B. Diveyev, I. Dorosh, I. Vikovych, T. Koval, I. Vysochan

DIFFERENT TYPE VIBRATION ABSORBERS DESIGN FOR ELONGATED CONSOLE STRUCTURES

The main aim of this paper is different type dynamic vibration absorbers investigation and optimization. As the model of many actual systems in the literature, Timoshenko beams with various supporting conditions and DVA's of various type are used. Methods of decomposition and numerical synthesis are considered on the basis of the adaptive schemes. Design of elongated elements of machines and buildings in view of their interaction with system of dynamic vibration absorbers is under discussion. A technique is developed to give the optimal DVA's for the elimination of excessive vibration in sinusoidal and impact forced Timoshenko beams system.

Key words: *noise and vibration, elongated elements, dynamic vibration absorbers, optimization, Timoshenko beam.*

Eq. 17. Tabl. 1. Fig. 4. Ref. 24.

Introduction. Noise and vibration are of concern with many mechanical systems including industrial machines, home appliances, transportation vehicles, and building structures. Many such structures are comprised of beam like console elements. The vibration of beam systems can be reduced by the use of passive damping, once the system parameters have been identified.

Machines will typically introduce both acoustic and vibration energy into any fluids or structures surrounding the machinery. This is dangerous for both for its construction strength and human health. From two general classes of tools used to assess and optimize machines acoustic performance: test based methods and Computer Aided Engineering based methods, the second should be discussed in this paper. Large elongated elements, particularly such elements as big masts of fire machines or derricks elongated elements of agricultural machines, are dynamically unbalanced during operation due to their exposure to various factors. It is often impossible to balance this elements to reduce the vibration to an acceptable level.

A tuned mass damper (TMD), or dynamic vibration absorber (DVA), is found to be an efficient, reliable and low-cost suppression device for vibrations caused by harmonic or narrow-band excitations. In the classical theory of DVA, the primary structure is modeled as a spring-mass system; however, other models also have high interesting research and engineering application. In particular, the pendulum type system occurring as a model of a solid body with a fixed fulcrum point can play an important role in many fields such as machinery, transportation and civil engineering. The effect of a DVA on a pendulum structure with the impact masses can be very different from that on a spring-mass system.

The paper contemplates the provision of dynamic vibration absorbers (DVA) or any number of such absorbers [1, 2]. Such originally designed absorbers reduce vibration selectively in maximum vibration mode without introducing vibration in other modes. In order to determine the optimal parameters of an absorber the need for complete modeling of machine dynamics is obvious. Present research has developed a modern prediction and control methodology, based on a complex continuum theory and the application of special frequency characteristics of structures.

The two most popular computational methods used in structural dynamics are: the finite element method (FEM) and the boundary element method (BEM). While investigating higher frequency ranges for acoustic applications and using finite elements, structures are decomposed into smaller and smaller elements. The mesh size is chosen so that its largest dimension does not exceed the wavelength of the vibration. Going in this direction, when dealing with complex and large structures, the number of elements often becomes prohibitive. The calculation of eigenvalues in the range of medium frequency becomes cumbersome and time consuming.

Since the dynamic characteristics of some structural systems may be predicted by using a beam carrying single or multiple concentrated elements, the literature concerned is plenty. In [3] the vibration analysis of a uniform cantilever beam with point masses by an analytical-and-numerical-combined method is performed The frequency equations of a Bernoulli–Euler beam to which several spring–mass systems are attached in span were investigated in [4]. The approach presented in [5] was based on the method which divided the beam into segments from the point attached to the spring–mass system. For the vibration analysis of beams with various attachments, various classical analytical methods are presented to solve the similar problems [6-10]. The hybrid methods and lumped-mass (model) transfer matrix method are one of the known approaches in early years [11–14]. From reviews of the existing literature [3–14], one finds that the information regarding the vibration analysis of a non-uniform beam with various boundary conditions and carrying multiple sets of pendulum type concentrated elements is rare, thus, the purpose of this paper is to extend the theories of [15-24] to the presented structures.

Basic equations for discrete-continuum modelling. Problem of vibration fields modelling of complicated designs deformation and strain is considered for the purposes of dynamic absorption. The problem is solved on the basis of modified method of modal synthesis. The basis of these methods is in deriving solving set of equations in a normal form at minimum application of matrix operations. The essence of the first method consists in reviewing knots of junctions as compact discrete elements A_i^n for which inertial properties are taken into account without reviewing their strain, and massive connected parts - as deformable elements A_i^c, their inertion being taken into account on the basis of modal expansion.

For every point X=(x, y, z) of A_i^c we have

$$U_i(t,X)=\begin{bmatrix} q_{1i}(t)\varphi_{1i}(X) \\ \dots \\ q_{ni}(t)\varphi_{ni}(X) \end{bmatrix} . \tag{1}$$

Here $\varphi_{1i}(X),\dots,\varphi_{ni}(X)$ are coordinate functions, $q_{1i}(t),\dots,$ $q_{ni}(t)$ – corresponding independent time functions. By variation of strain U_i^c and kinetic K_i^c energies for A_i^c we have

$$\delta U_i^c = (K_i^{uc}\cdot q_i)^T \cdot \delta q_i \ , \ \delta K_i^c = (M_i^{uc}\cdot q_i)^T\cdot \delta q_i \ . \qquad q_i = [q_{1i}, q_{2i},\dots,q_{ni}]^T \tag{2}$$

By variation of strain U_i^n and kinetic K_i^n energies for connecting and attached discrete element A_i^n we have

$$\delta U_i^n = k_{ij}(q_{ij}^n(t) - q_j(t)\varphi_j(X_{ij}))\times(\delta q_{ij}^n(t) - \delta q_j(t)\varphi_j(X_{ij})). \tag{3}$$

Here X_{ij} are point of contact of discrete element A_i^n and continual element A_j^c and k_{ij} – corresponding rigidity of connection. For the mass-less joints of continual elements we must add to the strain energy such terms

$$\delta U_i^n = k_{ij}(q_i(t)\varphi_i(X_{ij}) - q_j(t)\varphi_j(X_{ij})) \quad \cdot(\delta q_i(t)\varphi_i(X_{ij}) - \delta q_j(t)\varphi_j(X_{ij})). \tag{4}$$

Kinetic energy variation of discrete one-mass element A_i^n is

$$\delta K_i^n = m_i \ \dot{q}_i^n \cdot \delta \dot{q}_i^n \ . \tag{5}$$

By Hamilton-Ostrogradsky variation equation

$$\int_{t_0}^{t_1} (\delta U - \delta K)dt = 0 \ ,$$

equating terms by independent variation parameters in (2-5) we obtain [15-24]

$$(M \ \ddot{q} \ + \overline{K} \ \cdot q \ )\cdot \delta q \ = 0\,, \tag{6}$$

a set of ordinary differential equations.

Beam modeling. For the beam modeling let us consider no uniform Timoshenko beam. The kinematical hypothesis are (for pure bending) are

$$U(X,Y,Z,t) = \gamma(x,t)\cdot Z, \quad W(X,Y,Z,t) = w(x,t). \tag{7}$$

By substitution of (7) into the variation Hamilton-Ostrogradsky equation

$$\int_0^L \left(EI \frac{\partial \gamma}{\partial x} \delta \frac{\partial \gamma}{\partial x} + GF\left(\gamma + \frac{\partial W}{\partial x} \right)\delta\gamma + \rho I \frac{\partial^2 \gamma}{\partial t^2}\delta\gamma + GF\left(\gamma + \frac{\partial W}{\partial x} \right)\delta \frac{\partial W}{\partial x} + \rho F \frac{\partial^2 W}{\partial t^2}\delta W \right) dx = F \qquad (8)$$

and taking the power series expansion for the functions

$$\gamma(x,t) = \sum_1^N q_i(t)\gamma_i(x), \quad w(x,t) = \sum_1^N p_i(t)\gamma_i(x), \qquad (9)$$

we obtain a set of ordinary differential equations for unknown time dependent functions (written in matrix form)

$$[M]\frac{d^2 \vec{r}}{dt^2} + [C]\vec{r} = \vec{f} \quad . \qquad (10)$$

Here [M] and [C] are well known mass and rigidity matrix, $\vec{r} = \left(\vec{q}, \vec{p} \right)$ – vector of unknown functions, $\vec{f}$ vector of outer forces. Vectors F or f consists of two parts 1: Fe or f e –beam dynamic loading; 2: Fz or f z – beam DVA connections terms ($F = F_e + F_Z$ *or* $f = f_e + f_Z$).

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Pendulum – system modeling. Let us consider DVA-beam system. The first –ordinary mass elastically attached to the end of the beam and second – ordinary massive pendulum attached to the end of the beam (Fig. 1)

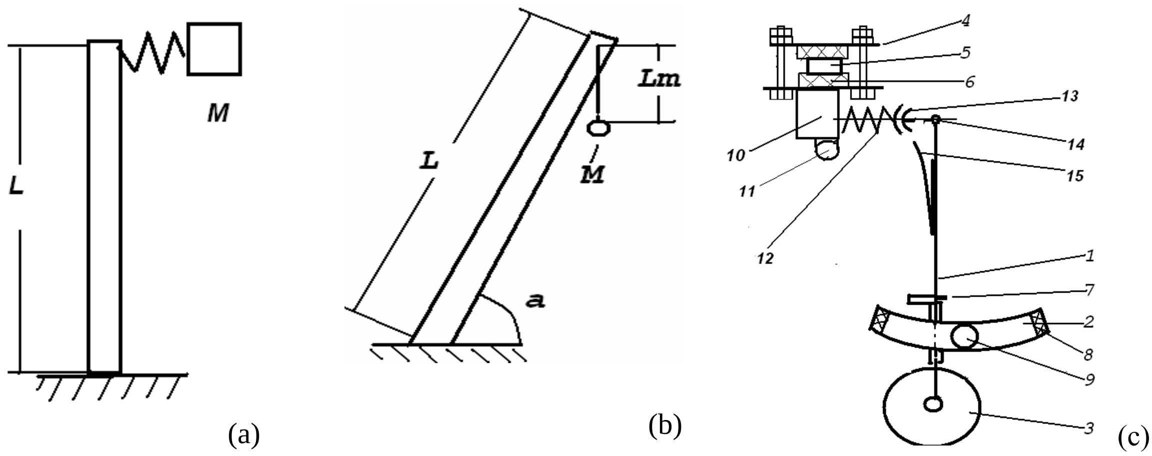


Fig. 1. a) **ordinary mass DVA ;** *b)* **–mast-single pendulum system;** *c)* **–pendulum with the additional elements**

The additional variations of the kinetic and potential energies caused by elastically suspended pendulum are

$$\delta K_m = M\left(\frac{\partial X_m}{\partial x}\delta\left(\frac{\partial X_m}{\partial x} \right) + \frac{\partial Y_m}{\partial x}\delta\left(\frac{\partial Y_m}{\partial x} \right) \right), \qquad (11)$$

$$\delta U_m = KL\delta L + Mg \,, \quad L = \sqrt{\left(X_m + W\cos(\alpha) \right)^2 + \left(L_0 - Y_m + W\sin(\alpha) \right)^2}$$

,

$$\delta U_m = K\frac{dL}{L}[\left(W + L_0 + \cos(\alpha)X_m - \sin(\alpha)Ym \right)\delta W + \left(W\cos(\alpha) + X_m \right)\delta X_m + \left(-L_0 - W\sin(\alpha) + Y_m \right)\delta Y_m] \,. (12)$$

Here M is a concentrated DVA mass, W – tip beam deflection (in normal to the beam direction) Xm, Ym – DVAa's rejections in horizontal end vertical directions, L,α – geometrical parameters (Fig.1b). Combined now the set of equation for beam (10) and (11, 12) we obtain the complete system of dynamic equations

$$[M_R]\frac{d^2\vec{R}}{dt^2}+[C_R]\vec{R}=\vec{f}\ . \tag{13}$$

Here $[M_R]$ and $[C_R]$ are complete mass and rigidity matrix, $\vec{r}=\left(\vec{q},\vec{p},X_m,Y_m\right)$ – complete vector of unknown functions, $\vec{f}$ the same vector of outer forces.

In Fig. 1c the pendulum type DVA with the additional elements is presented: 9 – an additional impact mass in container 2 with elastic elements 8; 12 – additional linear spring and 13 – additional friction damper. Anti-shock system consist of elements 4-6, 11, 15.
The additional variations of the kinetic and potential energies caused by impact mass

$$K_{amx}=M_X\left(\frac{dx_x}{dt}^2+2\frac{dx_a}{dt}\left(-x_x\sin\varphi\frac{d\varphi}{dt}+\cos\varphi\frac{dx_x}{dt}\right)+2L\frac{dx_x}{dt}\frac{d\varphi}{dt}\right) \tag{14}$$

$$\delta U_x=-M_X\ g\sin\varphi\,\delta x_x-M_X\left(L\sin\varphi+x_x\cos\varphi\right)\delta\varphi$$

The additional elastic energy of elastic elements is

$$\delta U_v=-Mm\,K_v(x_x-A)\quad |x_x|>A;\qquad \delta U_v=0\quad |x_x|<A \tag{15}$$

Numerical results, optimization. Let us at first consider the case of elastically connected DVA. The rigidity of elastic element is *k* and mass is *m.* In Fig. 2. results are presented for impact loading of beam with the elastically connected DVA. The mass of the tapered beam was 150 kg and length 15 m. Pendulum DVA is appropriately optimized (Fig. 1b). In Fig. 2. the results of optimization of first DVA (Fig. 1a) are presented by elastic and damping parameters. In Fig.3 the results of optimization of pendulum DVA by the length and the mass of the pendulum are presented. The evaluation function was the maximum tip beam deflection under 5s.

$$F_e=\max\nolimits_{T>5c}(W(T)) \tag{16}$$

DVA are appropriately optimized by genetic algorithms near the beam first eigen-frequency.

$$CiL=Max(A(f)),\ 0.7Hz<\mathrm{f}<1.45\mathrm{Hz} \tag{17}$$

The process of optimization for the DVA (Fig. 1c) is presented in Tabl. 1.

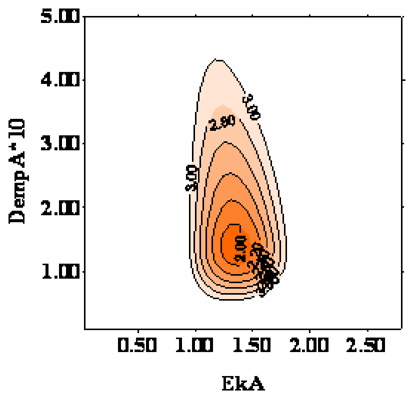


Fig. 2. **The evaluation functions for elastically clamped DVA: M=10 kg**

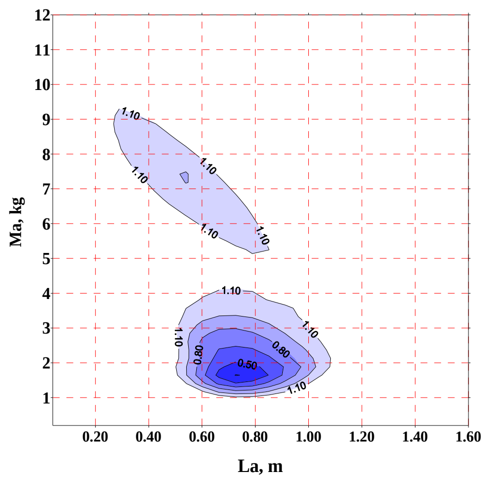


Fig. 3. **Evaluation functions map for DVA with the pendulum length La, mass Ma**

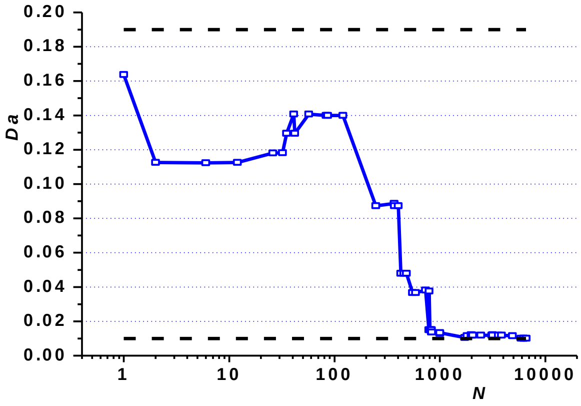
Table 1.

N = 2									
Mx	.179E+01	A	.228E+02	Dx	.143E+00	DGx	.153E+00	CiL	.182E+00
L	.290E+02	DM	.331E+00	Da	.563E-01	Ka	.825E+03		
N = 366									
Mx	.127E+01	A	.222E+02	Dx	.108E+00	DGx	.321E-01	CiL	.116E+00
L	.215E+02	DM	.377E+00	Da	.126E-01	Ka	.280E+03		
N = 2452									
Mx	.138E+01	A	.226E+02	Dx	.283E+00	DGx	.193E+00	CiL	.112E+00
L	.208E+02	DM	.423E+00	Da	.155E-01	Ka	.263E+03		

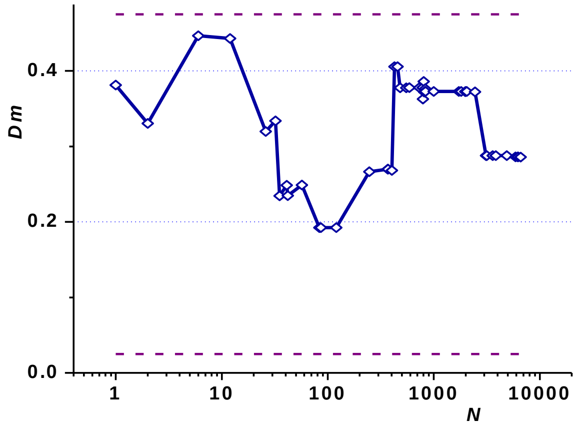
Here: Mx – additional impact mass, A – clearens, Dx – inner viscose-elastic damping in container, DGx – damping in elastic elements, L – pendulum length, DM – equivalent damping in frictional element, Da – damping in linear spring, Ka – rigidly of linear spring, CiL - the evaluation function.

In Fig. 4. the results of DVA optimization is presented in graphical forms.

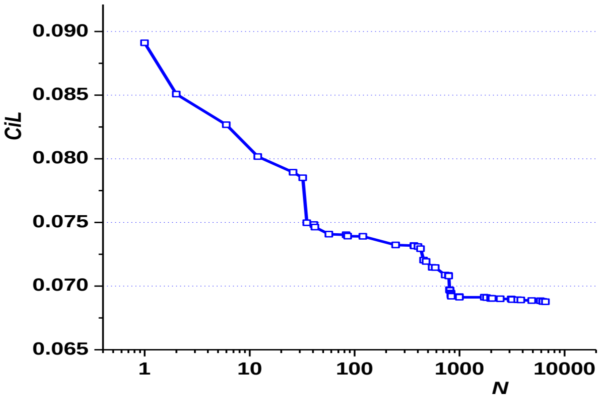
Concluding remarks. As the model of many actual systems, Timoshenko tapered beams with console supporting conditions and DVA of various type are used. However, in these applications the DVA's are frequently assumed to be elastically clamped. In the present study, a pendulum type DVA attached to the tip of a cantilevered beam thus composing the system is under study. The dynamic equation of this combined system is derived. Comparison of the numerical results with the elastically clamped DVA and pendulum type DVA case reveals the fact that this second is more preferable for some parameter combinations. The more compact pendulum-type DVA with additional elastic, damping and impact elements present better vibroabsorbing properties in the wide frequency range.



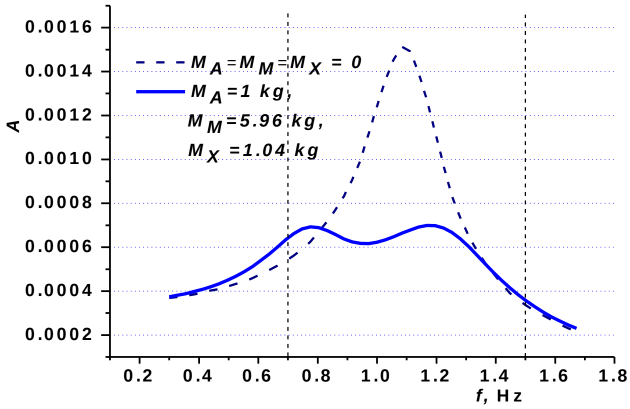
(a)



(b)



(c)



(d)

Fig. 4. **The results of DVA optimization presented in graphical forms:** *a)* **damping in linear spring 12 (see Fig. 1c);** *b* **Dm – equivalent damping in the additional friction damper 13;** *b)* **The evaluation function,** *d)* **result of optimization in the frequency range**

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