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RADIATION OF SPATIALLY LIMITED INHOMOGENEOUS PLASMA

The efficiency of the transformation of the surface wave energy of a longitudinally inhomogeneous cylindrical plasma column into radiation is investigated. Plasma is limited by a dielectric shell. The analysis was carried out by the method of spectral decomposition of the field by a complete set of functions, including surface and spatial waves of the plasma column. A system of integro-differential equations for coefficients of decomposition is derived. These coefficients determine the amplitudes of the transmitted, reflected and scattered waves, as well as radiation patterns. The system of equations is solved for an arbitrary longitudinal change in the density of the plasma. The dependences of the transformation coefficients of the surface wave energy on the plasma density gradient, the electric length of the plasma inhomogeneity section, the electric radius of the plasma cylinder, the dielectric constant and the dielectric thickness are calculated. Examples are given where the part of the energy of a surface wave, which is transformed into a radiation at sharp angles, is 35%. The padiation patterns are pointed and have one beam. The maximum of radiation is formed at an angle, that equal several degrees relative to the direction of propagation of the surface wave. The width of the beam decreases, and its position shifts to zero, when the density gradient of the plasma increases.The influence of dielectric properties on the radiation characteristics is investigated.

Keywords: *Cylindrical plasma antenna, spectral decomposition method, radiation, surface waves.*

Introduction

The cylindrical column of low-temperature plasma can be used as a transmitting antenna, as shown in [1–2]. The surface wave is arisen at the end of a dielectric waveguide filled with a plasma,as shown in the experiments, described in these papers.

At the same time, part of the energy of the wave is spent on the creation of a plasma whose density decreases as it is removed from the butt-end. It is precisely because of the longitudinal heterogeneity of the plasma density that the plasma antenna is radiated. Such inhomogeneities in plasma-dielectric waveguides are always present in the real conditions of the experiment. Therefore, it is necessary to study the dependence of the coefficients of the transformation of the energy of a surface wave on radiation from the degree of inhomogeneity of the plasma density, for the proper understanding of the radiation process in cylindrical plasma antennas and their construction.

The analysis of this inhomogeneity is fundamentally important, since it depends on the efficiency of the operational performance of any plasma waveguide antenna, including the cylindrical one. The transformation of an axially symmetric surface wave in a plasma antenna is also investigated. Such antenna is a cylindrical column of isotropic cold plasma, bounded by a dielectric. The density of the plasma varies in the longitudinal direction. In addition, the rate of change in the density of the plasma in the longitudinal direction can be arbitrary. This circumstance has required the use of numerical methods.

A method for solving such problems was developed by V.V. Shevchenko is described in the monograph [3]. It is based on Sturm-Liouville theory [4]. According to this method, the complete field is expanded by a complete set of functions of a plasma cylinder, which includes surface and spatial waves of open systems.

The coefficients of a decomposition via inhomogeneities, depend on the longitudinal coordination. They can be presented as the system of integro-differential equations. The influence of the dielectric on the radiation characteristics of antennas is investigated. The results of the investigation of a planar longitudinally inhomogeneous plasma antenna are presented at the article [5].

1. Basic equations

Fig. 1 shows an unrestricted, plasma-filled dielectric cylinder along the axis z dielectric cylinder filled with plasma. In this figure, a – is the radius of the plasma column, b –is the radius of a homogeneous dielectric cylinder, z and ρ – are the cylindrical coordinates.

When performing $0 \le z \le L$ the density of the plasma is inhomogeneous. The permittivity of dielectric is equal to ε_d, and the permittivity of plasma is equal to $\varepsilon_p = 1 - \omega_p^2 / \omega^2$.

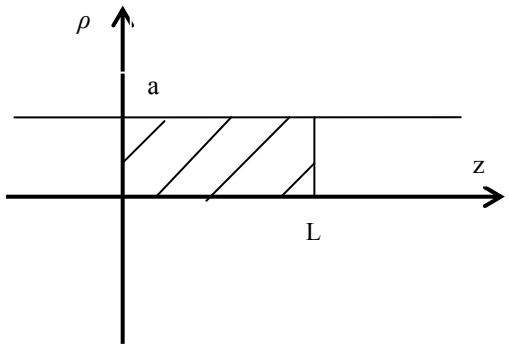


Fig. 1. Plasma column of radius a, enclosed in a dielectric cylinder with outer radius of a shell b. The region is shaded, where the plasma density $n_e(z)$ decreases, and its dielectric permittivity $\varepsilon_p(z)$ increases, as shown in fig. 3

In this formula ω – is the angular frequency of the wave, $\omega_p^2 = 4\pi n_e e^2 / m_e$ – is the plasma frequency, n_e – the electron density, e, m_e – the charge, and the mass of the electron, respectively. At the beginning, we consider a plasma cylinder with a homogeneous density n_e. The time dependence of the field components is determined by the factor $\exp(-i\omega t)$. In such a dielectric-limited cylindrical structure, an axially symmetric slow surface E wave can exist, with components $E_{z0}(\rho,z), E_{\rho 0}(\rho,z), H_{\varphi 0}(\rho,z)$. The components of the electric field of this wave are expressed through the azimuthal component of the magnetic field $H_{\varphi 0}(\rho,z)$, where φ – is the azimuthal coordinate. This component of the field depends on the coordinates as follows:

$$H_{\varphi 0}(\rho,z) = \Psi_0(\rho)\exp\left(\pm i k_z^0 z\right), \qquad (1)$$

where k_z^0 – is a wave vector of a surface wave, $\Psi_0(y)$ – is, so-called, surface wave cross section function. The signs + and - in the formula 1, correspond to the wave propagation along the axis z, and in the opposite direction. The dependence of the components $E_{z0}(\rho,z), E_{\rho 0}(\rho,z)$ on the coordinate z is the same as in (1).

The function $\Psi_0(y)$, from (1), must satisfy the boundary conditions on the boundary of the plasma columnar column with a dielectric, and a dielectric with a vacuum.

In addition, it must decrease at infinity. Firstly, we consider the case, when $k < k_z^0 < k\sqrt{\varepsilon_d}$. From the Maxwell's equations in cylindrical coordinates and boundary conditions we obtain formulas for $\Psi_0(y)$:

$$\Psi_0(\rho) = \frac{1}{N_0}\begin{cases} K_1\left(\kappa_0^0\rho\right), & b \le \rho < \infty, \\ \Delta_{1b}^0 J_1(\kappa_d^0\rho) + \Delta_{2b}^0 N_1(\kappa_d^0\rho), & a \le \rho \le b, \\ \dfrac{\Delta_{1b}^0}{\Delta_{1a}^0} I_1\left(\kappa^0\rho\right), & 0 \le \rho \le a, \end{cases} \qquad (2)$$

where $(\kappa_0^0)^2 = (k_z^0)^2 - k^2 > 0$, $k^2 = \omega^2 / c^2$, $(\kappa^0)^2 = \omega_p^2 / c^2 + (\kappa_0^0)^2 > 0$, $(\kappa_d^0)^2 = k^2\varepsilon_d - (k_z^0)^2$, $K_1(x)$, $I_1(x)$ – are MacDonald function and Bessel function of the imaginary argument, respectively; N_0 – is the normalization factor, which is found from the normalization condition:

$$\int_0^\infty d\rho \frac{1}{\varepsilon_p(\rho)}\left(\Psi_0(\rho)\right)^2 = 1. \qquad (3)$$

The coefficients, in (2), are determined by the following relations:

$$\Delta_{1a}^0 = -\frac{\pi\kappa_d^0 a}{2}\cdot \left(\frac{\kappa^0\varepsilon_d}{\kappa_d^0\varepsilon_p} I_0(\kappa^0 a)N_1(\kappa_d^0 a) - I_1(\kappa^0 a)N_0(\kappa_d^0 a)\right),$$

$$\Delta_{1b}^0 = \frac{\pi\kappa_d^0 b}{2}\cdot \left(\frac{\kappa^0\varepsilon_d}{\kappa_d^0\varepsilon_p} K_0(\kappa^0 a)N_1(\kappa_d^0 a) + K_1(\kappa^0 a)N_0(\kappa_d^0 a)\right),$$

$$\Delta_{2b}^0 = -\frac{\pi\kappa_d^0 b}{2}\cdot \left(\frac{\kappa_0^0\varepsilon_d}{\kappa_d^0} K_0(\kappa_0^0 b)J_1(\kappa_d^0 b) + K_1(\kappa_0^0 b)J_0(\kappa_d^0 b)\right).$$

Applying the boundary conditions to the function (2), we obtain the following dispersion equation for the surface E wave:

$$\frac{\kappa^0\varepsilon_d I_0\left(\kappa^0 a\right)N_1\left(\kappa_d^0 a\right) - \kappa_d^0\varepsilon_p I_1\left(\kappa^0 a\right)N_0\left(\kappa_d^0 a\right)}{\kappa^0\varepsilon_d I_0\left(\kappa_0^0 a\right)J_1\left(\kappa_d^0 a\right) - \kappa_d^0\varepsilon_p I_1\left(\kappa^0 a\right)J_1\left(\kappa_d^0 a\right)} = \frac{\kappa_0^0\varepsilon_d K_0\left(\kappa_0^0 b\right)N_1\left(\kappa_d^0 b\right) + \kappa_d^0 K_1\left(\kappa_0^0 b\right)N_0\left(\kappa_d^0 b\right)}{\kappa_0^0\varepsilon_d K_0\left(\kappa_0^0 b\right)J_1\left(\kappa_d^0 b\right) + \kappa_d^0 K_1\left(\kappa_0^0 b\right)J_0\left(\kappa_d^0 b\right)}, \qquad (4)$$

where $J_{0,1}(x)$, $N_{.0,1}(x)$ – are Bessel and Neumann functions.

In general, equation (4) has several solutions with respect to k_z^0, when quantities $\varepsilon_p, \varepsilon_d, a, b$ are given.

Each of these solutions corresponds to a surface wave. Under the condition of $k\sqrt{\varepsilon_d} < k_z^0$, the surface waves does not exist. Formally, this follows from the

fact that the dispersion equation for this case has no solution

In addition to the surface wave (1), there are also axially symmetric solutions of the Maxwell's equations corresponding to the spatial waves. The spatial waves are fast.

They exist under the condition $\kappa_0^2 = k^2 - k_z^2 > 0$, where k_z – is the wave number, κ_0 is the transverse wave number. Spatial E-waves have components $E_z(\rho,z), E_\rho(\rho,z), H_\varphi(\rho,z)$. The azimuthal component of the magnetic field of the wave can be represented in the form:

$$H_\varphi(\rho,z,\kappa_o) = \Psi(\rho,\kappa_0)\exp(\pm ik_z z). \quad (5)$$

In formula (5), the dependence on the transverse wave number κ_0 is singled out. The cross-sectional function $\Psi(\rho,\kappa_0)$ of the spatial E-wave is determined by the expressions:

$$\Psi(\rho,\kappa_0) = \frac{1}{N(\kappa_0)}\cdot$$
$$\cdot\begin{cases} u_5 J_1(\kappa_0\rho) + u_6 N_1(\kappa_0\rho), & b \le \rho < \infty, \\ u_1 J_1(\kappa_d\rho) + u_2 N_1(\kappa_d\rho), & a \le \rho \le b, \\ I_1(\kappa\rho), & 0 \le \rho \le a, \end{cases} \quad (6)$$

where

$$u_1 = -\frac{\pi\kappa_d a}{2}\left\{\frac{\kappa\varepsilon_d}{\kappa_d\varepsilon_p} I_0(\kappa a) N_1(\kappa_d a) - I_1(\kappa a) N_0(\kappa_d a)\right\},$$

$$u_2 = \frac{\pi\kappa_d a}{2}\left\{\frac{\kappa\varepsilon_d}{\kappa_d\varepsilon_p} I_0(\kappa a) J_1(\kappa_d a) - I_1(\kappa a) J_0(\kappa_d a)\right\},$$

$$u_3 = u_1 J_0(\kappa_d b) + u_2 N_0(\kappa_d b),$$

$$u_4 = u_1 J_1(\kappa_d b) + u_2 N_1(\kappa_d b),$$

$$u_5 = -\frac{\pi\kappa_0 b}{2}\left\{\frac{\kappa_d}{\kappa_0\varepsilon_d} u_3 N_1(\kappa_0 b) - u_4 N_0(\kappa_0 b)\right\},$$

$$u_6 = -\frac{\pi\kappa_0 b}{2}\left\{\frac{\kappa_d}{\kappa_0\varepsilon_d} u_3 N_1(\kappa_0 b) - u_4 N_0(\kappa_0 b)\right\},$$

$$\kappa^2 = \omega_p^2 / c^2 - \kappa_0^2, \qquad \kappa_d^2 = k^2\varepsilon_d - k_z^2,$$

$$(\kappa_d^0)^2 = k^2\varepsilon_d - (k_z^0)^2.$$

The normalization factor $N(\kappa_0)$ is not given for brevity. The function (6) satisfies the above mention boundary conditions. It slowly decreases with $\rho \to \infty$, and therefore is normalized to δ function:

$$\int_0^\infty d\rho \frac{1}{\varepsilon_p(\rho)} \Psi(\rho,\kappa_0)\Psi(\rho,\tilde{\kappa}_0) = \delta(\kappa_0 - \tilde{\kappa}_0). \quad (7)$$

Cross-sectional functions $\Psi_0(y)$ and $\Psi(\rho,\kappa_0)$ are orthogonal to each other:

$$\int_0^\infty d\rho \frac{1}{\varepsilon_p(\rho)} \Psi_0(\rho)\Psi(\rho,\kappa_0) = 0. \quad (8)$$

We represent the component of the total field $H_\varphi(\rho,z)$ in a homogeneous plasma layer and a dielectric in the form of a spectral expansion with respect to a complete set of functions $H_{\varphi 0}(\rho,z)$ (1) и $H_\varphi(\rho,z,\kappa_0)$:

$$H_\varphi(\rho,z) = \left(B_+^0 \exp(ik_z^0 z) + B_-^0 \exp(-ik_z^0 z)\right)\Psi_0(\rho) + \\ + \int_0^\infty d\kappa_0 \begin{pmatrix} B_+(\kappa_0)\exp(ik_z z) + \\ + B_-(\kappa_0)\exp(-ik_z z) \end{pmatrix} \Psi(\rho,\kappa_0), \quad (9)$$

where the first term corresponds to the surface waves, and the second term corresponds to the superposition of the spatial waves.

Now let the plasma density in the section $0 \le z \le L$, as shown in fig. 1, the plasma density n_e is nonuniform in the longitudinal direction, that is, depending on the coordinate z. In accordance with the method of spectral decomposition, the coefficients $B_\pm^0, B_\pm$ in the expansion (9) become functions of z. The constant of propagation $k_z^0(z)$ also depends on z. It is a solution of a dispersion equation containing an inhomogeneous density $n_e(z)$, and functions of a cross section $\Psi_0(\rho,z)$ and $\Psi(\rho,z,\kappa_0)$.

In this case, the dielectric is assumed to be homogeneous. For the following it is convenient to make the following substitution:

$$B_\pm^0(z) = \frac{\exp\left(\pm i(\gamma_0(z) - k_z^0 z)\right)}{\sqrt{k_z^0}} D_\pm^0(z),$$

$$B_\pm(\kappa_0,z) = \frac{1}{k_z} D_\pm(\kappa_0,z), \ \gamma_0(z) = \int_0^z dz k_z^0(z), \quad (10)$$

where $D_\pm^0(z), D_\pm(\kappa_0,z)$ – are the new amplitudes of surface and spatial waves. Substituting (9) with amplitudes (10) into the Maxwell's equations and using the conditions of normalization and orthogonality (3; 7–8), we obtain a system of four integro-differential equations for the amplitudes $D_\pm^0(z), D_\pm(\kappa_0,z)$. These equations can be transformed into the following system of integral equations:

$$D_+^0(z) = 1 + \int_0^z dz\left\{A_1 D_+^0(z) + A_2 D_-^0(z)\right\} + \\ + \int_0^z dz \int_0^\infty d\tilde{\kappa}_0 \left\{A_3 D_+(\tilde{\kappa}_0,z) + A_4 D_-(\tilde{\kappa}_0,z)\right\}, \quad (11)$$

$$D_-^0(z)=\int_L^z dz\left\{(A_2)^* D_+^0(z)+A_1 D_-^0(z)\right\}+$$
$$+\int_L^z dz\int_0^\infty d\tilde{\kappa}_0\left\{(A_4)^* D_+(\tilde{\kappa}_0,z)+(A_3)^* D_-(\tilde{\kappa}_0,z)\right\}, \quad (12)$$

$$D_+(\kappa_0,z)=\int_0^z dz\left\{A_5 D_+^0(z)+A_6 D_-^0(z)\right\}+$$
$$+\int_0^z dz\int_0^\infty d\tilde{\kappa}_0\left\{A_7 D_+(\tilde{\kappa}_0,z)+A_8 D_-(\tilde{\kappa}_0,z)\right\}, \quad (13)$$

$$D_-(\kappa_0,z)=\int_L^z dz\left\{(A_6)^* D_+^0(z)+(A_5)^* D_-^0(z)\right\}+$$
$$+\int_L^z dz\int_0^\infty d\tilde{\kappa}_0\left\{(A_8)^* D_+(\tilde{\kappa}_0,z)+(A_7)^* D_-(\tilde{\kappa}_0,z)\right\}. \quad (14)$$

Equations (11–14) satisfy the boundary conditions:

$$\left.\begin{aligned} &D_+^0(z=0)=1, \quad D_-^0(z=L)=0,\\ &D_+(z=0)=0, \quad D_-(z=L)=0.\end{aligned}\right\} \quad (15)$$

The relations (15) correspond to a surface wave that runs from -∞ to the nonuniform plasma region $[0,L]$.

The coefficients A_i, in equations (11–14) have the meaning of complex coupling coefficients among the transmitted surface waves and the reflected waves and also other types of radiation waves.

This relationship arises from the longitudinal inhomogeneity of the plasma density. The dependence A_i on the arguments is indicated below in the formulas (19–25).

It is convenient to expand the complex amplitudes $D_\pm^0(z), D_\pm(\kappa_0,z)$ in the following manner into real and imaginary parts:

$$D_+^0(z)=f_1(z)+ig_1(z),\ D_-^0(z)=f_2(z)+ig_2(z), \quad (16)$$

$$D_+(\kappa_0,z)=f_3(\kappa_0,z)+ig_3(\kappa_0 z),$$
$$D_-(\kappa_0,z)=f_4(\kappa_0,z)+ig_4(\kappa_0 z). \quad (17)$$

The integral in the expansion (10) describes the radiation. To find its asymptotic behavior by the saddle point method, where $kr\to\infty$, and r - the distance from the segment of the inhomogeneity of the plasma layer to the observation point, let us proceed, following the works [3.5], to the complex variable of integration ϑ by the formulas $\kappa_0=k\sin\vartheta$, $k_z=k\cos\vartheta$. In this case, the singularity in the integrals (10; 11–14) is removed, under conditions $\kappa_0=k$. The integration contour C with respect to a complex variable ϑ consists of two parts C_1 and C_2 as shown in fig. 2 [3; 5]. Note, that the point $\kappa_0=k$ corresponds to the point $\mathrm{Im}(\vartheta)=0,\quad \mathrm{Re}(\vartheta)=\pi/2$ in fig 2. There are two saddle points on the contour. This $\vartheta=\theta$ and $\vartheta=\pi-\theta$, where θ – is the angle of inclination of the radius of the observation point vector to the axis z in the plane (ρ,z). They are located on the contour C_1, which corresponds to the following interval of variable change κ_0: $0\le\kappa_0\le k$. The saddle point $\vartheta=\theta$ determines the radiation at acute angles θ, and the point $\vartheta=\pi-\theta$, at obtuse angles θ. Since both points are located on the contour C_1, we confine ourselves only to this segment of the contour C when solving the system of integral equations (11–14).

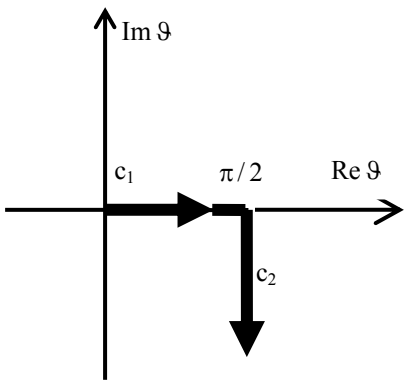


Fig. 2. The contour of integration $C=C_1+C_2$ with respect to a complex variable ϑ

In other words, we assume in (11–14) that

$$\int_0^\infty d\tilde{\kappa}_0\{\ \}\approx\int_0^k d\tilde{\kappa}_0\{\ \}=k\int_0^{\pi/2} d\tilde{\vartheta}\cos\tilde{\vartheta}\{\ \}. \quad (18)$$

The assumption (18), allows us to decompose the coefficients A_i into real and imaginary parts:

$$A_1(z)=a_1(z),\quad A_2(z)=a_2(z)-ib_2(z), \quad (19)$$

$$A_3(\tilde{\kappa}_0,z)=a_3(\tilde{\kappa}_0,z)-ib_3(\tilde{\kappa}_0,z),$$
$$A_4(\tilde{\kappa}_0,z)=a_4(\tilde{\kappa}_0,z)-ib_4(\tilde{\kappa}_0,z), \quad (20)$$
$$A_5(\tilde{\kappa}_0,z)=a_5(\tilde{\kappa}_0,z)+ib_5(\tilde{\kappa}_0,z),$$
$$A_6(\tilde{\kappa}_0,z)=a_6(\tilde{\kappa}_0,z)-ib_6(\tilde{\kappa}_0,z), \quad (21)$$
$$A_7(\tilde{\kappa}_0,\kappa_0,z)=a_7(\tilde{\kappa}_0,\kappa_0,z)+ib_7(\tilde{\kappa}_0,\kappa_0,z),$$
$$A_8(\tilde{\kappa}_0,\kappa_0,z)=a_8(\tilde{\kappa}_0,\kappa_0,z)-ib_8(\tilde{\kappa}_0,\kappa_0,z), \quad (22)$$

where $a_1(z)=-J_0(z)$,

$$a_2(z)=S_0(z)\cos(2\gamma_0(z)),$$
$$b_2(z)=S_0(z)\sin(2\gamma_0(z)),$$
$$S_0(z)=\frac{\left(k_z^0\right)_z'}{2k_z^0},$$
$$a_3(\tilde{\kappa}_0,z)=-\frac{k_z^0+\tilde{k}_z}{2\sqrt{k_z^0\tilde{k}_z}}J_1(\tilde{\kappa}_0,z)\cos\left(\gamma_0-\tilde{k}_z z\right),$$

$$b_3(\tilde{\kappa}_0,z)=-\frac{k_z^0+\tilde{k}_z}{2\sqrt{k_z^0\tilde{k}_z}}J_1\left(\tilde{\kappa}_0,z\right)\sin\left(\gamma_0-\tilde{k}_z z\right),$$

$$a_4(\tilde{\kappa}_0,z)=-\frac{k_z^0-\tilde{k}_z}{2\sqrt{k_z^0\tilde{k}_z}}J_1\left(\tilde{\kappa}_0,z\right)\cos\left(\gamma_0+\tilde{k}_z z\right),$$

$$b_4(\tilde{\kappa}_0,z)=-\frac{k_z^0-\tilde{k}_z}{2\sqrt{k_z^0\tilde{k}_z}}J_1\left(\tilde{\kappa}_0,z\right)\sin\left(\gamma_0+\tilde{k}_z z\right),$$

$$a_5(\kappa_0 z)=-\frac{k_z+k_z^0}{2\sqrt{k_z^0}}J_2\left(\kappa_0,z\right)\cos\left(\gamma_0-k_z z\right),$$

$$b_5(\kappa_0 z)=-\frac{k_z+k_z^0}{2\sqrt{k_z^0}}J_2\left(\kappa_0,z\right)\sin\left(\gamma_0-k_z z\right),$$

$$a_6(\kappa_0,z)=-\frac{k_z-k_z^0}{2\sqrt{k_z^0}}J_2\left(\kappa_0,z\right)\cos\left(\gamma_0+k_z z\right),$$

$$b_6(\kappa_0,z)=-\frac{k_z-k_z^0}{2\sqrt{k_z^0}}J_2\left(\kappa_0,z\right)\sin\left(\gamma_0+k_z z\right),$$

$$a_7(\tilde{\kappa}_0,\kappa_0,z)=-\frac{k_z+\tilde{k}_z}{2\tilde{k}_z}J_3\left(\tilde{\kappa}_0,\kappa_0,z\right)\cos\left((\tilde{k}_z-k_z)z\right),$$

$$b_7(\tilde{\kappa}_0,\kappa_0,z)=-\frac{k_z+\tilde{k}_z}{2\tilde{k}_z}J_3\left(\tilde{\kappa}_0,\kappa_0,z\right)\sin\left((\tilde{k}_z-k_z)z\right),$$

$$a_8(\tilde{\kappa}_0,\kappa_0,z)=-\frac{k_z-\tilde{k}_z}{2\tilde{k}_z}J_3\left(\tilde{\kappa}_0,\kappa_0,z\right)\cos\left((\tilde{k}_z+k_z)z\right),$$

$$b_8(\tilde{\kappa}_0,\kappa_0,z)=-\frac{k_z-\tilde{k}_z}{2\tilde{k}_z}J_3\left(\tilde{\kappa}_0,\kappa_0,z\right)\sin\left((\tilde{k}_z+k_z)z\right).$$

The functions $J_0(z)$, $J_1(\kappa_0,z)$, $J_2(\kappa_0,z)$, $J_3(\tilde{\kappa}_0,\kappa_0,z)$ are defined by integrals

$$J_0(z)=\int_0^\infty d\rho\frac{1}{\varepsilon_p(\rho)}\Psi_0(\rho,z)\left(\Psi_0(\rho,z)\right)_z',$$

$$J_1(\kappa_0,z)=\int_0^\infty d\rho\frac{1}{\varepsilon_p(\rho)}\Psi_0(\rho,z)\left(\Psi(\rho,\kappa_0,z)\right)_z', \quad (23)$$

$$J_2(\kappa_0,z)=\int_0^\infty d\rho\frac{1}{\varepsilon_p(\rho)}\Psi(\rho,\kappa_0,z)\left(\Psi_0(\rho,z)\right)_z',$$

$$J_3(\tilde{\kappa}_0,\kappa_0,z)=\int_0^\infty d\rho\frac{1}{\varepsilon_p(\rho)}\Psi(\rho,\kappa_0,z)\left(\Psi(\rho,\tilde{\kappa}_0,z)\right)_z'. \quad (24)$$

The dependence of these integrals on the z coordinate arises from the dependence of the root of the dispersion equation k_z^0 and the plasma frequency ω_p, which enter into the function (2; 6). In the final form, the formulas for these integrals are cumbersome and are not given for brevity. Note, however, that the integrals (23–24) should be calculated directly from the Bessel equation. Finally, the integral equations for the real functions f_i, g_i $(i=1,2,3,4)$ in (16–17), taking into account the expansion of the coefficients A_i into real and imaginary parts, take the form:

$$\begin{aligned}f_1(z)&=1+\int_0^z d\tilde{z}\left\{\alpha_1(\tilde{z})f_1(\tilde{z})+\alpha_2(\tilde{z})f_2(\tilde{z})+\beta_2(\tilde{z})g_2(\tilde{z})\right\}+\\&\int_0^z d\tilde{z}\int_0^{\pi/2}d\tilde{\vartheta}\{\alpha_3\left(\tilde{\vartheta},\tilde{z}\right)f_3\left(\tilde{\vartheta},\tilde{z}\right)+\beta_3\left(\tilde{\vartheta},\tilde{z}\right)g_3\left(\tilde{\vartheta},\tilde{z}\right)+\alpha_4\left(\tilde{\vartheta},\tilde{z}\right)f_4\left(\tilde{\vartheta},\tilde{z}\right)+\\&+\beta_4\left(\tilde{\vartheta},\tilde{z}\right)g_4\left(\tilde{\vartheta},\tilde{z}\right)\},\end{aligned} \quad (25)$$

$$\begin{aligned}g_1(z)&=\int_0^z d\tilde{z}\left\{\alpha_1(\tilde{z})g_1(\tilde{z})+\alpha_2(\tilde{z})g_2(\tilde{z})-\beta_2(\tilde{z})f_2(\tilde{z})\right\}+\\&\int_0^z d\tilde{z}\int_0^{\pi/2}d\tilde{\vartheta}\{\alpha_3\left(\tilde{\vartheta},\tilde{z}\right)g_3\left(\tilde{\vartheta},\tilde{z}\right)-\beta_3\left(\tilde{\vartheta},\tilde{z}\right)f_3\left(\tilde{\vartheta},\tilde{z}\right)+\\&+\alpha_4\left(\tilde{\vartheta},\tilde{z}\right)g_4\left(\tilde{\vartheta},\tilde{z}\right)-\beta_4\left(\tilde{\vartheta},\tilde{z}\right)f_4\left(\tilde{\vartheta},\tilde{z}\right)\},\end{aligned} \quad (26)$$

$$\begin{aligned}g_2(z)&=\int_1^z d\tilde{z}\left\{\alpha_2(\tilde{z})g_1(\tilde{z})+\beta_2(\tilde{z})f_1(\tilde{z})+\alpha_1(\tilde{z})g_2(\tilde{z})\right\}+\\&\int_1^z d\tilde{z}\int_0^{\pi/2}d\tilde{\vartheta}\{\alpha_4\left(\tilde{\vartheta},\tilde{z}\right)g_3\left(\tilde{\vartheta},\tilde{z}\right)+\beta_4\left(\tilde{\vartheta},\tilde{z}\right)f_3\left(\tilde{\vartheta},\tilde{z}\right)+\\&+\alpha_3\left(\tilde{\vartheta},\tilde{z}\right)g_4\left(\tilde{\vartheta},\tilde{z}\right)+\beta_3\left(\tilde{\vartheta},\tilde{z}\right)f_4\left(\tilde{\vartheta},\tilde{z}\right),\end{aligned} \quad (28)$$

$$\begin{aligned}f_2(z)&=\int_1^z d\tilde{z}\left\{\alpha_2(\tilde{z})f_1(\tilde{z})-\beta_2(\tilde{z})g_1(\tilde{z})+\alpha_1(\tilde{z})f_2(\tilde{z})\right\}+\\&\int_1^z d\tilde{z}\int_0^{\pi/2}d\tilde{\vartheta}\{\alpha_4\left(\tilde{\vartheta},\tilde{z}\right)f_3\left(\tilde{\vartheta},\tilde{z}\right)-\beta_4\left(\tilde{\vartheta},\tilde{z}\right)g_3\left(\tilde{\vartheta},\tilde{z}\right)+\\&+\alpha_3\left(\tilde{\vartheta},\tilde{z}\right)f_4\left(\tilde{\vartheta},\tilde{z}\right)-\beta_3\left(\tilde{\vartheta},\tilde{z}\right)g_4\left(\tilde{\vartheta},\tilde{z}\right)\},\end{aligned} \quad (27)$$

$$\begin{aligned}f_3(\vartheta,z)&=\int_0^z d\tilde{z}\left\{\alpha_5(\vartheta,\tilde{z})f_1(\tilde{z})-\beta_5(\vartheta,\tilde{z})g_1(\tilde{z})+\right.\\&\left.+\alpha_6(\vartheta,\tilde{z})f_2(\tilde{z})+\beta_6(\vartheta,\tilde{z})g_2(\tilde{z})\right\}+\\&\int_0^z d\tilde{z}\int_0^{\pi 2}dz\{\alpha_7\left(\tilde{\vartheta},\vartheta,z\right)f_3\left(\tilde{\vartheta},z\right)-\beta_7\left(\tilde{\vartheta},\vartheta,z\right)g_3\left(\tilde{\vartheta},z\right)+\\&\alpha_8(\tilde{\vartheta},\vartheta,\tilde{z})f_4(\tilde{\vartheta},\tilde{z})+\beta_8(\tilde{\vartheta},\vartheta,\tilde{z})g_4(\tilde{\vartheta},\tilde{z})\},\end{aligned} \quad (29)$$

$$g_3(\vartheta,z)=\int_0^z d\tilde{z}\{\beta_5(\vartheta,\tilde{z})f_1(\tilde{z})+$$
$$+\alpha_5(\vartheta,\tilde{z})g_1(\tilde{z})+\alpha_6(\vartheta,\tilde{z})g_2(\tilde{z})-\beta_6(\vartheta,\tilde{z})f_2(\tilde{z})\ \}+$$
$$\int_0^z d\tilde{z}\int_0^{\pi 2} d\tilde{\vartheta}\left\{\alpha_7\left(\tilde{\vartheta},\vartheta,\tilde{z}\right)g_3\left(\tilde{\vartheta},\tilde{z}\right)+\beta_7\left(\tilde{\vartheta},\vartheta,\tilde{z}\right)f_3\left(\tilde{\vartheta},\tilde{z}\right)+\right. \quad (30)$$
$$\left.+\alpha_8(\tilde{\vartheta},\vartheta,\tilde{z})g_4(\tilde{\vartheta},\tilde{z})-\beta_8(\tilde{\vartheta},\vartheta,\tilde{z})f_4(\tilde{\vartheta},\tilde{z})\right\},$$

$$f_4(\vartheta,z)=\int_1^z d\tilde{z}\{\alpha_6(\vartheta,\tilde{z})f_1(\tilde{z})-\beta_6(\vartheta,\tilde{z})g_1(\tilde{z})+$$
$$\alpha_5(\vartheta,\tilde{z})f_2(\tilde{z})+\beta_5(\vartheta,\tilde{z})g_2(\tilde{z})\ \}+$$
$$\int_1^z d\tilde{z}\int_0^{\pi/2} d\tilde{\vartheta}\{\alpha_8\left(\tilde{\vartheta},\vartheta,\tilde{z}\right)f_3\left(\tilde{\vartheta},\tilde{z}\right)-\beta_8\left(\tilde{\vartheta},\vartheta,\tilde{z}\right)g_3\left(\tilde{\vartheta},\tilde{z}\right)+ \quad (31)$$
$$+\alpha_7(\tilde{\vartheta},\vartheta,\tilde{z})f_4(\tilde{\vartheta},\tilde{z})+\beta_7(\tilde{\vartheta},\vartheta,\tilde{z})g_4(\tilde{\vartheta},\tilde{z}),$$

$$g_4(\vartheta,z)=\int_1^z d\tilde{z}\{\alpha_6(\vartheta,\tilde{z})g_1(\tilde{z})+\beta_6(\vartheta,\tilde{z})f_1(\tilde{z})+$$
$$+\alpha_5(\vartheta,\tilde{z})g_2(\tilde{z})-\beta_5(\vartheta,\tilde{z})f_2(\tilde{z})\}+$$
$$+\int_L^z dz\int_0^{\pi/2} d\tilde{\vartheta}\left\{\begin{array}{l}\alpha_8\left(\tilde{\vartheta},\vartheta,\tilde{z}\right)g_3\left(\tilde{\vartheta},\tilde{z}\right)+\\+\beta_8\left(\tilde{\vartheta},\vartheta,\tilde{z}\right)f_3\left(\tilde{\vartheta},\vartheta,\tilde{z}\right)+\end{array}\right. \quad (32)$$
$$\left.+\alpha_7(\tilde{\vartheta},\vartheta,\tilde{z})g_4(\tilde{\vartheta},\tilde{z})-\beta_7(\tilde{\vartheta},\vartheta,\tilde{z})f_4(\tilde{\vartheta},\tilde{z})\right\},$$

where $\tilde{z}=z/L$ $(0\le\tilde{z}\le 1)$, and the dimensionless coefficients α_i,β_i $(i=1-8)$ are related to the coefficients a_i, b_i $(i=1-8)$ by the relations:

$$a_1(z)=\frac{\alpha_1(\tilde{z})}{L},\ a_2(z)=\frac{\alpha_2(\tilde{z})}{L},\ b_2(z)=\frac{\beta_2(\tilde{z})}{L},$$
$$a_3(\kappa_0,z)=\frac{\alpha_3(\vartheta,\tilde{z})}{k_z L},\ b_3(\kappa_0,z)=\frac{\beta_3(\vartheta,\tilde{z})}{k_z L},$$
$$a_4(\kappa_0,z)=\frac{\alpha_4(\vartheta,\tilde{z})}{k_z L},\ b_4(\kappa_0,z)=\frac{\beta_4(\vartheta,\tilde{z})}{k_z L},$$
$$a_5(\kappa_0,z)=\frac{\alpha_5(\vartheta,\tilde{z})}{L},\ b_5(\kappa_0,z)=\frac{\beta_5(\vartheta,\tilde{z})}{L},$$
$$a_6(\kappa_0,z)=\frac{\alpha_6(\vartheta,\tilde{z})}{L},\ b_6(\kappa_0,z)=\frac{\beta_6(\vartheta,\tilde{z})}{L},$$
$$a_7(\tilde{\kappa}_0,\kappa_0,z)=\frac{\alpha_3(\tilde{\vartheta},\vartheta,\tilde{z})}{\tilde{k}_z L},\ b_7(\tilde{\kappa}_0,\kappa_0,z)=\frac{\beta_7(\tilde{\vartheta},\vartheta,\tilde{z})}{\tilde{k}_z L},$$
$$a_8(\tilde{\kappa}_0,\kappa_0,z)=\frac{\alpha_8(\tilde{\vartheta},\vartheta,\tilde{z})}{\tilde{k}_z L},\ b_8(\tilde{\kappa}_0,\kappa_0,z)=\frac{\beta_8(\tilde{\vartheta},\vartheta,\tilde{z})}{\tilde{k}_z L}.$$

Having found the asymptotic form of the integral in the expansion (10) and the corresponding Poynting vector, we obtain the angular distribution of the radiation power at acute angles $P^+(\theta)$ and obtuse angles $P^-(\theta)$:

$$\frac{dP^+(\theta)}{d\theta}=\frac{c}{4k}\left(f_3^2(\theta,L)+g_3^2(\theta,L)\right),$$

$$\frac{dP^-(\theta)}{d\theta}=\frac{c}{4k}\left(f_4^2(\pi-\theta,0)+g_4^2(\pi-\theta,0)\right). \quad (33)$$

The amplitudes $f_1(z)$, $g_1(z)$, $f_2(z)$, $g_2(z)$ determine the surface wave field and the corresponding Poynting vector, which allows to find the powers of both $P_0^+(z)$ and $P_0^-(z)$ the transmitted and reflected surface waves, respectively.

The transmission η_0^+ and reflection η_0^- coefficients for a surface wave are equal to:

$$\eta_0^+=\frac{P_0^+(z\to\infty)}{P_0^+(z\to-\infty)}=\left(f_1^2(L)+g_1^2(L)\right),$$
$$\eta_0^-=\frac{P_0^-(z\to\infty)}{P_0^+(z\to-\infty)}=\left(f_2^2(0)+g_2^2(0)\right). \quad (34)$$

Relations (33–34) allow us to find radiation patterns, as well as the coefficients η^+ and η^- transformations of the energy of the surface wave into the radiation energy at acute and obtuse angles, respectively:

$$\eta^+=\frac{1}{P_0^+(z\to-\infty)}\int_0^{\pi/2}d\theta\frac{dP^+(\theta)}{d\theta}=$$
$$=\int_0^{\pi/2}d\theta\left(f_3^2(\theta,L)+g_3^2(\theta,L)\right), \quad (35)$$

$$\eta^-=\frac{1}{P_0^+(z\to-\infty)}\int_\pi^{\pi/2}d\theta\frac{dP^-(\theta)}{d\theta}=$$
$$=\int_\pi^{\pi/2}d\theta\left(f_4^2(\pi-\theta,0)+g_4^2(\pi-\theta,0)\right). \quad (36)$$

2. Results of the computations

The system of integral equations (25–32) was solved numerically by the method of successive approximations. The zeroth approximation for the amplitudes is given by the following relations:

$$f_1(z)=1,\ g_1(z)=f_2(z)=g_2(z)=$$
$$=f_3(\vartheta,z)=g_3(\vartheta,z)=f_4(\vartheta,z)=g_4(\vartheta,z)=0, \quad (37)$$

Conditions (37) correspond to the passage of a surface wave through a section with an inhomogeneous plasma without distortion.

In the present paper it is assumed that the function $n_e(\tilde{z})$ is such that the dielectric permittivity $\varepsilon_p\left(\tilde{z}\right)=1-\omega_p^2(\tilde{z})/\omega^2$ increases linearly as shown in fig. 3. The dependence is chosen in the form:

$$\varepsilon_p\left(\tilde{z}\right)=-1-\varepsilon_0-\varepsilon'+\varepsilon'\tilde{z},$$
$$-1-\varepsilon'-\varepsilon_0\le\varepsilon_p(\tilde{z})\le-1-\varepsilon_0, \quad (38)$$

where $\varepsilon_0>0$ и $\varepsilon'>0$. The parameter ε' is equal to the gradient of the dielectric constant of the plasma:

$$\varepsilon' = \left(\varepsilon_p\right)'_{\tilde{z}}. \qquad (39)$$

It is clear from fig. 3 that ε_0 determines the function jump for $\varepsilon_p(\tilde{z})$ where $\tilde{z} = 1$. Emphasize that the function $\varepsilon_p(\tilde{z})$ can be arbitrary. The solution of the problem is completely determined by the electric radius of the plasma cylinder a/L, the electrical length of the inhomogeneity section L/λ, the relative wall thickness of the dielectric cylinder b/a, the dielectric permittivity ε_d and the parameters $\varepsilon_0, \varepsilon'$ of the function $\varepsilon_p(\tilde{z})$.

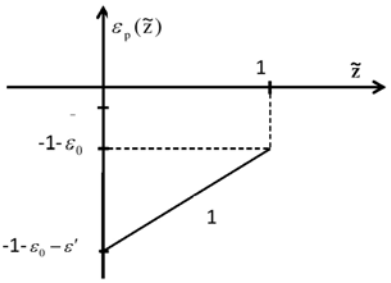


Fig. 3. Dependence of the permittivity of the plasma $\varepsilon_p(\tilde{z})$, where $\tilde{z} = z/L$ on the section with longitudinal inhomogeneity of the plasma density (line 1)

The accuracy of the computations is determined by the difference approximation of the integrals in equations (25–32) and by the validity of the approximation (18). Accuracy was controlled with the help of obvious equality:

$$\eta_0^+ + \eta_0^- + \eta^+ + \eta^- = 1. \qquad (40)$$

In the present paper we confined ourselves to the parameters of the problem when it is possible to use the substitution (26). For large values of the parameters a/λ, b/a, and the absolute value of the dielectric permittivity of the plasma $|\varepsilon_p|$ in the antenna, there is more than one surface wave. In this case, the method of spectral decomposition is extremely complicated so that it becomes practically inapplicable.

Physically, this complication is due to the fact that it is necessary to take into account the coupling of all waves to each other, including surface and spatial waves.

Mathematically, this leads to an increase in the number of equations in a system of equations of the type (25–32). Therefore, we confined ourselves to the values of the parameters a/λ and b/a for which only one surface wave exists in the antenna.

Fig. 4–8 show the results of calculations of the surface wave energy transformation coefficients η_0^+, η_0^- (34), η^+ (35), η^- (36), where ε_0 = 10, a/λ = 0.1, depending on the plasma permittivity gradient ε' for $L/\lambda = 2$ for different values of the parameters ε_d and b/a. The transformation coefficients shown in fig. 4–6, correspond to the value $\varepsilon_d = 2{,}5$ (polyethylene, polystyrene), and in fig. 7–8 – $\varepsilon_d = 10$ (sapphire). These substances have a sufficiently small tangent of the loss angle, which is approximately equal $\mathrm{tg}\delta \approx 10^{-4}$. From fig. 4–8, that it is possible to calculate the coefficients: η_0^+, η_0^-, η^+, η^- with sufficient accuracy for a very rapid change in the plasma density. At ε_0 =10, and ε'=10000, we get from the formula $\varepsilon_p = 1 - \omega_p^2/\omega^2$, that the plasma density at the end of the inhomogeneity region decreases by about 100 compared to the density by $\tilde{z} = 0$.

It is seen that the energy of the surface wave is mainly converted into radiation energy at acute angles and the energy of the surface wave transmitted through the inhomogeneity.

The coefficient η^+ increases with the gradient of the dielectric permittivity ε' and reaches values of $\eta^+ \approx$ 15–35%, depending on the geometric parameters and physical properties of the antenna. The function $\eta^+(\varepsilon')$ monotonically grows, therefore at ε' > 10000, radiation is possible with even greater efficiency. With increasing ε', however, it is necessary to reduce the steps of integration into (25–32), which requires an inordinately large computer resource. From a comparison of not only fig. 4 and 5, but also fig. 7 and 8, it is seen, that with increasing of the relative thickness of the wall of the dielectric b/a, the coefficient η^+ decreases.

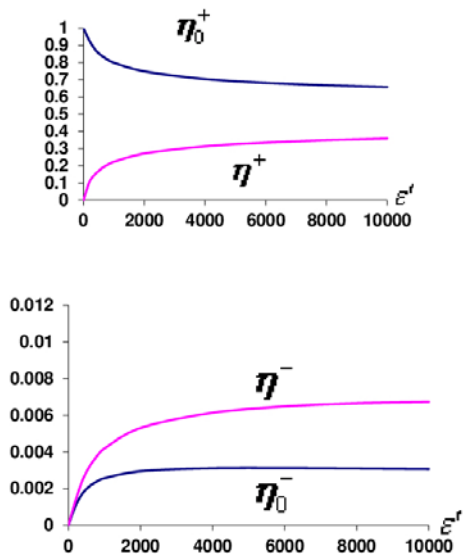


Fig. 4. Dependence of the energy transformation coefficients of the surface wave on ε' for ε_0 = 10, L/λ = 2, a/λ =0.1, b/a = 1.01, ε_d = 2.5

Physically, this is explained as follows. The surface wave is partially propagated along the dielectric. This part of the wave propagates without distortion, since the dielectric is homogeneous. The transformation of the wave occurs only on plasma inhomogeneities. The thicker the dielectric, the more uniform the plasma antenna, which leads to a weakening of the transformation of the surface wave into radiation.

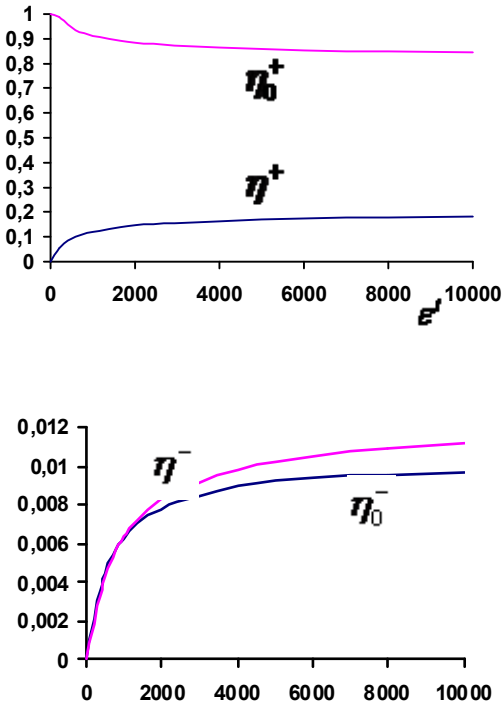


Fig. 5. Dependence of the energy transformation coefficients of the surface wave on ε' for $\varepsilon_0 = 10$, $L/\lambda = 2$, $a/\lambda = 0.1$, $b/a = 1.2$, $\varepsilon_d = 2.5$

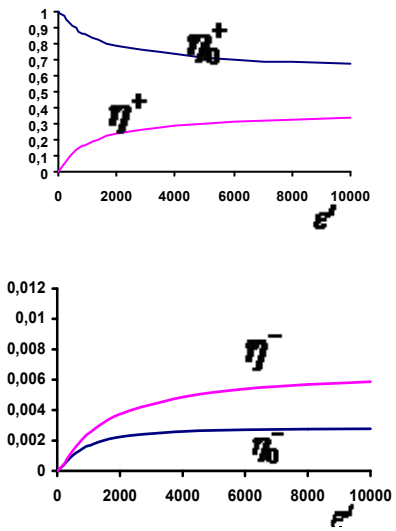


Fig. 6. Dependence of the energy transformation coefficients of the surface wave on ε' for $\varepsilon_0 = 10$, $L/\lambda = 5$, $a/\lambda = 0.1$, $b/a = 1.01$, $\varepsilon_d = 2.5$

The functions $\eta^+(\varepsilon')$ and $\eta_0^+(\varepsilon')$ slightly change when the electrical length of the inhomogeneity part is changed L/λ and are determined mainly by the parameter b/a. This is illustrated in fig. 6. The fraction of the energy of the reflected surface and backward scattered waves is very small. With an increase in the jump in the permittivity at the end of the inhomogeneity region determined by the parameter ε_0, the efficiency of the transformation of the energy of the surface wave into radiation, that is, the parameter η^+, where all other conditions being equal, decreases.

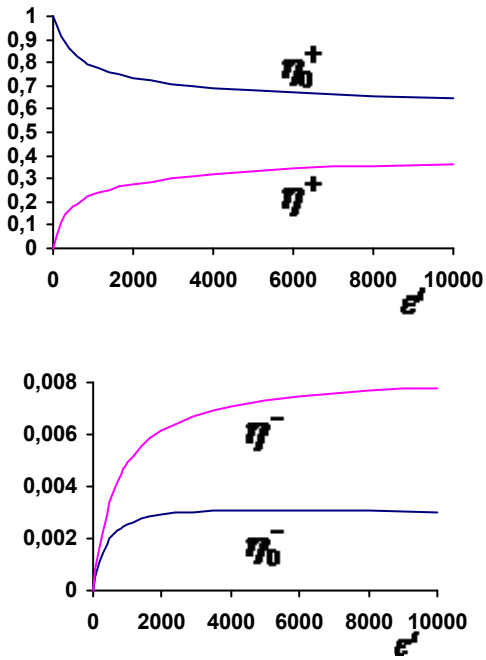


Fig. 7. Dependence of the energy transformation coefficients of the surface wave on ε' for $\varepsilon_0 = 10$, $L/\lambda = 2$, $a/\lambda = 0.1$, $b/a = 1.01$, $\varepsilon_d = 10$

Therefore, for effective antenna operation, the plasma density at the end of the plasma inhomogeneity region should be small, as far as possible, compared with the density at the beginning of the inhomogeneity region.

Examples of normalized radiation patterns, are shown in fig. 9, where $L/\lambda = 2$, $a/\lambda = 0.1$ for different values of the permittivity ε_d and parameter b/a. It can be seen, that the normalized rediation patterns has one lobe, and it is highly directional with a maximum beam located at small angles θ_m to the axis z. As the thickness of the dielectric b/a and the dielectric permittivity ε_d increase, the surface wave slows down due to the dielectric shell of the plasma antenna. As the deceleration increases, the maximum radiation angle increases and approaches the value $\pi/2$.

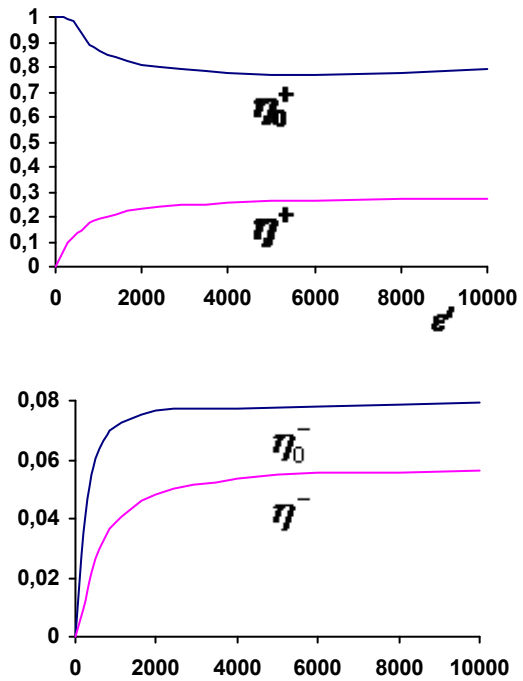
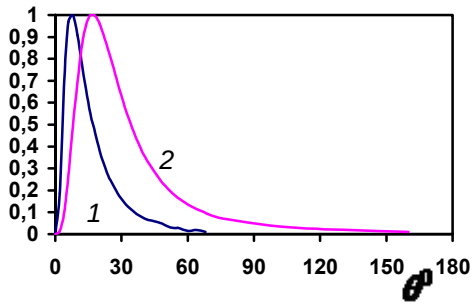
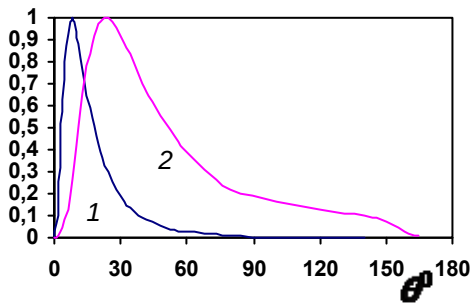


Fig. 8. Dependence of the energy transformation coefficients of the surface wave on ε' for $\varepsilon_0 = 10$, $L/\lambda = 2$, $a/\lambda = 0.1$, $b/a = 1.2$, $\varepsilon_d = 10$



a – $\varepsilon_d = 2.5$



b – $\varepsilon_d = 10$

Fig. 9. Normalized radiation patterns calculated for $L/\lambda = 2$, $a/\lambda = 0.1$, $\varepsilon_0 = 10$ and $\varepsilon' = 10000$, for different values of ε_d. The line 1 corresponds to $b/a = 1.01$ and lines 2 corresponds to $b/a = 1.2$. On the abscissa axis, angles are set in degrees

This is a common property of traveling wave antennas [6–12]. This property explains the increase θ_m in the half-widths of the normalized radiation patterns, with increasing values of b/a and ε_d.

In addition, with an increase in the longitudinal velocity of the plasma density and a decrease in the electric thickness of the plasma cylinder, the width of the normalized radiation patterns, and the angle θ_m decrease.

Thus, the plasma cylinder in question with a strong longitudinal inhomogeneity is the basis for creating an effective plasma antenna with a sharply directed paraxial radiation. In such a plasma antenna, the energy introduced into the antenna is converted into radiation at acute angles with high efficiency.

In addition, the positive property of such an antenna is the absence of sidelobes in its directional pattern.

Conclusion

The efficiency of transformation of a surface axially symmetric wave into radiation in a cylindrical plasma column bounded by a dielectric cylinder is studied. The plasma is longitudinally inhomogeneous.

For the analysis of such systems, the spectral decomposition method is effective.

According to this method, the electromagnetic field is decomposed into a complete set of functions, including surface and volume waves. A system of integral equations for the expansion coefficients is obtained.

These coefficients determine the efficiency of the transformation of the energy of the surface wave into radiation energy and the radiation pattern. Longitudinal heterogeneity can be arbitrary, including a very strong.

Calculations are performed for different values of the electric radius of the cylinder, the relative thickness of the dielectric, the electrical length of the plasma inhomogeneity section, the dielectric permittivity of the plasma, and the dielectric.

It is shown, that as the gradient of the dielectric permittivity increases, the part of the surface wave energy converted into radiation, increases and reaches 35%.

The normalized radiation patterns and the coefficients of transformation of the energy of the surface wave into the radiation energy depend only slightly on the electrical length of the region of plasma inhomogeneity.

The normalized radiation patterns have a single beam, located at a small angle to the plane of the plasma layer. This angle, as well as the width of the beam, decrease with increasing rate of change of the function $\varepsilon_p(\tilde{z})$ and are equal to several degrees.

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ВИПРОМІНЮВАННЯ ПРОСТОРОВО ОБМЕЖЕНОЇ НЕОДНОРІДНОЇ ПЛАЗМИ

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Досліджено ефективність перетворення енергії поверхневої хвилі поздовжньо неоднорідного циліндричного плазмового стовпа у випромінювання. Плазма обмежена діелектричною оболонкою. Аналіз проведено методом спектрального розкладання по повному набору функцій, що містить у собі поверхневі та просторові хвилі плазмового стовпа. Виводиться система інтегро-диференційних рівнянь для коефіцієнтів розкладання. Ці коефіцієнти визначають амплітуду розсіяної хвилі, а також амплітуди поверхневих хвиль що пройшли неоднорідність або відбились від неї. Система рівнянь розв'язується для довільного змінення щільності плазми. Обчислено залежності коефіцієнтів трансформації енергії поверхневої хвилі від градієнта щільності плазми, електричної довжини ділянки неоднорідності плазми, електричного радіуса плазмового циліндра, діелектричної проникності та товщини діелектрика. Наведено приклади, коли частка енергії поверхневої хвилі, що перетворюється у випромінювання під гострими кутами, досягає 35%. Діаграми направленості є гостро направленими и мають одну пелюстку. Максимум випромінювання відповідає куту в декілька градусів по відношенню до напрямку розповсюдження поверхневої хвилі. Ширина пелюстки зменшується, а її положення зсувається до 0° при збільшенні градієнта щільності плазми. Досліджується вплив властивостей діелектрика на характеристики випромінювання.

Ключові слова: *циліндрична плазмова антена, метод спектрального розкладання, випромінювання, поверхневі хвілі.*

ИЗЛУЧЕНИЕ ПРОСТРАНСТВЕННО ОГРАНИЧЕННОЙ НЕОДНОРОДНОЙ ПЛАЗМЫ

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Исследована эффективность преобразования энергии поверхностной волны продольно неоднородного цилиндрического плазменного столба в излучение. Плазма ограничена диэлектрической оболочкой. Анализ проведен методом спектрального разложения поля по полному набору функций, включающему в себя поверхностные и пространственные волны плазменного столба. Выводится система интегро-дифференциальных уравнений для коэффициентов разложения. Эти коэффициенты определяют амплитуды прошедшей, отраженной и рассеянной волн, а также диаграммы направленности излучения. Система уравнений решается для произвольного продольного изменения плотности плазмы. Вычислены зависимости коэффициентов трансформации энергии поверхностной волны от градиента плотности плазмы, электрической длины участка неоднородности плазмы, электрического радиуса плазменного цилиндра, диэлектрической проницаемости и толщины диэлектрика. Приведены примеры, когда доля энергии поверхностной волны, которая трансформируется в излучение под острыми углами, составляет 35%. Диаграммы направленности являются остронаправленными и имеют один лепесток. Максимум излучения приходится на угол в несколько градусов по отношению к направлению распространения поверхностной волны. Ширина лепестка уменьшается, а его положение сдвигается к 0^0 при увеличении градиента плотности плазмы. Исследуется влияние свойств диэлектрика на характеристики излучения.

Ключевые слова: *цилиндрическая плазменная антенна, метод спектрального разложения, излучение, поверхностные волны.*