ENHANCING THE INQUIRY-BASED LEARNING VIA REFORMULATING CLASSICAL PROBLEMS AND DYNAMIC SOFTWARE

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Ми представляємо динамічні дидактичні сценарії, що базуються на динамічному ППЗ, наприклад GeoGebra, за допомогою якого можливі експерименти з геометричними об’єктами. Акцент здійснено на школярах у ролі дослідників, які працюють з динамічними конструкціями, формулюють гіпотези і самостійно досягають суті властивостей, після чого доводять їх.

Мы представляем динамические дидактические сценарии, базированные на динамическом софтуере, например как GeoGebra, с помощью которого возможны эксперименты с геометрическими объектами. Акцент поставлен на школьниках в роли исследователей, которые работают с динамическими конструкциями, формулируют гипотезы и самостоятельно достигают до сути свойств, после чего доказывают их.

We present dynamic scenarios based on dynamic software (such as GeoGebra) in which various experiments with geometric objects can be performed. The focus is on putting the learners in the role of investigators who are expected to explore the dynamic constructions, to formulate conjectures and then – to prove them as theorems of their own.

Don’t preach facts, stimulate acts!
Paul Halmos

1. Introduction.

The great art of teaching mathematics has long traditions but the development of digital technologies present mathematics educators with real challenges – how to create a class culture making the best use of these technologies so that the students could behave like working mathematicians, i.e. to play with mathematical ideas and to communicate their findings. To create such a class culture by designing and developing computer environments of exploratory type (Geomland, Elica) and then experiment with new principles of teaching has been the goal of a long-term research in Bulgaria dating from the early 80s [viz. 1-7].

A series of good practices was reported [8-9] in which the teachers had managed to overcome the sterility of the preaching style: “Look how clever I am and what good solution of the problem I know” or “Here are some theorems discovered by mathematics geniuses and you should learn their proofs”. The teachers involved in the pilot experiments integrating the computer environments of laboratory type in mathematics classes got convinced that mathematical thinking is not purely “formal”, that it involves generalizing from observed cases, inductive arguments, recognizing a mathematical concept in a specific situation or extracting this notion from it. And even more important – that to teach guessing and conjecturing is vital for conveying the real spirit of mathematics in a school setting.

This awareness is in harmony with Polya’s advice to the pre-service math education students: teachers should not ask the questions but kids should ask the questions….The ideas should be born in the students’ mind and the teacher should act as a midwife.

2. The inquiry based learning spreading in Europe.

With the advent of powerful modern computers and specially designed educational software for mathematical experiments a way was opened for the inquiry-based learning in many European countries [10]. Developing and implementing didactical concepts and strategies for the use of dynamic software was recognized as crucial for the mathematics education and a number of European Projects are focusing on this, e.g.

- “InnoMathEd - Innovations in Mathematics Education on European Level" and
- “Fibonacci - Disseminating inquiry-based science and mathematics education in Europe” [11-13]

Special attention in both projects is given to the development and the dissemination of the so called dynamic scenarios based on dynamic software (such as GeoGebra) in which various experiments with geometric objects can be performed. The focus is on putting the learners in the role of investigators who are expected to explore the dynamic constructions, to formulate conjectures and then – to prove them as theorems of their own.

3. A dynamic scenario on parallelograms.

To illustrate these ideas we shall consider the dynamic scenario Parallelograms for 7th grade making use of GeoGebra [14].

To explore the properties of the parallelogram a dynamic construction is offered to the students with options for showing/hiding some of its elements together with their measurements. Students are expected to formulate their hypotheses related to the sides, the angles and the diagonals of the parallelogram.

The rigorous proofs will be done in several consecutive classes but what matters the most is the fact that the students will have experienced the joy of the discovery and will have formulated themselves the theorems to be proven.

Many of the classical problems end with the phrase: Prove that…Such formulation could be compared with revealing the mystery on the first page of a criminal novel… To prepare the mathematical ground for explorations and possible discoveries on behalf of the students we offer another formulation.

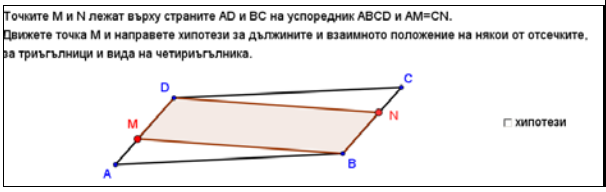
Consider for instance the problem:

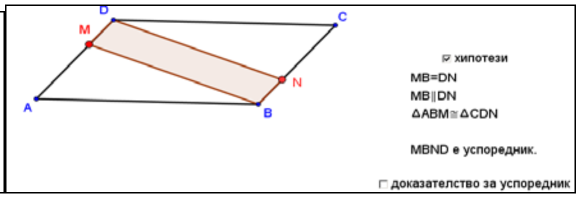
The points M and N are on the sides AD and BC of the parallelogram $ABCD$, and $AM = CN$. Prove that $\Delta ABM \cong \Delta CDN$

We find it more appropriate to replace the “Prove that” part by the following

Move point M and formulate your conjecture about the lengths and the mutual position of some segments in the construction, about some of the triangles and about the type of the quadrilateral.

The dynamic construction accompanying this problem enables the students to move not only the point M along the side AD but also the whole parallelogram so as to get various specific parallelograms. Thus, by manipulating the construction and observing what varies and what stays unchanged, they can formulate their hypotheses.





This style is followed through the whole module. To stimulate students to prove specific theorems we first prepare the setting for them and encourage them to explore the situation and make conjectures:

As an example let us consider the following exploratory problem:

Let O be the intersection point of the diagonals of the parallelogram $ABCD$. A line passing through O meets AB and DC in the points E and F. Make conjectures.

Some of the possible conjectures we expect from the students include: $EO = FO$; $\Delta AEO \cong \Delta CFO$; $AECF$ is a parallelogram; EF divides the parallelogram in two parts of equal area.

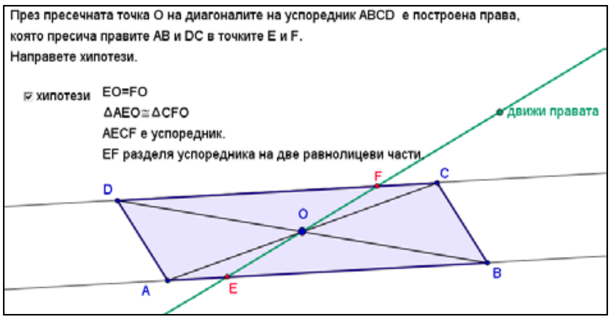
Then the following problems are offered:

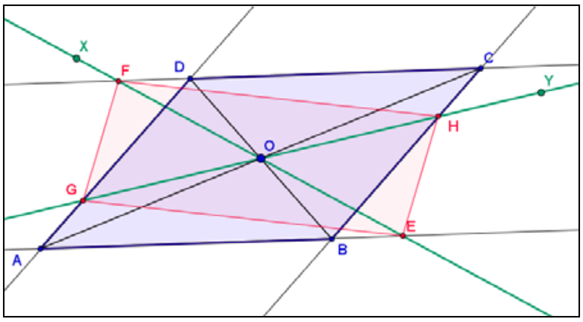
Let O be the intersection point of the diagonals of the parallelogram $ABCD$. Two lines OX and OY .are drawn through O so that OX intersects AB and DC in the points E and F, and the line OY intersects AD и BC in the points G and H. Proved that $EHFG$ is a parallelogram.

Prove that every line through the intersection point of the diagonals of a parallelogram divides it in two parts of equal area.

Given two parallelograms construct a line which divides them in two parts of equal area.

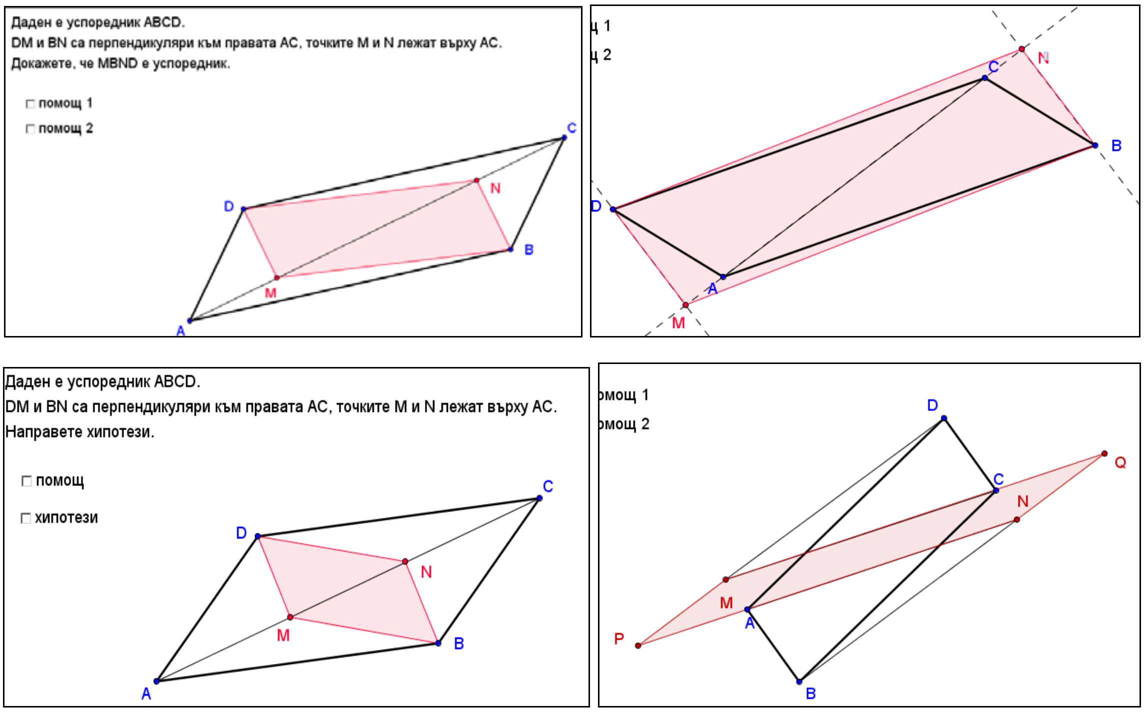
Prove that if a line divides a parallelogram in two parts of equal area it passes through the intersection point of its diagonals.





One of the specifics behind the explorations of dynamic construction is the fact that they extend the set of figures under consideration (which is usually not restricted in the problems but is treated in specific cases). For instance, points on the sides of a figure rather than on the lines containing the segments are considered. Or, the problems about altitudes would take into account only the case of internal rather than external altitudes (because of the difference in the proofs).

Such examples are shown on the figures below:

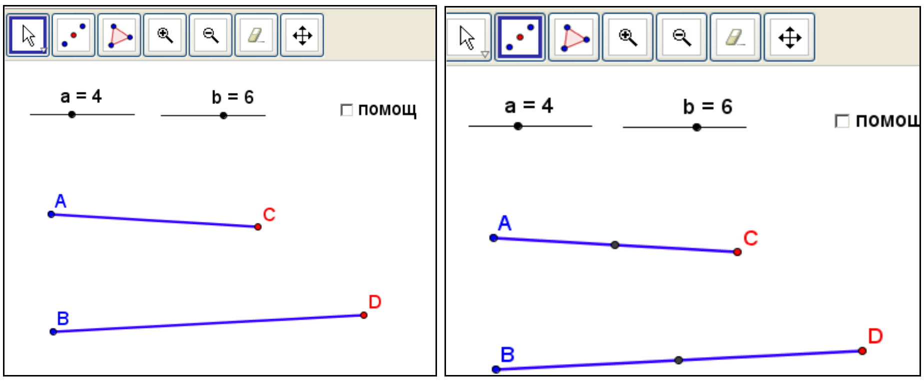


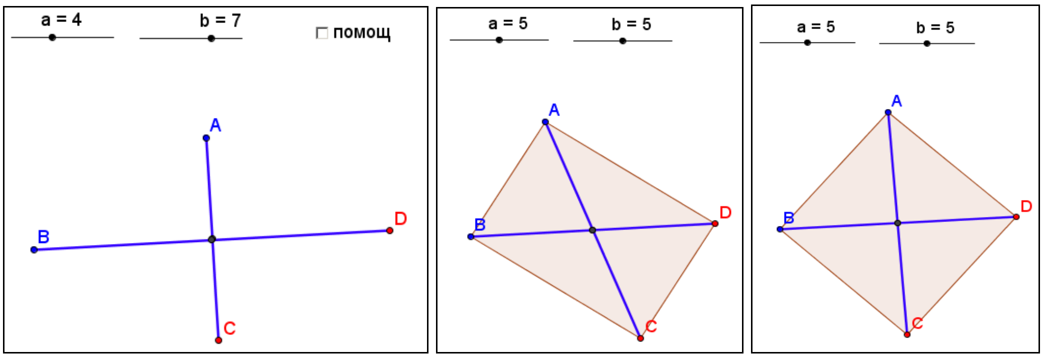
The dynamic software enables the formulation of problems whose solving needs the students to apply properties they have already discovered.

Consider for example the sufficient condition used in the following formulation: Place the segments AC and BD so that they are diagonals of a parallelogram.

Here only part of the tools has been left and the students have to figure out that they have to construct the midpoints of the given segments and to make them coincide. Thus the students have to manipulate with the diagonals until getting the desired figure.

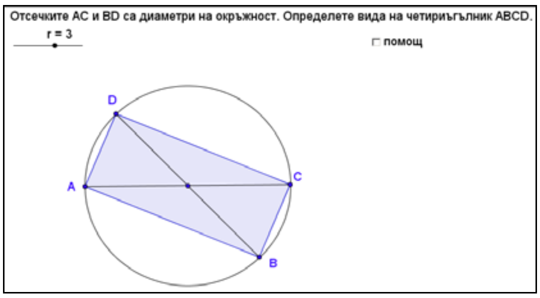
A similar problem is given in a following lesson related to the topic Types of parallelograms. For its solving the students will be expected to use specific conditions for specific types of parallelograms [15].

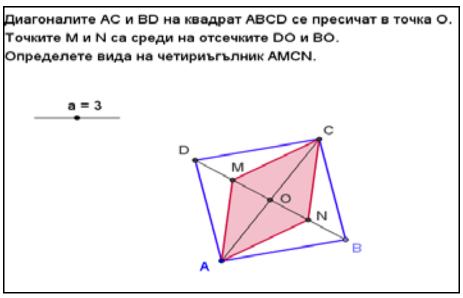


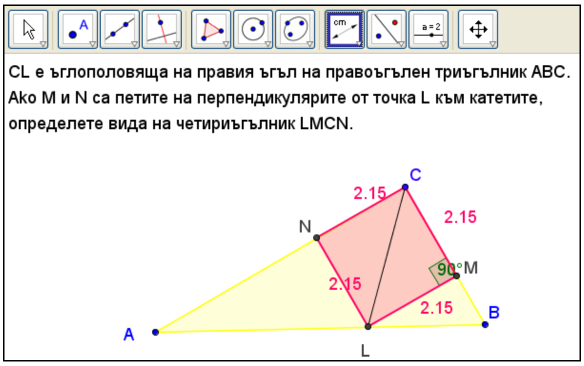


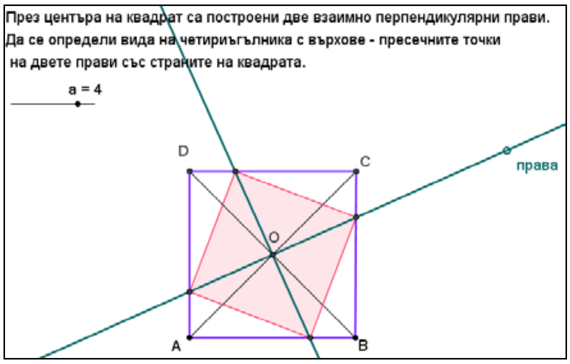
The next group of problems from the module Types of parallelograms deals with determining the type of a quadrilateral in typical situations. Thanks to the dynamics of the constructions the students could observe and explore a whole class of figures possessing specific properties. Here, unlike the style in the traditional textbooks and the collections of problems, the type of the quadrilateral is not specified in advance. Based on their explorations the students first formulate conjectures which afterwards they are motivated to prove. We present in the table below the classical formulations of some problems together with new formulations in exploratory style.

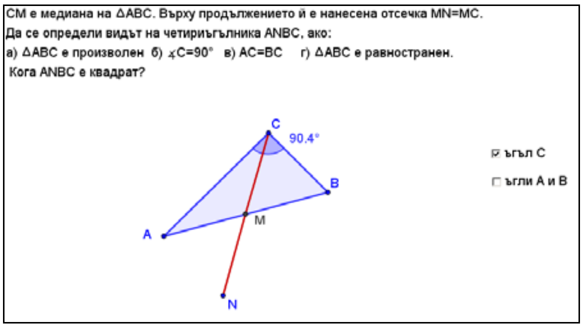
The classical formulation	**An exploration-enhancing formulation**
• The segments AC and BD are diameters of a circle.	
Prove that $ABCD$ is a rectangle.	What is the type of the quadrilateral $ABCD$?
• The diagonals AC and BD of a square meet at a point O. The points M and N are midpoints of the segments DO and BO.	
Prove that $AMCN$ is a square.	What is the type of the quadrilateral $AMCN$?.
• Let CL be the angular bisector of the right angle of the triangle ABC, and M and N be the feet of the perpendiculars from the point L to the legs.	
Prove that $LMCN$ is a square.	What is the type of the quadrilateral $LMCN$.
• Two perpendicular lines are passing through the centre of a square.	
Prove that the quadrilateral with vertices the intersection points of the two lines with the sides of the square is also a square..	Determine the type of the quadrilateral with vertices the intersection points of the two lines with the sides of the square
• CM is a median of ΔABC. The segment $MN = MC$ is on the median's extension.	
Prove that $ANCB$ is a parallelogram.	What is the type of the quadrilateral $ANCB$ if: a) ΔABC is arbitrary; b) $\angle C = 90°$; c) $AC = BC$; d) ΔABC equilateral? When is $ANCB$ a square?

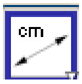
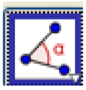










In some cases the students could measure segments and angles by means of special buttons (the buttons  and  in the case of GeoGebra) so as to formulate their conjectures

Other examples could be found in [16-18].

4. Conclusions

In conclusion, the development of resources making use of dynamic constructions is just an element of the dynamic mathematics education. The discoveries, the representations and the implementation of mathematical objects and ideas could be related to the enhancement of the creative potential of learners by providing appropriate conditions and our on-going efforts are in this direction.

Our long-term experience with implementing the "learning by discovering" style in the context of exploratory computer environments has proven that the students (12-15-year of age) readily adopt it – this style responds to their natural wish to learn rather than to be taught.

Teachers, on the other hand, have problems mainly with changing the traditional style of preaching facts. When educating teachers in the frames of the InnoMathEd and the Fibonacci projects we saw that they acquire sufficiently fast the technical skills needed for working with it. They enjoy the richness of resources including dynamic scenarios and express their readiness to implement them in class setting, proposing sometimes their own modifications or even own scenarios [19]. However, a problem we often face when the teachers present their projects at the end of the course is that they do not take advantage of the potential of the dynamic software for explorations and inquiry based learning but rather use it for illustrations and visualizations still in the traditional style of “you see that…”

Changing the style of teaching and seeing the role of the teacher as one of a facilitator and a partner in a research process requires ongoing efforts on behalf of the teacher educators. These efforts include preparing a good ground for exploration activities including re-formulation of some classical problems so as to stimulate acts. Furthermore, we should not stop there – “you do, you understand” says the old Chinese proverb. That is true but why not extend it to “you explore, you invent”… It is often the case that the teachers react with: O-o-h, the inspectors would not be happy with this ‘waste of time” – we have to cover the curriculum, the students have to cover the tests, etc. And they are right if we accept that education is about knowing the right answers…

We are optimists in our belief that the assessment and evaluation mechanisms will reach the level of recognizing the achievements of learners who are able to approach learning as a task of discovering rather than “learning about”, the reward being the discovery itself [20]. Till then we, in our role of teachers’ educators, have to do our best to become that type of learners ourselves.

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