N} \tilde{r}_{ij}$ must be positive. Instead of (15), if the matrix $\mathbf{R}$ contains zero entries (say, the minimal risk has been evaluated to zero), the rule

$$r_{ij}^{\langle \bullet \rangle} = \frac{r_{ij} + \gamma}{\max\limits_{q=\overline{1,M}} \max\limits_{t=\overline{1,N}} r_{qt} + \gamma} \text{ by } \gamma > 0 \tag{18}$$

gives the positive matrix $\mathbf{R}^{\langle \bullet \rangle} = [r_{ij}^{\langle \bullet \rangle}]_{M \times N}$ with $r_{ij}^{\langle \bullet \rangle} \in (0;1]$. For $\mathbf{R} > 0$ the rule (18) is stated simpler:

$$r_{ij}^{\langle \bullet \rangle} = \frac{r_{ij}}{\max\limits_{q=\overline{1,M}} \max\limits_{t=\overline{1,N}} r_{qt}}, \tag{19}$$

where we do not need to justify a selection of some $\gamma > 0$.

When decision making criteria use matrices $\tilde{\mathbf{R}}$, $\tilde{\mathbf{F}}$, $\mathbf{P}$, $\tilde{\mathbf{R}}_\mathbf{P}$, $\mathbf{R}^{\langle \bullet \rangle}$, the expected (estimated by a criterion) risk not depending upon states comes within segment $[0;1]$ having no units of measurement. Let $r_h(x_i)$ be the risk estimated by the h-th criterion for the alternative x_i. Then

$$X^* = \arg \min_{\{x_i\}_{i=1}^M} \sum_{h=1}^{H} \lambda_h r_h(x_i) \tag{20}$$

is a single OAS with the h-th criterion weight

$$\lambda_h > 0 \text{ by } \sum_{h=1}^{H} \lambda_h = 1 \tag{21}$$

where $H \in \mathbb{N} \backslash \{1\}$ is a number of criteria involved to solve a DMP.

As an example, consider 5×8 risk decision matrix

$$\mathbf{R} = \begin{bmatrix} 6 & 2 & 2 & 5 & 2 & 3 & 1 & 3 \\ 3 & 4 & 0 & 2 & 0 & 3 & 6 & 4 \\ 5 & 2 & 4 & 8 & 1 & 2 & 5 & 4 \\ 1 & 4 & 5 & 1 & 1 & 2 & 5 & 1 \\ 4 & 1 & 2 & 2 & 4 & 6 & 1 & 1 \end{bmatrix} \quad (22)$$

wherein the minimax criterion gives a single optimal alternative, namely OAS is $X^* = \{x_4\}$. However, the Savage criterion by its regret matrix

$$\mathbf{F} = \begin{bmatrix} 5 & 1 & 2 & 4 & 2 & 1 & 0 & 2 \\ 2 & 3 & 0 & 1 & 0 & 1 & 5 & 3 \\ 4 & 1 & 4 & 7 & 1 & 0 & 4 & 3 \\ 0 & 3 & 5 & 0 & 1 & 0 & 4 & 0 \\ 3 & 0 & 2 & 1 & 4 & 4 & 0 & 0 \end{bmatrix} \quad (23)$$

gives $X^* = \{x_5\}$. Moreover, having added 1 to matrix (22), we get positive matrix wherein the product criterion gives five products 36288, 8400, 145800, 8640, 12600 correspondingly for alternatives $\{x_i\}_{i=1}^{5}$ and thus $X^* = \{x_2\}$. This is an instance where DMP with (22) has three different OASs by three criteria.

For disambiguation, hybridize those criteria according to normalization (15) and (18) by $\gamma = 1$:

$$\tilde{\mathbf{R}} = \begin{bmatrix} 3/4 & 1/4 & 1/4 & 5/8 & 1/4 & 3/8 & 1/8 & 3/8 \\ 3/8 & 1/2 & 0 & 1/4 & 0 & 3/8 & 3/4 & 1/2 \\ 5/8 & 1/4 & 1/2 & 1 & 1/8 & 1/4 & 5/8 & 1/2 \\ 1/8 & 1/2 & 5/8 & 1/8 & 1/8 & 1/4 & 5/8 & 1/8 \\ 1/2 & 1/8 & 1/4 & 1/4 & 1/2 & 3/4 & 1/8 & 1/8 \end{bmatrix}, \quad (24)$$

the corresponding regret matrix is

$$\tilde{\mathbf{F}} = \begin{bmatrix} 5/8 & 1/8 & 1/4 & 1/2 & 1/4 & 1/8 & 0 & 1/4 \\ 1/4 & 3/8 & 0 & 1/8 & 0 & 1/8 & 5/8 & 3/8 \\ 1/2 & 1/8 & 1/2 & 7/8 & 1/8 & 0 & 1/2 & 3/8 \\ 0 & 3/8 & 5/8 & 0 & 1/8 & 0 & 1/2 & 0 \\ 3/8 & 0 & 1/4 & 1/8 & 1/2 & 1/2 & 0 & 0 \end{bmatrix} \quad (25)$$

and

$$\mathbf{R}^{\langle \bullet \rangle} = \begin{bmatrix} 7/9 & 1/3 & 1/3 & 2/3 & 1/3 & 4/9 & 2/9 & 4/9 \\ 4/9 & 5/9 & 1/9 & 1/3 & 1/9 & 4/9 & 7/9 & 5/9 \\ 2/3 & 1/3 & 5/9 & 1 & 2/9 & 1/3 & 2/3 & 5/9 \\ 2/9 & 5/9 & 2/3 & 2/9 & 2/9 & 1/3 & 2/3 & 2/9 \\ 5/9 & 2/9 & 1/3 & 1/3 & 5/9 & 7/9 & 2/9 & 2/9 \end{bmatrix}.$$

Without any priorities, weights (21) can be put equal. And (20) is stated by $\{\lambda_h = 1/3\}_{h=1}^{3}$ as

$$X^* = \arg \min_{\{x_i\}_{i=1}^{5}} \sum_{h=1}^{3} \lambda_h r_h(x_i) = \arg \min_{\{x_i\}_{i=1}^{5}} \sum_{h=1}^{3} r_h(x_i).$$

The minimax, Savage, and product criteria are indexed by $h = 1$, $h = 2$, $h = 3$, respectively:

$$\{r_1(x_i)\}_{i=1}^{5} = \{3/4,\ 3/4,\ 1,\ 5/8,\ 3/4\},$$

$$\{r_2(x_i)\}_{i=1}^{5} = \{5/8,\ 5/8,\ 7/8,\ 5/8,\ 1/2\},$$

$$\{r_3(x_i)\}_{i=1}^{5} =$$

$$= \{448/3^{12},\ 2800/3^{15},\ 200/3^{10},\ 320/3^{13},\ 1400/3^{14}\} \approx$$

$$\approx \{0.000843,\ 0.000195,\ 0.003387,\ 0.000201,$$

$$0.000293\},$$

wherein the truncation error is insignificant. Finally, the hybridization gives the single OAS

$$X^* = \arg \min_{\{x_i\}_{i=1}^{5}} \{1.375843,\ 1.375195,\ 1.878387,$$

$$1.250201,\ 1.250293\} = \{x_4\}$$

whose single optimal alternative coincides with the minimax one.

It might seem that $X^* = \{x_4\}$ is the solution. But let think of how SCNRM (25) was calculated. It was deduced from the normalized risk decision matrix (24). However, SCNRM could be calculated straightforwardly by normalizing the origin regret matrix (23), using the standardization rule identical to (15). Denote such an SCNRM by $\tilde{\mathbf{F}}^{\langle 1 \rangle}$. In the being considered example,

$$\tilde{\mathbf{F}}^{\langle 1 \rangle} = \begin{bmatrix} 5/7 & 1/7 & 2/7 & 4/7 & 2/7 & 1/7 & 0 & 2/7 \\ 2/7 & 3/7 & 0 & 1/7 & 0 & 1/7 & 5/7 & 3/7 \\ 4/7 & 1/7 & 4/7 & 1 & 1/7 & 0 & 4/7 & 3/7 \\ 0 & 3/7 & 5/7 & 0 & 1/7 & 0 & 4/7 & 0 \\ 3/7 & 0 & 2/7 & 1/7 & 4/7 & 4/7 & 0 & 0 \end{bmatrix} \quad (26)$$

and, with SCNRM (26),

$$\{r_2(x_i)\}_{i=1}^{5} = \{5/7,\ 5/7,\ 1,\ 5/7,\ 4/7\},$$

whereupon we get diverse OAS:

$$X^* = \arg\min_{\{x_i\}_{i=1}^{5}} \{1.465129,\ 1.464481,\ 2.003387,\ 1.339486,\ 1.321721\} = \{x_5\}.$$

Having no preference to $\tilde{\mathbf{F}}$ and $\tilde{\mathbf{F}}^{\langle 1\rangle}$, these both ought to be regarded while each of them produces different result. Therefore, if $h_0 \in \{\overline{1,H}\}$ in (20) corresponds to the Savage criterion then

$$X^* = \arg\min_{\{x_i\}_{i=1}^{M}} \left\{ \sum_{h\in\{\overline{1,H}\}\backslash\{h_0\}} \lambda_h r_h(x_i) + \frac{1}{2}\lambda_{h_0} r_{h_0}(x_i) + \frac{1}{2}\lambda_{h_0} r_{h_0}^{\langle 1\rangle}(x_i) \right\} \quad (27)$$

by the risk $r_{h_0}(x_i)$ estimated via SCNRM $\tilde{\mathbf{F}}$ and the risk $r_{h_0}^{\langle 1\rangle}(x_i)$ estimated via SCNRM $\tilde{\mathbf{F}}^{\langle 1\rangle}$. Obviously, OAS (27) is more general than (20).

For the example of the risk decision matrix (22) with (27), we write

$$X^* = \arg\min_{\{x_i\}_{i=1}^{5}} \left\{ r_1(x_i) + r_3(x_i) + \frac{1}{2} r_2(x_i) + \frac{1}{2} r_2^{\langle 1\rangle}(x_i) \right\}$$

by denotation

$$\{r_2^{\langle 1\rangle}(x_i)\}_{i=1}^{5} = \{5/7,\ 5/7,\ 1,\ 5/7,\ 4/7\}$$

related to SCNRM $\tilde{\mathbf{F}}^{\langle 1\rangle}$. And now

$$X^* = \arg\min_{\{x_i\}_{i=1}^{5}} \{1.420486,\ 1.419838,\ 1.940887,\ 1.294844,\ 1.286007\} = \{x_5\}.$$

Heretofore, we did not pay attention to values $\{r_3(x_i)\}_{i=1}^{5}$ which are very small. And this is a distinctive feature of the expected risk by the product criterion over the normalized matrix $\mathbf{R}^{\langle\bullet\rangle}$ — when number of states increases, the expected risk badly decreases not influencing on the grand total. In the example, those expected risks can be rounded even to zero, but the truncation error is still insignificant. To prevent this drawback of the product criterion normalization, the following expected risks are better to use:

$$\tilde{r}_h(x_i) = \frac{r_h(x_i)}{\max\limits_{q=\overline{1,M}} r_h(x_q)} \quad \forall\, i = \overline{1,M} \text{ and } \forall\, h = \overline{1,H}. \quad (28)$$

Normalization (28) implies that

$$r_h(x_i) \geqslant 0 \quad \forall\, i = \overline{1,M} \text{ and } \forall\, h = \overline{1,H}$$

what always can be done in processing matrices $\tilde{\mathbf{R}}$, $\tilde{\mathbf{F}}$, $\tilde{\mathbf{F}}^{\langle 1\rangle}$, $\mathbf{P}$, $\tilde{\mathbf{R}}_{\mathbf{P}}$, $\mathbf{R}^{\langle\bullet\rangle}$ even if matrix $\mathbf{R}$ has negative entries. But if the expected risks $\{\{r_h(x_i)\}_{i=1}^{M}\}_{h=1}^{H}$ are deduced bypassing the standardization rules (15) and (17) then, instead of (28), the normalized expected risks are

$$\tilde{r}_h(x_i) = \frac{r_h(x_i) - \min\limits_{q=\overline{1,M}} r_h(x_q)}{\max\limits_{q=\overline{1,M}} r_h(x_q) - \min\limits_{q=\overline{1,M}} r_h(x_q)} \quad \forall\, i = \overline{1,M} \text{ and } \forall\, h = \overline{1,H}. \quad (29)$$

Normalization (29) implies that, for every h-th criterion, $\exists\, i_0 \in \{\overline{1,M}\}$ such that $\tilde{r}_h(x_{i_0}) = 0$ and $\exists\, i_1 \in \{\overline{1,M}\}$ such that $\tilde{r}_h(x_{i_1}) = 1$. This relieves from selecting or combination between $\tilde{\mathbf{F}}$ and $\tilde{\mathbf{F}}^{\langle 1\rangle}$ allowing to re-state (20) as

$$X^* = \arg\min_{\{x_i\}_{i=1}^{M}} \sum_{h=1}^{H} \lambda_h \tilde{r}_h(x_i) \quad (30)$$

with the h-th criterion weight (21).

Completing the example of the risk decision matrix (22), we get

$$\{\tilde{r}_1(x_i)\}_{i=1}^{5} = \{3/4,\ 3/4,\ 1,\ 5/8,\ 3/4\},$$

$$\{\tilde{r}_2(x_i)\}_{i=1}^{5} = \{5/7,\ 5/7,\ 1,\ 5/7,\ 5/7\},$$

$$\{\tilde{r}_3(x_i)\}_{i=1}^{5} = \{56/225,\ 14/243,\ 1,\ 8/135,\ 7/81\},$$

and OAS

$$X^* = \arg\min_{\{x_i\}_{i=1}^{5}} \{1.713175,\ 1.521899,\ 3,\ 1.398545,\ 1.407848\} = \{x_4\} \quad (31)$$

is the ultimately best solution. Note that the minimax and Savage criterion came too close with their risks 1.398545 and 1.407848, although the product criterion appeared far behind them.

Discussion

MSDMP and its formalization can be imagined as a stratification of a finite series of DMPs with their matrices. Each layer is a DMP matrix. The reduction into a DMP is similar to scalarization in solving multicriteria problems. The algorithm in Figure 2 has two sides. The first one is that it relies on statistics supposing probabilities $\{p_k\}_{k=1}^{K}$ are known. This also often assumes that there is a long-term statistical trend, enough for practicing OASs $\{X_k^*\}_{k=1}^{K}$ where X_z^* is chosen if (14) is true in the current round. The second side is far more real: probabilities $\{p_k\}_{k=1}^{K}$ cannot be evaluated as points or they are just unknown, and there is a short-term statistical trend for metastates of MSDMP. In this way, a union-like DMP with set of alternatives (13) and set of states (10) is the most relevant. A short-term statistical trend nonetheless implies DMP with the set of alternatives (9) and set of states (10) when the inclusion (8) turns true.

Cases in which (1) or (2) turn true are practically impossible unless DMPs have very weak relation. Nevertheless, such "scattered" DMPs may be assigned rather with probabilities $\{p_k\}_{k=1}^{K}$ by (3) than those DMPs which have stronger relation to each other what actually impedes distinguishing related DMPs. Despite any relation strength, an OAS by (5) is rarely possible requiring at least the condition (7).

Decision making criteria hybridization aims at disambiguation as well. Sometimes normalization to matrices $\tilde{\mathbf{R}}$, $\tilde{\mathbf{F}}$, $\tilde{\mathbf{F}}^{\langle 1\rangle}$, $\tilde{\mathbf{R}}_{\mathbf{P}}$, $\mathbf{R}^{\langle\bullet\rangle}$ is needed to compare expected risks as they are. Then formulas (20) and (27) could be useful. Normalizing expected risks by (29), meritoriously, brings to simple hybridization effect by (30). That requires only weights $\{\lambda_h\}_{h=1}^{H}$ whose values, in statistically poor cases of DMP, are set identical: $\{\lambda_h = H^{-1}\}_{h=1}^{H}$.

In most practical events, probability-based criteria (say, Germeyer, modal, minimal variance, maximal probability, etc.) are not reliable. This is caused by the stochastic matrix (16) is influenced with a great deal of factors and badly varies as time goes by. So when (30) is constructed, weights corresponding to probability-based criteria could be taken smaller.

For non-risk matrices, those normalization rules fit also. Only $\gamma > 0$ must be justified such that $\mathbf{R}^{\langle\bullet\rangle} > 0$ when the rule (19) is non-applicable. For gain (profit) matrices, minimum in (30) is substituted with maximum. And expected gains are weighted as usually, but, if the minimal variance criterion is included, minimal variance expected values are taken with minuses. The same concerns Savage criterion regret expected values.

Conclusions

The represented multiple state problem reduction in Fig. 2 and decision making criteria hybridization by (30) both provide a researcher with the one DMP having the single OAS, which usually contains less elements than OAS by any other approaches. Here, a problem of selecting a unique decision from the OAS is not solved. But, with sufficiently great number of criteria involved in hybridization, OAS is believed to contain just one element, that unique decision. This is a manifestation of the law of large numbers transfigured into the law of multiple approaches (criteria). The greater number of criteria is involved, the more reliable decisions by the statement (30) are.

In addition to improved substantiation of optimality, unification and normalization allow to rank alternatives at higher reliability and validity [14, 15]. For instance, after the solution (31), alternatives are ranked as follows:

$$x_4 \succ x_5 \succ x_2 \succ x_1 \succ x_3 . \qquad (32)$$

Besides, if the matrix (22) characterized a five-criteria problem, then it might be solved via scalarization either by weights (if the risk features importance of alternative)

{0.18948, 0.168324, 0.331805, 0.154681, 0.15571}

or by weights (if the risk features non-importance of alternative)

{0.194606, 0.219065, 0.111131, 0.238387, 0.236811}

which would correspond to alternatives in the ranking (32).

In this way, further work is going to be connected with multiple criteria which are applied to solving multicriteria problems.

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В.В. Романюк

РЕДУКЦІЯ БАГАТОПОЗИЦІЙНИХ ЗАДАЧ І ГІБРИДИЗАЦІЯ КРИТЕРІЇВ ПРИЙНЯТТЯ РІШЕНЬ

Проблематика. Оскільки прийняття рішень завжди зачіпає багато підходів й евристик, а також недостатня статистика і хід часу можуть породжувати цілі послідовності задач прийняття рішень, то розглядається задача врахування множинних станів і критеріїв.

Мета дослідження. Розробка методу редукції загальної задачі прийняття рішень з множинними станами поряд з урахуванням множинних критеріїв через їх гібридизацію для однозначного розв'язання єдиної задачі прийняття рішень.

Методика реалізації. Пропонується алгоритм зведення скінченної множини задач прийняття рішень до єдиної задачі прийняття рішень. Також формалізується гібридизація критеріїв прийняття рішень, яка дає змогу отримати єдину множину оптимальних альтернатив.

Результати дослідження. На практиці ця множина містить лише одну альтернативу. Тут, завдяки дії закону великих чисел (множинних критеріїв), чим більше число критеріїв, що залучаються до гібридизації, тим більш надійним, згідно зі сформульованим виразом, виходить рішення.

Висновки. Представлені редукція багатопозиційних задач і гібридизація критеріїв прийняття рішень забезпечують для дослідника одну задачу прийняття рішень, число оптимальних розв'язків якої має бути меншим, ніж за будь-якими іншими підходами. Також це дає змогу ранжувати альтернативи з більшими надійністю та достовірністю. Крім того, утворюються надійні ваги (пріоритети) для скаляризації багатокритеріальних задач.

Ключові слова: задача прийняття рішень; багатопозиційна задача; редукція; гібридизація критеріїв.

В.В. Романюк

РЕДУКЦИЯ МНОГОПОЗИЦИОННЫХ ЗАДАЧ И ГИБРИДИЗАЦИЯ КРИТЕРИЕВ ПРИНЯТИЯ РЕШЕНИЙ

Проблематика. Поскольку принятие решений всегда затрагивает много подходов и эвристик, а также недостаточная статистика и течение времени могут порождать целые последовательности задач принятия решений, то рассматривается задача учета множественных состояний и критериев.

Цель исследования. Разработка метода редукции общей задачи принятия решений с множественными состояниями наряду с учетом множественных критериев путем их гибридизации для однозначного решения единственной задачи принятия решений.

Методика реализации. Предлагается алгоритм приведения конечного множества задач принятия решений к единственной задаче принятия решений. Также формализуется гибридизация критериев принятия решений, позволяющая получить единственное множество оптимальных альтернатив.

Результаты исследования. На практике это множество содержит всего лишь единственную альтернативу. Здесь, благодаря действию закона больших чисел (множественных критериев), чем больше число критериев, вовлекаемых в гибридизацию, тем более надежным, согласно сформулированному выражению, выходит решение.

Выводы. Представленные редукция многопозиционных задач и гибридизация критериев принятия решений обеспечивают для исследователя одну задачу принятия решений, число оптимальных решений которой должно быть меньше, чем согласно любым другим подходам. Также это позволяет ранжировать альтернативы с большими надежностью и достоверностью. Кроме того, создаются надежные веса (приоритеты) для скаляризации многокритериальных задач.

Ключевые слова: задача принятия решений; многопозиционная задача; редукция; гибридизация критериев.

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