YDK 004.942+674.047

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# MATHEMATICAL MODELING OF STRESSED-STRAINED RELAXATION FIELDS IN WOOD DRYING PROCESS

*Developed methods of synthesis and analysis of stressed-strained state of wood in drying process with taking account of its rheological behaviour, which allowed to set and quantitatively to describe influence conformities of gigroscopic moisture, its gradient and form of connection with material, structural anisotroption, geometrical dimensions and density on distribution of viscous-elastic and residual strains for falling and constant speeds of wood drying with constant speeds of wood drying with constant and changeable potentials of mass heat transfer.*

Keywords: *modeling, stressed-strained state, wood drying, changeable potentials of mass heat transfer, finite element method.*

Actuality of problem

Development of drying methods and analysis of stressed-strained relaxation processes in capillary-porous materials with changeable potentials of mass heat transfer assists decision of important science technical problem, concerned with rational choice and usage of technological processes of hydrothermic wood and wood composites treatment with at the same time supplying necessary quality indexes of these materials. Solving of this problem complicates because of high hydrophobicity, considerable changebility of structural and physic mathematical properties in anisotropy directions.That’s why researches of temperature-moistural fields influence on strains distribution or deformation in wood depending on anisotropy of its physic mathematical properties.

Models that describe such processes are too complicated for analytical searching for its analytical decision. It stipulates development of numerous algorithms and applicable software.

Analysis of known results

In works [1, 2] on base of termodynamics of irreversible processes was proposed system of differential equations that describe associate stressed-strained relaxation and mass heat transfer processes in capillary-porous colloidal materials. Among works dedicated to problem of two dimensional distribution of temperature-moistural fields numerical modeling in wood drying process with constant mass heat transfer coefficients we can name [3, 4]. In researches [5] was made modelling of anisotropic and nonlinear dependent from physic mathematical properties of material, field temperature and moisture content. This article continues these researches and proposes application of finite element method for numerical modeling of stressed-strained relaxational and mass heat transfer fields distribution in wood drying process.

Task formulation. Physic mathematical model.

Two dimensional model is expedient to examine also from considering that lumber dimensions along fibres are always bigger than across. Nonstationary task of heat moisture change and task of stressed-strained relaxational fields distribution are considered for drying time changing on interval $\tau \in [0,\theta]$ in region $\Omega = \{\mathbf{x} = (x_1, x_2) : x_i \in [0,a]\times[0,b], i = 1,2\}$, that is presented by rectangular wooden beam with the center in the beginning of coordinates ( fig 1).

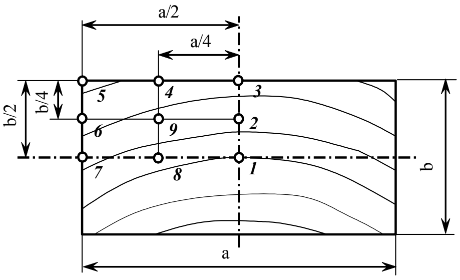



Fig 1. scheme of wooden beam cross section (a,b – halfs of geometric dimensions and location of characteristic points in the section of the material

Temperature distribution $T(x_1,x_2,\tau)$ and moisture content $U(x_1,x_2,\tau)$ in the case of absence of gradient of general pressure is described by the system of differential equations in partial derivative with appropriate initial and boundary conditions:

$$\begin{aligned} c\rho\frac{\partial T}{\partial \tau} &= \frac{\partial}{\partial x_1}\left(\lambda_1\frac{\partial T}{\partial x_1}\right)+\frac{\partial}{\partial x_2}\left(\lambda_2\frac{\partial T}{\partial x_2}\right)+\varepsilon\rho_0 r\frac{\partial U}{\partial \tau}; \\ \frac{\partial U}{\partial \tau} &= \frac{\partial}{\partial x_1}\left(a_1\frac{\partial U}{\partial x_1}\right)+\frac{\partial}{\partial x_2}\left(a_2\frac{\partial U}{\partial x_2}\right)+\frac{\partial}{\partial x_1}\left(a_1\delta\frac{\partial T}{\partial x_1}\right)+\frac{\partial}{\partial x_2}\left(a_2\delta\frac{\partial T}{\partial x_2}\right). \end{aligned} \tag{1}$$

Initial conditions:

$$T\big|_{\tau=0}=T_0;\ U\big|_{\tau=0}=U_0. \tag{2}$$

Boundary conditions:

$$\begin{aligned} &\lambda_i\frac{\partial T}{\partial x}\bigg|_{x_i=l_i}+\rho_0(1-\varepsilon)\beta_i(U\big|_{x_i=l_i}-U_p)=\alpha_i(t_c-T\big|_{x_i=l_i}); \quad \frac{\partial T}{\partial x}\bigg|_{x_i=0}=0; \\ &\left(a_i\frac{\partial U}{\partial x}+a_i\delta\frac{\partial T}{\partial x}\right)\bigg|_{x_i=l_i}=\beta_i(U_p-U\big|_{x_i=l_i}); \quad \left(\alpha_i\frac{\partial U}{\partial x}+\alpha_i\delta\frac{\partial T}{\partial x}\right)\bigg|_{x_i=0}=0, \quad i=1,2, \end{aligned} \tag{3}$$

where $T_0(x_1,x_2)$, $U_0(x_1,x_2)$ – initial temperature distribution and moisture content in material; $u_p(T,\varphi)$ – equilibrium moisture; $c(T,U)$ – heat capacity; $\rho(U)$ – density; $\lambda_1(T,U)$, $\lambda_2(T,U)$ – heat conductivity coefficient in anisotropy directions; $\varepsilon$ – phase changing coefficient; $\rho_0$ – basic density; $r$ – specific heat of evaporation; $\delta(T,U)$ – thermogradient coefficient; $a_1(T,U)$, $a_2(T,U)$ – hydraulic conductivity coefficients in anisotropy directions; $\alpha_1(t_c,v)$, $\alpha_2(t_c,v)$ – heat exchange coefficients and $\beta_1(t_c,\varphi,v)$, $\beta_2(t_c,\varphi,v)$ – moisture exchange coefficients, that depend on $t_c$, $\varphi$ and $v$ – ambient temperature, relative air moisture and speed of drying agent movement accordingly.

It is significant that for numerical decision of the equations system (1) – (3) were used dependences of heat physical characteristics of wood from temperature and moisture with the help of approximation dependences [3] at the moment of calculation on time $\tau$.

Algorithm of numerical decision of initial-boundary task (1)-(3) was considered in previous article co-authors [5]. In this article were also given results of numerical finding of temperature distribution $T(x_1,x_2,\tau)$ and moisture content $U(x_1,x_2,\tau)$, that's why later we shall consider these functions as known in region $\Omega$ and every moment of time $\tau\in[0,\theta]$, let's formulate task for determination of stressed-strained state of wood in drying process. It's necessary to find displacement vector components $\mathbf{u}=(u_1,u_2)^T$, that satisfies in region $\Omega$ equation of equilibrium:

$$\mathbf{B}^T \mathrm{y}=0 \tag{4}$$

Boundary conditions ( that take into account the symmetry of the task region $\Omega$ ) are:

$$u_i\big|_{x_i=0}=0;, \tag{5}$$

$$\sigma_{ii}\big|_{x_i=1_i}=0. \tag{6}$$

Here are set notations: $\mathbf{y}=(\sigma_{11},\sigma_{22},\sigma_{12})^T$ – strains component vector, $\mathbf{B}$ – matrix of differential operators

$$\mathbf{B}^T=\begin{bmatrix} \dfrac{\partial}{\partial x_1} & 0 & \dfrac{\partial}{\partial x_2} \\ 0 & \dfrac{\partial}{\partial x_2} & \dfrac{\partial}{\partial x_1} \end{bmatrix}.$$

Correlation between movements and vector of deformation $\mathbf{e}=(\varepsilon_{11},\varepsilon_{22},\varepsilon_{12})^T$ is written this way:

$$\mathbf{e}=\mathbf{Bu}. \tag{7}$$

For describing deformation processes in viscous elastic bodies, to which concerns wood, was used hereditary theory of elasticity [6]. It's correlation describes connection between components of strains and deformations in wood drying process, that is in tensorial form with the help of formula

$$\sigma(t) = C\left(\varepsilon - \varepsilon_T\right) - C\int_0^t R(t,\tau)\varepsilon(\tau)d\tau, \tag{8}$$

or in scalar form with the help of formulas:

$$\sigma_{11}(t) = C_{11}\left(\varepsilon_{11} - \varepsilon_{T1}\right) - C_{11}\int_0^t R_{11}(t,\tau)\varepsilon_{11}(\tau)d\tau + C_{12}\left(\varepsilon_{22} - \varepsilon_{T2}\right) - C_{12}\int_0^t R_{12}(t,\tau)\varepsilon_{22}(\tau)d\tau;$$

$$\sigma_{22}(t) = C_{21}\left(\varepsilon_{11} - \varepsilon_{T1}\right) - C_{21}\int_0^t R_{21}(t,\tau)\varepsilon_{11}(\tau)d\tau + C_{22}\left(\varepsilon_{22} - \varepsilon_{T2}\right) - C_{22}\int_0^t R_{22}(t,\tau)\varepsilon_{22}(\tau)d\tau;$$

$$\sigma_{12}(t) = 2C_{33}\left(\varepsilon_{12} - \varepsilon_{T3}\right) - 2C_{33}\int_0^t R_{33}(t,\tau)\varepsilon_{12}(\tau)d\tau,$$

where $\varepsilon_T = \begin{bmatrix} \varepsilon_{T1} \\ \varepsilon_{T2} \\ 0 \end{bmatrix} = \begin{bmatrix} \alpha_1\Delta T + \beta_1\Delta U \\ \alpha_2\Delta T + \beta_2\Delta U \\ 0 \end{bmatrix}$ – deformations vector, that caused by changeable gradients of temperature $\Delta T$ and moisture content $\Delta U$ accordingly. Exactly these deformations are the main source of strains formation in wood during drying process. For anisotropic body in case of plain stressed-strained state, elasticity matrix is given by

$$\mathbf{C} = \begin{bmatrix} \dfrac{E_{11}}{1-\upsilon_1\upsilon_2} & \dfrac{\upsilon_1 E_{22}}{1-\upsilon_1\upsilon_2} & 0 \\ \dfrac{\upsilon_1 E_{22}}{1-\upsilon_1\upsilon_2} & \dfrac{E_{22}}{1-\upsilon_1\upsilon_2} & 0 \\ 0 & 0 & \mu \end{bmatrix}.$$

Here $E_{11}$, $E_{22}$ – modulus of elasticity, $\upsilon_1$, $\upsilon_2$ – Puasson coefficients, $\mu$ – modulus of rigidity.

In this task was taken into account that elasticity m atrix coefficients depend on temperature's value and on moisture content of material. Relaxation nuclei $R(t,\tau)$ is given by

$$R = R(t,\tau) = R_1(t-\tau)\cdot R_2(\tau) = \left[\sum_{j=1}^{L}\eta_j e^{-\chi_j(t-\tau)}\right]\cdot\left[\sum_{j=1}^{L}\mu_j e^{-\kappa_j(\tau-\tau_0)}\right], \tag{9}$$

where parameters $\eta_j$, $\chi_j$, $\mu_j$, $\kappa_j$, $\tau_0$, $L$ determine from minimum of quadratic deviation of experimental curves $\varepsilon(T,U,\tau)$.

Research results of deformation creeping and reverse creeping along fibres [7] allowed to plot rheological wood behaviour functions with taking into account accumulated residual deformations, that are necessary for calculation of stressed-strained lumber state in wood drying process. That's why, when we substitute correlation (7) into formula (8), and then into equation (4), we get equilibrium equations that are similar to Lyame equations, where part of additional volume forces play gradients of temperature and moisture content. So, if to add to the equations (4), (7), (8) and boundary conditions (5), (6) initial condition given by

$$u_i\big|_{\tau=0} = 0, \tag{10}$$

then we will get nonstationary task for stressed-strained state determination of dried wood.

Variational task formulation

For the task category to which belongs written above non-stationary task of stressed-strained state determination, is popular formulation on basis on Lagranzh principle (principle of a minimum of full potential energy) [8], that claims the following. Among the acceptable movings $\mathbf{u}$ of wood as viscous elastic body, which belong to space

$$H_A = \{\mathbf{u} = (u_1, u_2)^T : u_i\big|_{x_i=0} = 0, u_i \in W_2^1(\Omega), i = 1,2\},$$

are transfers that meet the location of equilibrium and give minimal value to functional of Lagranzh

$$\Pi(u) = \tfrac{1}{2}\int_\Omega \mathbf{e}^T\mathbf{C}\mathbf{e}d\Omega + \int_\Omega \mathbf{e}^T\mathbf{C}\int_0^t \mathbf{R}(t,\tau)\mathbf{e}(\tau)d\tau d\Omega - \int_\Omega \mathbf{e}^T\mathbf{C}(\mathbf{б}\Delta T + \mathbf{в}\Delta U)d\Omega. \quad (11)$$

when to substitute into functional (11) expressions (7), (8), we'll get

$$\Pi(u) = \tfrac{1}{2}\int_\Omega \mathbf{u}^T\mathbf{B}^T\mathbf{C}\mathbf{B}\mathbf{u}d\Omega + \int_\Omega \mathbf{u}^T\mathbf{B}^T\mathbf{C}\int_0^t \mathbf{R}(t,\tau)\mathbf{B}\mathbf{u}d\tau d\Omega - \int_\Omega \mathbf{u}^T\mathbf{B}^T\mathbf{C}(\mathbf{б}\Delta T + \mathbf{в}\Delta U)d\Omega, \quad (12)$$

Decision of task about minimum of functional (12) with the help of finite element method is searched in finite subspace $S_N$ of energetic space $H_A$. Basic functions are determined on quadrangles, that cover with grid region $\Omega$ and intersect between each other. In that case transfers on each element express through nodal values of transfers. So, we have:

$$u_1(\mathbf{x},\tau) = \sum_{i=1}^{N} u_{1i}(\tau)\varphi_i(\mathbf{x})\,;\; u_2(\mathbf{x},\tau) = \sum_{i=1}^{N} u_{2i}(\tau)\varphi_i(\mathbf{x}). \quad (13)$$

Let's input dissection for time using the rule $t_k = \tau_k = k\Delta\tau$, $\Delta\tau = \dfrac{\theta}{S}$, where $S$ – integer, and mark $\mathbf{u}_k = \{u_1(\tau_k), u_2(\tau_k)\}^T$. When to put correlation (13) into functional (12) and sum all finite elements, from minimum conditions of functional (12) $\delta\Pi = 0$, then we'll get on each step by time, system of linear algebraic(al) equations as:

$$\begin{aligned}&\tfrac{1}{2}\int_\Omega \mathbf{B}^T\mathbf{C}\mathbf{B}\mathbf{u}_k d\Omega + \tfrac{\Delta\tau}{2}\int_\Omega \mathbf{B}^T\mathbf{C}\mathbf{R}(t_k,\tau_k)\mathbf{B}\mathbf{u}_k d\Omega = \\ &= \int_\Omega \mathbf{u}^T\mathbf{B}^T\mathbf{C}(\mathbf{б}\Delta T + \mathbf{в}\Delta U)d\Omega - \sum_i^{k-1}\Delta\tau\int_\Omega \mathbf{B}^T\mathbf{C}\mathbf{R}(t_k,\tau_i)\mathbf{B}\mathbf{u}_i d\Omega.\end{aligned} \quad (14)$$

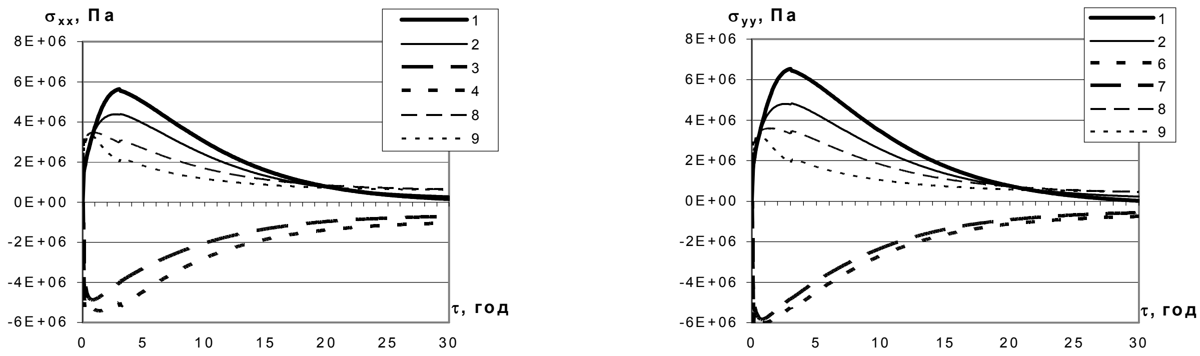
Correlation (14) makes it clear that calculated on $k$-step transfer vector components $\mathbf{u}_k$ (in the left part) depend on gradient ot temperature and moisture and from previous states of system (in the right part). Numeral calculations are realized on the object-oriented language of programming Java.

Numeric modeling results analysis

Numeral experiment of determination of dynamics of stressed-strained relaxational and heat mass exchange fields in wood drying process on the basis of the resulted algorithm of realization of physic mathematical model (1) – (9) it is resulted for wood of pine-tree ($\rho_0$ = 490 kg/м$^3$) with initial values of moisture U = 0,3% and temperature $T_0$ = 20$^0$C. ( For research of influence of wood geometric dimensions on it's stressed-strained state in drying process were taken such lumber dimensions (a = b = 25 mm; a = 2b = 50 mm; 2a = b = 50 mm). Drying agent parameters are: $t_c$ = 75$^0$C; Δt = 24$^0$C; φ = 0,3; V = 2 m/s.

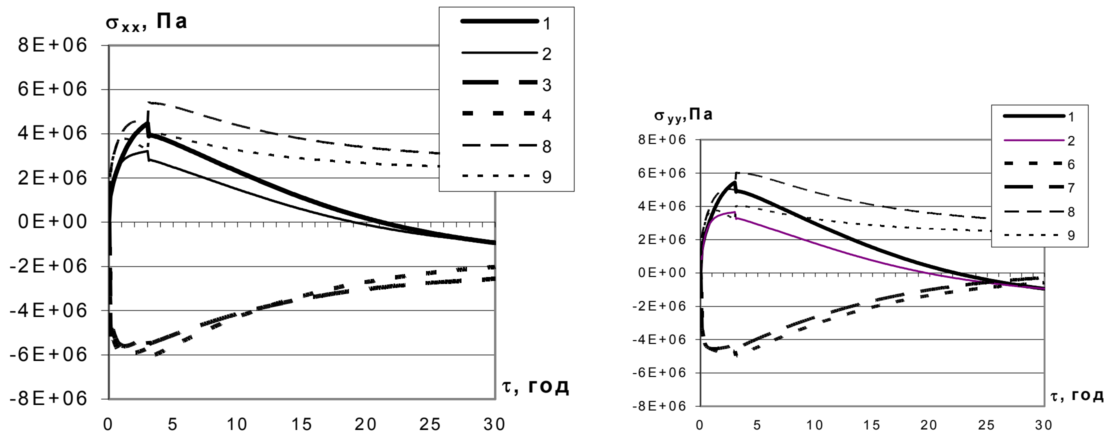
In connection with that, both moisture distribution and stressed-strained, are symmetric in relation to the center of (fig. 1), let's consider a few characteristic points for the reflection of strains dynamics in the process of moving away a moisture.

In points on a surface as a result of the rapid moving of moisture from superficial layers there are stretchings (negative) strains $\sigma_{xx}$ in points 3, 4 (fig. 2) and $\sigma_{yy}$ – in points 6, 7 , yet in an irregular period begin to fall. Strains value $\sigma_{xx}$ differ from value $\sigma_{yy}$ for correlation of width a to thickness b of lumber b/a = 1 only due to the anisotropy of properties (fig. 3). With the flow of drying time these strains fall to the minimum value which is different from a zero. The change of sign is not observed and after completion of drying process these strains though are small, but stretchings after character. The eventual size of strains on a surface is directly related to geometry of material. Judging from fig.3 for dimension 25×50 mm strains on surface $\sigma_{xx}$ proportional to strains on surface $\sigma_{yy}$ for dimension 50×25 мм. With decreasing of correlation b/a from 1 to 0,5 $\sigma_{xx}$ on surface increase approximately on 20% from -5 Mpascal to -6 Mpascal, and $\sigma_{yy}$ – decrease accordingly on 20% from -6 Mpascal to -5 Mpascal (fig. 3). The subsequent increase of correlation will result in the strains formation on a surface, which exceed the proper strains for correlation of b/a = 1. So, strains on a surface that are the least in a size will be for materials with correlation of width to the thickness of b/a from 0,5 to 1.




a b

Fig. 2. Strains dynamics in characteristic points of pine-tree wood lumber with dimension 25×25 mm during drying: a) $\sigma_{xx}$; b) $\sigma_{yy}$



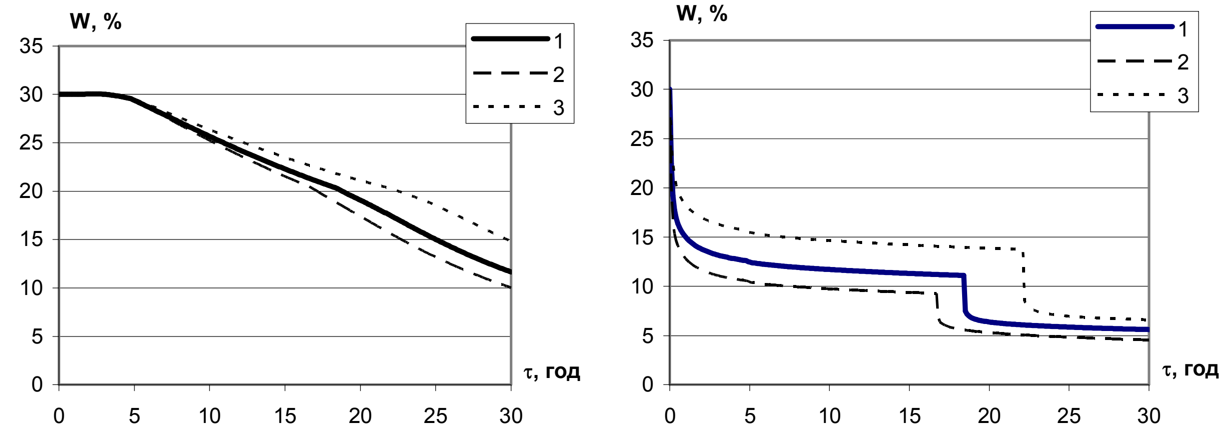

a b

Fig. 3. Strains dynamics in pine-tree wood with dimension 25×50 mm during drying in characteristic points: a) $\sigma_{xx}$; b) $\sigma_{yy}$

Unlike superficial layers, in the central layers of material at the beginning of drying moisture decreasing is quite insignificant, but deformations of shrinkage on a surface and stretchings strains which appear here, cause squeezing strains inside. In the irregular period strains inside grow quickly, though not so quickly as on a surface, and get their maximal value at $\tau = \tau_{F_0}$ (fig. 2,3) for all taken corelations b/a. With the flow of drying time strains indicated higher are fluently falling to the minimum value and depending on correlation of b/a follow to the zero (b/a = 1) or from squeezing go into stretchings (b/a≠ 1), that is that change a sign.

For research of relative moisture influence of drying agent on stressed-strained and moisture states of drying wood let's model the two-stage technological regime of drying of pine-tree wood by a size 25×50 mm, by changing relative moisture of drying agent on ±10% for each stage and leaving other parameters of technological process pursuant to normative documents.

At the beginning of drying process the change of relative moisture of drying agent does not influence on moisture inside material (fig. 4), and on the surface –differ on the size of change of equilibrium moisture (fig. 4,a). With the flow of time on the lumber surface moisture is evened to equilibrium, and in a center – begins decrease. At the reduced relative moisture of drying agent transfer to the next stage of the technological regime will happen quicker (fig.4,b, dependence 2).




a b

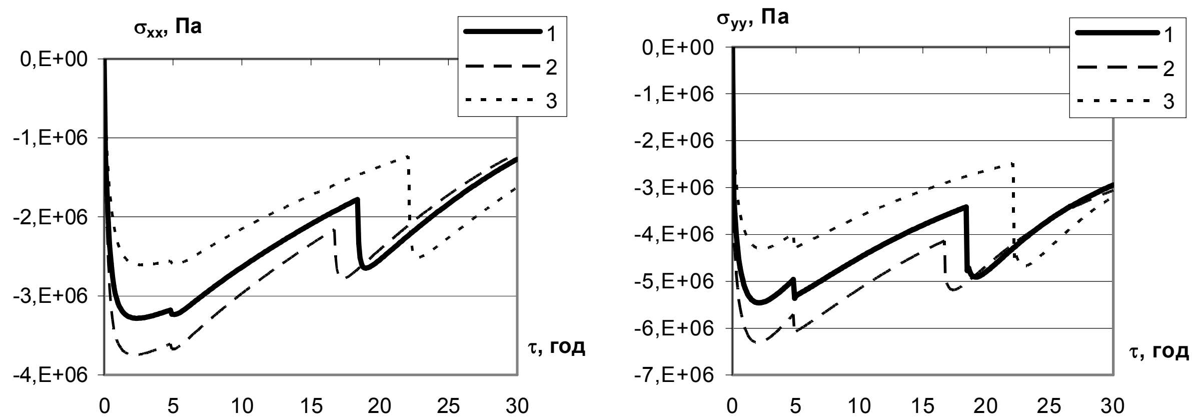
Fig. 4. Influence of relative moisture of drying agent on moisture dynamics in pine-tree wood with dimension 25×50 mm in the center 25×50 мм (a) and on the surface x=b/2 (b), де: 1 – relative moisture is after normative documents; 2 –relative moisture reduced on 10 %;
3 – increased relative moisture reduced on 10%

The change of relative moisture of drying agent of above all things stipulates bigger change of equilibrium moisture and moisture exchange coefficients in compare with the change of temperature.

That is why the strains dynamics at the beginning of drying process for the different values of relative moisture of drying agent will differ after the size of strains as follows: at decreased relative moisture to $\varphi$=0,52 accordingly to the technological regime for $\varphi$=0,62, that is approximately 19 %, the size of maximal strains on the surface $\sigma_{xx}$ will increase from -3,2 Mpascal to -3,7 Mpascal (fig. 5, a), and $\sigma_{yy}$ – from -5,3 Mpascal to -6,2 Mpascal (fig. 5, b), that is on 16-17In the center of lumber at mentioned above change of relative moisture strains will grow also: $\sigma_{xx}$ from 4,2 Mpascal to 4,8 Mpascal (fig. 5, a), and $\sigma_{yy}$ – from 4,7 Mpascal to 5,3 Mpascal (fig. 5, b), that is on 13-14%.

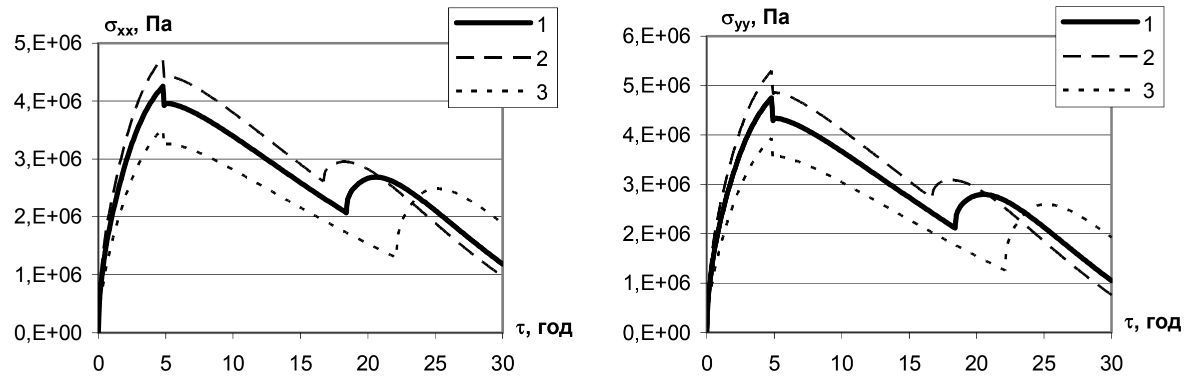
At increased relative moisture to $\varphi$=0,72 accordingly to the technological regime for $\varphi$=0,62, that is approximately 16 %, the size of maximal strains on the surface $\sigma_{xx}$ will decrease from -3,2 Mpascal to -2,6 Mpascal (fig. 5, a), and for $\sigma_{yy}$ – from -5,3 Mpascal to -4,2 Mpascal (fig. 5, b ), that is on 20-23%.

In the center of lumber at mentioned above change of relative moisture strains will fall also: $\sigma_{xx}$ from 4,2 Mpascal to 3,4 Mpascal (fig. 6, a), and $\sigma_{yy}$ – from 4,7 Mpascal to 3,9 Mpascal (fig. 6, b), that is on 20-23%.




a b

Fig. 5. Influence of relative moisture of drying agent on strains $\sigma_{xx}$ on surface when x=a/2 (a) and $\sigma_{yy}$ on surface when y=b/2 (b) pine-tree wood lumber with dimension 25×50 mm, where 1 – relative moisture is after normative documents; 2 –relative moisture reduced on 10 %; 3 – increased relative moisture reduced on 10%




a b

Fig. 6 Influence of relative moisture of drying agent on strains $\sigma_{xx}$ (a) і $\sigma_{yy}$ (b) in the center of lumber with dimension 25×50 mm, where 1 – relative moisture is after normative documents; 2 –relative moisture reduced on 10 %; 3 – increased relative moisture reduced on 10%

So, change of relative moisture of agent in a bigger or smaller side from a value, recommended in normative documents, at the beginning of technological process of drying does not influence on moisture in the center of lumber, and predefined accordingly the change of equilibrium moisture and moisture exchange coefficients considerably change the moisture value in superficial layers. The size of overfall of moisture between center and surface directly influences on the size of strains. At decreasing relative moisture of drying agent an overfall of moisture will be bigger, and consequently the size of strains will grow in all of volume of material. At increasing relative moisture of agent – the size of strains will fall accordingly.

1. Соколовський Я.І. Взаємозв'язок деформаційно-релаксаційних і тепломасообмінних процесів у капілярно-пористих тілах //Доповіді НАН України. - 1998. - №9. - С. 76-80.
2. Соколовский Я.И. Взагтосвязь деформационно-релаксационных и тепломасообменных процессов при сушке капилярно-пористых тел //Прикладная механика. - 1998. - №9. - С. 101-107.
3. Дендюк В.М., Поберейко В.П., Соколовський Я.І. Застосування методу кінцевих елементів для розрахунку нестаціонарних полів вологоперенесення у висушуваній деревині //Лісове господарство, лісова, паперова і деревообробна промисловість. – 2003. – Вип.28. -С.100-106.
4. Соколовський Я.І., Бакалець А.В. Розрахунок анізотропних нестаціонарних температурно-, вологісних полів у висушуваній деревині методом скінчених елементів // Лісове господарство, лісова, паперова і деревообробна промисловість. - 2004. - Вип. 29. - С. 109-117.
5. Бакалець А.В., Соколовський Я.І. Моделювання нелінійних тепломасообмінних процесів у висушуваній деревині методом скінченних елементів // Зб. наук.-техн. праць «Комп'ютерниї інженерія та інформаційні технології». - НУ «Львівська політехніка». - Львів. - 2005. - Вип. 543. - С. 58-63.
6. Илюшин А.А. Победря Б.Е. Основи математической теории термовязкоупругости. — М.:Наука, 1970.-280с.
7. Соколовський Я.І., Андрашек Й.В. Методика та результати експериментальних досліджень реологічної поведінки деревини //Науковий вісник: Зб. наук.-техн. праць – Львів: УкрДЛТУ. — 1999. - Вип. 9.13. - С. 15-26.
8. Кузьменко В.І. Вступ до методу скінченних елементів. -Дніпропетровськ, 2002. - 50с.
9. Уголєв Б.Н. й др. Контроль напряжений при сушке древесины. - М.: Лесная пром-сть, 1980. - 208с.