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DEVELOPMENT OF OPTIMUM NAVIGATION INFORMATION PROCESSING ALGORITHM

The purpose of a navigation system or a complex of navigation systems of any object is achieving of navigation parameters vector as a result of the initial measurements and their further processing which are further used in controlled systems of the object motion. The achieving of navigation parameters in such systems is based on the "ideal work" algorithm, which, with without abnormal primary information sensor system (measurers of velocity and course and also acceleration and angels that position the gyroscopes) and in the assumption of zero-error of these sensors ensures the error-free parameterisation. The implementation of the "ideal work" algorithm ensures the system unexcitability by useful signal, which is the valid values of navigation parameters. Despite the fact that the indifference of system errors from the true motion of the object, taking into account the multiplicative component errors, the unintentional smoothing of the high-frequency objects moving through the inertia of the sensors is carried out approximate, the algorithm of the system of calculating coordinates and the inertial system tend to bring as close as possible to the "ideal work" algorithm. For this purpose, the mathematical devise for the development of an optimal algorithm for the navigation information processing is grounded in the article. The optimal algorithm for processing of navigation information is proposed to provide the necessary accuracy of determining the position of an object in the route of movement.

***Keywords**: the navigation system, the navigation information, the processing algorithm, the position of an object*, *the route of movement.*

Introduction

Target setting. Inertial navigation systems are the basis of navigation complexes of modern motion objects. This is because they provide complete information about navigation trajectory parameters – yaw angle, trim, bank, acceleration, speed and location details. However, they are completely autonomous, that is, they do not require outside information. Due to the ability to determine the high precision flight attitude and furnish data with high frequency, inertial navigation systems have no alternatives for today [1–2].

Analysis of literature. The development of inertial navigation framework began in the 1930s of the last century. It should be noted that a significant role in the theoretical foundations of inertial navigation is played by the theory of stability of mechanical systems, in which mathematician Lyapunov had made the grate contributions [3–4].

Practical implementation of inertial navigation methods often faces significant obstacles needed to provide high accuracy and reliability of all devices at prescribed weight and dimensions.

Increasing the accuracy of motion objects navigation is due to enhancements both measuring equipment and mathematical support information processing solutions [5–11]. Therefore, assignments directed towards achievement of high level of navigation accuracy should be realised by using of the navigation information processing effective techniques and taking into account the influence of the irregularity figure of the Earth. It is clear that the complication of navigation algorithms is appropriate only in the conditions of small instrument errors of navigation systems, which are provided today only in some cases [12–14].

In that regard actual relevant scientific research in the spheres of navigation and control appears – methods and models synthesis of marine navigation increased activity under uncertainty.

The purpose of the article. Substantiation of the mathematical apparatus for the development of optimal algorithm for the processing of navigation information.

Main part

Based on the analysis of navigation systems and factors which affect the efficiency with which they function, output signal of navigation system $\tilde{\psi}$, which generates *l*-dimensional vector ψ of navigation parameters for the case of discrete time can be written as:

$$\tilde{\psi}(k) = \psi(k) + \Delta\psi(k) = \psi(k) + B_1(k)X_1(k), \quad (1)$$

where $X_1 = \left(\Delta\psi^T, \Delta^T\right)^T$ is dimensional vector r_1, which describes the navigation system errors and satisfy the system of equations:

$$X_1(k) = A_1\left[X_1(k-1), k\right] + W_1(k). \quad (2)$$

In expression (2) B_1 – [$l\times r_1$] – matrix [E | 0], while E is unit matrix [$l\times l$] м, 0 – zero matrix [$l\times r_1 - l$]. More generally vector $\Delta\psi$ is not a subvector of X_1. It is a linear combination of its components.

As an example, there is a relation

$$\Delta\psi = B_1X_1. \quad (3)$$

It is clear that on the basis of navigation system errors description satisfying the system of equations (2) and matrix B_1, the possible solutions of analysis of navigation accuracy parameters producing which is composed of $\Delta\psi$ random vector\`s definite properties. The situation qualitatively changes if simultaneously with navigation system which generates $\tilde{\psi}$ vector, at least one other navigation system functions, for the output signal Y of which is accordingly

$$Y(k) = F\left[\Delta\psi(k), k\right] + n(k),$$

where F is known, generally nonlinear, function depending from navigational parameters; $n = n_1 + n_2$ – measuring error, while n_1 and n_2 = V are slow variable and high-frequency (white noise) composite errors of this navigation system.

Let's consider, that

$$n_1(k) = B_2(k)X_2(k). \quad (4)$$

r_2 – dimensional vector X_2 satisfies the equation:

$$X_2(k) = A_2(k)X_2(k-1) + W_2(k). \quad (5)$$

When operating within several navigation systems (this particular case will be further considered), the signal Y(k) is a vector; all expressions resulting this signal are matrix. The expression for measuring with the use of (3) and (4) will be get in the form

$$Y(k) = F_1\left[X_1(k), k\right] + B_2(k)X_2(k) + V(k). \quad (6)$$

F_1 is known function X_1 and k is expressed in F and B_1. Let\`s input augmented r-dimensional vector $X = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$, $r = r_1 + r_2$ that satisfy the system of equation

$$X(k) = A\left[X(k-1), k\right] + W(k), \quad (7)$$

where $W = \begin{pmatrix} W_1 \\ W_2 \end{pmatrix}$, $A = \begin{pmatrix} A_1 & 0 \\ 0 & A_2 \end{pmatrix}$.

Rewrite (6) in the form

$$Y(k) = H\left[X(k), k\right] + V(k). \quad (8)$$

We shall notice, that (4) and (5) follow

$$H(0) = 0. \quad (9)$$

When there are at least two simultaneously operating navigation systems (the first system is usually continuous operating inertial like system or system of calculating coordinates; it is convenient to give the second system the value of correcting the first one) it is possible task setting to obtain an estimate of $\hat{\psi}(k)$ vector of navigation parameters on measurements Y(i) which are made basing on the interval $i \in [0, k]$, where 0 is the moment when the navigation systems have got started. The estimate should provide minimum differences $\hat{\psi}(k) - \psi(k)$.

It is easy to understand that solving of this task is equal derivation of estimate $\Delta\hat{\psi}$ vector $\hat{\psi} - \psi$, that provides the small quantity

$$\varepsilon(k) = \Delta\hat{\psi}(k) - \Delta\psi(k), \quad (10)$$

because with a taken as $\hat{\psi} = \tilde{\psi} - \Delta\hat{\psi}$, we get: $\hat{\psi} - \psi = \varepsilon$.

Given that ε is random vector because of randomness X and Y, its level can be characterised only as the average set of possible values X and Y. For this purpose, it is convenient to use covariance matrix $G_\varepsilon = M\left(\varepsilon\varepsilon^T\right)$, where M (·) provide averaging over X and Y.

Let $\hat{\psi}$ is optimal estimate (in contrast to any other estimate $\bar{\psi}$), if

$$\Delta_\varepsilon = \bar{G}_\varepsilon - G_\varepsilon \geq 0, \quad (11)$$

where $\bar{G}_\varepsilon = M\left[\left(\Delta\bar{\psi} - \Delta\psi\right)\left(\Delta\bar{\psi} - \Delta\psi\right)^T\right] = M\left(\bar{\varepsilon} \cdot \bar{\varepsilon}^T\right)$, $\Delta\bar{\psi} = \bar{\psi} - \psi$, $\bar{\varepsilon} = \Delta\bar{\psi} - \Delta\psi$; inequality means symmetric positive definite matrix Δε.

It should be noted that derivation of optimal estimate of navigation parameters error vector $\hat{\psi}$ by the criterion (11) is in agreement with the basic requirements of navigation practice. Using the properties of positive definite matrix [15], it should be mentioned that diagonal elements of matrix Δε з (11) are inseparable. Therefore, the root-mean-square errors of optimal estimates of all navigation parameters ψ are minimal. Hence, the popular name of the criterion (11) is root-mean-square criterion.

Further, in the allocation of error estimates vectors to a normal law of isosurface density form the dispersion volume. The equations of which have the next form

$$\frac{\varepsilon^T \varepsilon}{G_\varepsilon} = l^2 . \qquad (12)$$

At the fixed l the volume of ellipsoid is proportional to $|G_\varepsilon|^{1/2}$ and the probability that the vector ε will take the value inside the ellipsoid (12) is $1-\exp\left(-l^2/2\right)$ [16]. We now show that from (11) we get inequality

$$|G_\varepsilon| \le |\bar{G}_\varepsilon| , \qquad (13)$$

It determines the minimum size of the dispersion volume error of the optimal estimation.

Let`s convert the error vector ε and $\bar{\varepsilon}$ using non-degenerate matrix T in order that equality $G'_\varepsilon = TG_\varepsilon T^T = \mathrm{diag}(\lambda) = \lambda E$ should provide for vector covariance.

Note that first G'_ε and $\bar{G}'_\varepsilon = T\bar{G}_\varepsilon T^T$ also satisfy the inequality (11), secondly the inequality (as yet unidentified) for determinants $|G_\varepsilon|$ i $|\bar{G}_\varepsilon|$ is preserved for the determinants $|G'_\varepsilon|$ i $|\bar{G}'_\varepsilon|$, because (17) for C = AB we receive $|C| = |A| \times |B|$.

Let`s define covariation $\bar{\varepsilon}' = T\bar{\varepsilon}$ using

$$\bar{G}'_\varepsilon = \bar{U}^T \mathrm{diag}\left(\bar{\lambda}_i\right) \bar{U} , \qquad (14)$$

where $\bar{\lambda}_i$ – matrix $\bar{G}'$ eigenvalues.

The structure $\bar{G}'_\varepsilon$ allows to present it in the form $G'_\varepsilon = \bar{U}^T \mathrm{diag}(\lambda_i) \bar{U}$. Taking into account this fact and if plugging of equation (14) in (11) we arrive at inequality $\bar{\lambda}_i \ge \lambda$ whence it follows the inequality proof (13):

$$|\bar{G}'_\varepsilon| = |\bar{U}|^2 \prod_{i=1}^{r} \bar{\lambda}_i = \prod_{i=1}^{r} \bar{\lambda}_i \ge \lambda^r = |G'_\varepsilon| .$$

The following equation is used here: $\left|\bar{U}^T\right| \cdot |\bar{U}| = |\bar{U}|^2 = 1$.

We also note that the obtained properties of the criterion are also fulfilled for the sub-vector ε. It is quite enough in any vector a, that has a quadratic form, to leave non-zero elements of corresponding Δψ vector components that matter to us. That will make sure that using the common criterion in minimality of root-mean-square errors of estimate in any linear combinations of the components Δψ, minimality of the dispersion volume area error of object coordinates estimation.

To avoid methodological difficulties, related to the formation covariation matrix determinant and other forms of criteria (11) in the case of different navigation parameters physical dimensions let`s consider that these parameters are pre-reduced to one dimensions by some scaling conversions.

By rewriting (3) in the form Δψ = BX, where B = (B_1, O), O – zero-matrix of order $[l \times r_2]$, put the problem of X(k) vector estimate on measurements $Y_0^k = \left\{Y(i),\ i = \overline{0,k}\right\}$ according to the criterion

$$\Delta_\varepsilon(k) = \left[\bar{G}(k) - G(k)\right] \ge 0 . \qquad (15)$$

By analogy with (11) let`s introduce

$$G = M\left[\left(X-\hat{X}\right)\left(X-\hat{X}\right)^T\right],\ \bar{G} = M\left[\left(X-\bar{X}\right)\left(X-\bar{X}\right)^T\right],$$

where $\hat{X}$ is optimal, and $\bar{X}$ – any estimate X.

As noted above, the estimation $B\hat{X}$ of the linear combination of vector X components is the optimal estimation of the vector Δψ of the errors of navigation parameters.

$$\Delta\hat{\psi} = B\hat{X} . \qquad (16)$$

This shows that solution of optimal estimate X on measurements Y_0^k using (16) and $\hat{\psi} = \tilde{\psi} - \Delta\hat{\psi}$ ratio leads to an optimal estimation of navigation parameters vector. The “extension” of the task – the evaluation of high-dimension vector X to obtain the estimate of sub-vector Δψ usually lesser dimension ensures the possibility of the Markovian approach usage, which opens ways to simplify the filtration procedure. The results of this unit are general; under the vector X it is possible to understand and the subvector Δψ.

In the general theory of statistical filtration, the following result is one of the main: optimal according the criterion (15) the estimate of the form

$$\hat{X}(k) = M\left[X(k)/Y\right], \qquad (17)$$

where Y is – some synergy of measurements of vector X(k); symbol M(·/·) indicates conditional expectation. Let's show it.

Let $\bar{X}(k)$ is the estimate X(k), defined on measurements Y by any filter. To the error of the estimate $\bar{\varepsilon}(k) = \bar{X}(k) - X(k)$ we give the form

$$\bar{\varepsilon} = \bar{X} - X + \hat{X} - \hat{X} = \varepsilon + \delta , \qquad (18)$$

where ε – estimate of the error (17); $\delta(k) = \bar{X}(k) - \hat{X}(k)$.

Let`s write the covariance matrix of the estimate error $\bar{X}(k)$ by virtue of (18) in the form

$$\bar{G} = G + M\left(\varepsilon\delta^T\right) + M\left(\delta\varepsilon^T\right) + M\left(\delta\delta^T\right) . \qquad (19)$$

The mathematical expectation in this formula is taken on the strength of random variables (X)k and Y and needs to be defined sequentially: $M_{XY}(\ ) = M_Y\left[M_X(\ /Y)\right]$.

Using this rule, we can write:

$$X\left(\varepsilon\delta^T\right) = M_Y\left[M_X\left(\varepsilon\delta^T/Y\right)\right].$$

Additional error δ(k) – the difference of estimates – is the function of measurements and at fixed Y doesn`t depend on X(k). The same is true and for the estimate $\hat{X}(k)$. Therefore, δ and $\hat{X}$ can be brought outside the sign of conditional mathematical expectation:

$$M\left(\varepsilon\delta^T\right) = M_Y\left[M_X\left(X - \hat{X}/Y\right)\delta^T\right] =$$
$$= M_Y\left\{\left[M(X/Y) - \hat{X}\right]\delta^T\right\}.$$

Using (17), we obtain the equation,

$$M\left(\varepsilon\delta^T\right) = 0\,, \quad (20)$$

it allows us to conclude that there is no correlation of optimal estimate error X(k) on measurements Y with any function of these measurements.

Likewise it can be shown that the third addend in the expression (19) is also equal to zero. Now, we see that $\Delta(k) = \bar{G}(k) - G(k) = M\left[\delta(k)\delta^T(k)\right]$.

As $M\left(\delta\delta^T\right)$ is covariation matrix, which is always sufficiently definite, the estimate (17) meet the criteria (15), that has to be shown. The condition (20) is necessary and enough criteria for optimal estimate.

The population of random variables is completely described by total density of probability distribution. Let the population X_0^k (set of state vectors X(0), ..., X(k)) and Y_0^k (set of measurements Y(0), ... Y(k)) corresponds to density $f\left(x_0^k,\, y_0^k\right)$. Now we can write the estimate (17) in the form

$$\hat{X}(k) = \int x(k)\pi\left[x(k)\right]dx(k)\,, \quad (21)$$

where

$$\pi\left[x(k)\right] = \int f\left(x(k)/\ Y_0^k\right) = \frac{\int f\left(x_0^k,\, Y_0^k\right)dx_0^{k-1}}{\int f\left(x_0^k,\, Y_0^k\right)dx_0^k} \quad (22)$$

is a posterior density of random vector X(k).

Here and further integrals are understood as infinite integrals, and vector differential – as a product of the differentials of their components. In addition, we will use small letters to denote the density arguments, corresponding to random variables whose densities are considered. Use in (22) Y_0^k as density argument instead of y_0^k means the substitution in the function $f\left(x_0^k,\, y_0^k\right)$ obtained measurement realisation.

If there are no special assumptions about properties of sequences X(k) and Y(k), then multidimensional density $f\left(x_0^k,\, y_0^k\right)$ can be of very complicated structure, and prevents using formulas (21) and (22) as an algorithm for calculating the optimal estimate. In particular, the calculation of a posterior density according to the formula (22) does not have the properties of recurrence.

Optimal filtering of measurement results at Markovian error model. Signals of navigation systems are characterized by specific properties that allow a little progress in solving the tasks of optimal nonlinear filtration. These characteristics are related to the Markovian character of the navigation signals. Using them, we specify the expression (22) for posterior density $\pi\left[x(k)\right]$ and get recurrent relation for it.

It is known that a random sequence has Markovian property if it is described by a finite-difference stochastic equation [16]. In our case, we create the sequence $\xi(k) = \begin{pmatrix} X(k) \\ \tilde{Y}(k) \end{pmatrix}$, where $\tilde{Y}(k) = Y(k-1)$. Using the expressions (7) and (8), for this sequence, we can write the equation:

$$\xi(k) = B\left[\xi(k-1),\ k\right] + V_\xi(k)\,,$$

where

$$B\left[\xi(k-1), k\right] = \begin{pmatrix} A\left[X(k-1), k\right] \\ H\left[X(k-1), k-1\right] \end{pmatrix};\ V_\xi(k) = \begin{pmatrix} W(k) \\ V(k-1) \end{pmatrix}.$$

This fact allows us to define concretely the kind of joint density $f\left(x_0^k,\, y_0^k\right)$, which is the part of the expression (22), giving it through the transition density of Markovian sequence $\xi(k)$ [17]. They have some specifics which are connected with the fact that at first, the sequence X(k), satisfying the equation (7) is Markovian and secondly, measurements according to the ratio (8) depending only from X(k). Due to it transition density of sequence $\xi(k)$ in this case have the form

$$f\left[\xi(k)/\xi(k-1)\right] = f\left[x(k),\ \tilde{y}(k)/x(k-1),\ \tilde{y}(k-1)\right] \times$$
$$\times \prod_{i=1}^{k+1} f\left[x(i)/x(i-1)\right] =$$
$$= f\left[x(k)/x(k-1)\right] f\left[\tilde{y}(k)/\ \tilde{y}(k-1)\right].$$

Let`s write the expression for general density $f\left(\xi_0^{k+1}\right)$:

$$f\left(x_0^{k+1},\ \tilde{y}_0^{k+1}\right) = f\left[x(0),\ \tilde{y}(0)\right] \times$$
$$\times \prod_{i=1}^{k+1} f\left[x(i)/x(i-1)\right] f\left[\tilde{y}(i)/\tilde{y}(i-1)\right]. \quad (23)$$

Because we are interested in density $f\left(x_0^k,\, y_0^k\right)$ or $f\left(x_0^k,\, \tilde{y}_0^{k+1}\right)$, we integrate (23) on variables x(k + 1), $\tilde{y}(0)$ in infinite limits and substitute $\tilde{y}(k) = y(k-1)$. Then

$$f\left(x_0^k,\, y_0^k\right) = = f\left[x(0)\right]\prod_{i=1}^{k} f\left[x(i)/x(i-1)\right]\prod_{i=0}^{k} f\left[y(i)/x(i)\right]. \quad (24)$$

After it we get recurrence equation for the posterior density $\pi\left[x(k)\right]$. For this using the expression (24), let`s write at first the recurrence equation for posterior density:

$$f\left(x_0^k,\, y_0^k\right) = f\left[x(k)/x(k-1)\right]\times \times f\left[y(k)/x(k)\right] f\left[x_0^{k-1},\, y_0^{k-1}\right]. \quad (25)$$

Substituting (25) into (22), we obtain.

$$\pi\left[x(k)\right] = \frac{f\left[Y(k)/x(k)\right]}{\int f\left[Y(k)/x(k)\right]}\times \times\frac{\int f\left[x(k)/x(k-1)\right] f\left(x_0^{k-1},\, Y_0^{k-1}\right)dx_0^{k-1}}{\int f\left[x(k)/x(k-1)\right] f\left(x_0^{k-1},\, Y_0^{k-1}\right)dx_0^{k-1}dx(k)}.$$

Let`s split numerator and denominator in $\int f\left(x_0^{k-1},\, Y_0^{k-1}\right)dx_0^{k-1}$ that by virtue of equation

$$\frac{\int f\left(x_0^{k-1},\, Y_0^{k-1}\right)dx_0^{k-2}}{\int f\left(x_0^{k-1},\, Y_0^{k-1}\right)dx_0^{k-1}} = \pi\left[x(k-1)\right],$$

then

$$\pi\left[x(k)\right] = \frac{f\left[Y(k)/x(k)\right]}{\int f\left[Y(k)/x(k)\right]}\times \times\frac{\int f\left[x(k)/x(k-1)\right]\pi\left[x(k-1)\right]dx(k-1)}{\int f\left[x(k)/x(k-1)\right]\pi\left[x(k-1)\right]dx(k-1)dx(k)}. \quad (26)$$

Because

$$f\left[x(k)/x(k-1)\right]\pi\left[x(k-1)\right] = f\left[x(k),\, x(k-1)/Y_0^{k-1}\right]$$

integral, that is in numerator (26), become the density of measure

$$\tau\left[x(k)\right] = f\left[x(k)/Y^{k-1}\right] = = \int f\left[x(k)/x(k-1)\right]\pi\left[x(k-1)\right]dx(k-1), \quad (27)$$

thus we can write (26) in the form

$$\pi\left[x(k)\right] = \frac{f\left[Y(k)/x(k)\right]\tau\left[x(k)\right]}{\int f\left[Y(k)/x(k)\right]\tau\left[x(k)\right]dx(k)}. \quad (28)$$

The function $f\left[Y(k)/x(k)\right]$ by content is the function of state vector veracity on k step according to the obtained measurement results Y(k).

The initial condition for the recurrence equation given by (27), (28) is always prior density $f\left[x(0)\right]$ of the vector X(0) at this the result $\pi\left[x(0)\right] = f\left[x(0)/Y(0)\right]$ is the first step of recurrence equation.

The next step for defying of this equation is to obtain the expressions for density $f\left[x(k)/x(k-1)\right]$ and $f\left[Y(k)/x(k)\right]$ from the equation (7) and (8). As W(k) and V(k) Gaussian white noise with covariations Q(k) and R(k) accordingly, then we receive [16]:

$$f\left[x(k)/x(k-1)\right] = \frac{|Q|^{-1/2}}{|2\pi|^{-1/2}}\times \times\exp\left\{-\frac{1}{2}\left[x(k) - A\left[(k-1)\right]\right]^T \frac{x(k) - A\left[(k-1)\right]}{Q}\right\}; \quad (29)$$

$$f\left[Y(k)/x(k)\right] = f_V\left\{Y(k) - H\left[x(k)\right]\right\} = = \frac{|R|^{-1/2}}{|2\pi|^{-m/2}}\exp\left\{\begin{matrix} -\frac{1}{2}\left[Y(k) - H\left[x(k)\right]\right]^T \times \\ \times\frac{Y(k) - H\left[(k)\right]}{R}\end{matrix}\right\}. \quad (30)$$

Obtaining of the recurrence equation for the posterior density in the task of processing of navigation measurements is finished after substituting (29) in (27) and (30) in (28).

The denominator (28) is normalising constant for posterior density, that is denote by ρ(k). Because of the integrand in the denominator is density $f\left[x(k),\, y(k)/Y_0^{k-1}\right]$ under y(k) = Y(k) (here like in (22), making use of the notation Y instead of the density argument y means substituting of already achieved measurement)

$$\rho(k) = f\left[Y(k)/Y_0^{k-1}\right]. \quad (31)$$

Thus, the optimal algorithm for processing navigation information should work as follows.

1. Using the relations (27) and (28), when the further measurement result Y (k) is obtained, recalculate posterior density, then the vector X (k) optimal estimate is obtained from the formula (21), which is used for getting (according to the relation (16) estimate error of navigation parameter vector $\Delta\hat{\psi}(k)$ measured by the first system.

2. The relationship $\hat{\psi}(k) = \tilde{\psi}(k) - \Delta\hat{\psi}(k)$ completes the solution of obtaining an optimal estimate of navigation parameters vector after measurements Y_0^k.

3. To determine the accuracy of the estimates, the posterior covariation matrix of error vector estimate

$$P(k) = = \int\left[\hat{x}(k) - x(k)\right]\left[\hat{x}(k) - x(k)\right]^T \pi\left[x(k)\right]dx(k) \quad (32)$$

and obtained posterior covariation matrix of error vector estimate according to (10) and (16) $P_\varepsilon(k) = B(k)\cdot P(k)\cdot B^T(k)$ should be used.

4. The covariation matrixes $G_{\varepsilon}(k)$ and G (k) which are used to form the criterion of optimality (11), (15), differ from covariation matrixes $P_{\varepsilon}(k)$, $P(k)$, conditioned to the measurement results, which are realized Y_0^k, they allow the average with all possible values Y_0^k, as a random vector.

Markovian description of navigation systems errors, which ensured the transition from the ratio (22) for posterior density to the recurrence ratio (28), leaves, however, significant difficulties in realisation of filter that estimate on computer, "unable" to solve the functional equation correctly (28).

Conclusions

The practical solving of the optimal filtration problem is based on certain simplifications under task setting, that leads to the loss of optimal estimates of navigation parameters and consequently, increase the estimate error.

However, if sub-optimization of the filter proceeds from one or another simplification of the optimal procedure, it remains possible to provide small losses of accuracy of the implemented algorithms in comparison with the optimal algorithm, which provides the highest, potential precision.

Further research is proposed to carry out on the line of the development estimation methods for inertial navigation systems error signals.

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РОЗРОБКА ОПТИМАЛЬНОГО АЛГОРИТМУ ОБРОБКИ НАВІГАЦІЙНОЇ ІНФОРМАЦІЇ

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Показано, що призначенням навігаційної системи або комплексу навігаційних систем будь-якого об'єкта є вироблення в результаті проведення вихідних вимірювань і їх подальшої обробки вектора навігаційних параметрів, що використовується у подальшому в системах управління рухом об'єкта. Вироблення навігаційних параметрів у таких системах основане на алгоритмі «ідеальної роботи», який при безнадмірній системі датчиків первинної інформації (вимірювачів швидкості та курсу, а також прискорення та кутів, які задають положення гіроскопів) і в припущенні відсутності похибок цих датчиків забезпечує безпомилкове вироблення навігаційних параметрів. Реалізація алгоритму «ідеальної роботи» забезпечує незбуреність систем корисним сигналом, за який виступають дійсні значення навігаційних параметрів. Незважаючи на те, що незалежність помилок систем від істинного руху об'єкта з урахуванням мультиплікативних складових похибок, неминучого згладжування високочастотних переміщень об'єкта за рахунок інерційності датчиків виконується наближено, алгоритм роботи системи зчислення координат і інерціальної системи прагнуть максимально наблизити до алгоритму «ідеальної роботи». З цією метою у статті обґрунтований математичний апарат для розробки оптимального алгоритму обробки навігаційної інформації. Запропонований оптимальний алгоритм обробки навігаційної інформації для забезпечення необхідної точності визначення положення об'єкта на маршруті руху.

Ключові слова*: навігаційна система, навігаційна інформація, алгоритм обробки, позиція об'єкта, маршрут руху.*

РАЗРАБОТКА ОПТИМАЛЬНОГО АЛГОРИТМА ОБРАБОТКИ НАВИГАЦИОННОЙ ИНФОРМАЦИИ

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Показано, что назначением навигационной системы или комплекса навигационных систем любого объекта является выработка в результате проведения выходных измерений и их последующей обработки вектора навигационных параметров, используемого в дальнейшем в системах управления движением объекта. Выработка навигационных параметров в таких системах основано на алгоритме «идеальной работы», который при определенной системе датчиков первичной информации (измерителей скорости и курса, а также ускорение и углов, задающих положение гироскопов) и в предположении отсутствия погрешностей этих датчиков обеспечивает безошибочное выработки навигационных параметров. Реализация алгоритма «идеальной работы» обеспечивает невозбудимость систем полезным сигналом, за который выступают действительные значения навигационных параметров. Несмотря на то, что независимость ошибок систем от истинного движения объекта с учетом мультипликативных составляющих погрешностей, неизбежного сглаживания высокочастотных перемещений объекта за счет инерционности датчиков выполняется приближенно, алгоритм работы системы счисления координат и инерциальной системы стремятся максимально приблизить к алгоритму «идеальной работы». С этой целью в статье обоснован математический аппарат для разработки оптимального алгоритма обработки навигационной информации. Предложен оптимальный алгоритм обработки навигационной информации для обеспечения требуемой точности определения положения объекта на маршруте движения.

Ключевые слова*: навигационная система, навигационная информация, алгоритм обработки, позиция объекта, маршрут движения.*