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IMPROVING THE MATHEMATICAL MODEL OF THE DYNAMICS FOR UNDERWATER VEHICLES WITH ASYMMETRICAL HULLS

The dynamic model of motion UWV is constructed. It takes into account the asymmetry of hull's shells, infiltration processes and regular surface waves as a system of nonlinear of differential equations. Solution as a convergence sequence is received, convergence of this sequence as quadratic convergence is proved. Estimation error as a function of the magnitude of initial velocity of translational and rotational motions value of over time and other kinematics and dynamic parameters is obtained.
Key words: *underwater vehicles, model of motion, asymmetrical hulls' shells, recurrence sequence, convergence, error.*

Побудовано модель руху ПА, яка враховує асиметричність корпусів, інфільтраційні процеси та поверхневі хвилі у вигляді системи нелінійних диференційних рівнянь. Подано розв'язок у вигляді послідовності наближень, обґрунтовано збіжність цієї послідовності, доведено її двосторонність та квадратичність. Отримано оцінки похибки, як функції величини початкових швидкостей поступального та обертального рухів, величини проміжку часу та інших кінематичних і динамічних параметрів.
Ключеві слова: *підводний апарат, модель руху, асиметричні корпусні оболонки, рекурентна послідовність, збіжність, похибка.*

Построенна математическая модель движения ПА, которая учитывает ассиметричность корпусов, инфильтрационные процессы и поверхностные волны в виде системы нелинейных дифференциальных уравнений. Получено решение в виде последовательности приближений, обоснованна ее сходимость, доказана ее двосторонность и квадратичность. Получены оценки ошибки, как функции величины начальных скоростей поступательного и вращательного движения, величины промежутка времени и других кинематических и динамических параметров.
Ключевые слова: *подводный аппарат, модель движения, асимметричные корпусные оболочки, рекуррентная последовательность, сходимость, ошибка.*

The global operating experience of uninhabited underwater vehicles (UUWV) [1-7] during the execution of search and rescue – repairs and inspection in river and sea conditions proves, that changes in geometric characteristics and deviations of assumptions about the symmetry of the hull's shell significantly alter the dynamical properties of these underwater vehicles (UWV). These UWV can be classified into three main types: technological; inspection and monitoring; carriers of UUWV and equipment [8-15]. They include systems and accessories, manufacturing equipment and tool that changes the shape and their spatial arrangement of hulls thereon during operation, as well as the relative orientation of the individual parts of the body or board of industrial equipment. Producing, on the one hand, additional forces and moments, compensation of which is not possible [14-15] and on the other hand, making it difficult to account of the inertial properties for the automated control systems of technological operations of such devices, changes of ratio asymmetry and changes of geometric properties become the main reasons of uncontrolled dynamics. The lack of methods taking into account in mathematical models of these features formulate the preconditions for the error in the control of mechanical movements and thus leads to an emergency, or complete failure, or partial damage leads to specific units and equipment. In connection with the above, the task of constructing a mathematical model for the description of dynamics UWR and application its into decision making system is relevant [14-15; 18]. Specific requirements on reliability and safety of UWV convert task of design automatic control system or automated control systems of the technological and complex equipment that provides a workflow into sophisticated complex of scientific and technical problems. It in consequence of the diversity in design, the complexity of the kinematic and dynamic relationships of objects, specific conditions of movement of both the device and its main and auxiliary process equipment requires considerable labor to design control systems, feedback and internal sensors of states and actuators. However, its solution must to precede the construction of adequate models of UWR with asymmetric hulls, because so far in the literature such methods are none. The next reason is due to the need to simultaneously consider, when modeling changes such as inertial and hydrodynamic characteristics, which may be carried out only in the presence of appropriate methods for solving the system of nonlinear differential equations, that allow access to the vector shape representation models and systems of nonlinear matrix logic equations [7; 8; 10; 14-18]. Existing at present in the practice of design models are imperfect as they can be describe dynamic for a limited number of values of inertia moments and factors added mass and asymmetry of body doesn't take into account at all.

The main unresolved problem is to build a model of the UWR movement as one from the elements of the object management in underwater technology for automated intelligent control systems with associative memory, in which takes into account the asymmetry of hull's shell and infiltration processes and waves on the surface of water.

The purpose of this paper is derivation equations of motion of the UWV with asymmetrical hull and solution of the system of differential equations and its transformation in a convenient form for use in intelligent control systems with associative memory.

Derivation equations of motion of the UWV with asymmetrical hull, what is formed shell, which contains holes. In the recent decades a wide range of articles [1; 6; 7; 14-17] have been published, which dealt with various aspects creation of mathematical models of the calculation of individual characteristics and design of UWV management [1; 6; 7; 15], but nevertheless problem of the influence asymmetry effects on the form of equations of dynamics and values of the coefficients added mass set in [14; 16-18], remains not sufficiently studied and proven to engineering design procedures. It is particularly relevant for the job of UWV in which the relative asymmetric property of mass distribution is composed (30-40 %) from the total weight. This asymmetry is due to the heterogeneity of the density asymmetry in cross-sectional area of UWR and deviation ratio of the volume on both sides of the coordinate planes to the total volume, that is limited light hull and which is established of values in range (10-15 %).

Put the goal of this article to deduce equations of motion for the UWV with the influence of the effects of asymmetry of the hull, and to explore the solution of the equations of dynamics. Let notice the letter O point that is situated inside the hull of UWV, the reference point is chosen as origin the coordinate system $Oxyz$ and which is rigidly connected with the UWV. Suppose, that the UWV is absolutely solid body and the beginning of coordinate point – O moving forward with speed $\bar{v}_o$ and UWV rotates with angular velocity – $\bar{\Omega}$. Consider an arbitrary point – i of a rigid body, the position of which a given radius vector $\bar{r}_{oi}$ and which is allocated in the vicinity of element mass Δm_i, moving with a speed $\bar{v}_i$. Consider an inertial Cartesian coordinate system $Ox'y'z'$ in which the UWV moves. Denote the radius vector $\bar{r}_o$ defining the position in it of the origin of the coordinate system related $Oxyz$ (Fig. 1). Define vectors $\bar{K}_1, \bar{M}_1, \bar{L}_1$. and $\bar{K}, \bar{M}, \bar{L}$ – an impulse, momentum, angular momentum in the inertial and bounded with UWV coordinate system accordingly as the vector sum by the expressions:

$$\bar{K}_1 = \sum_{i=1}^{N} \Delta m_i \bar{v}_i = \sum_{i=1}^{N} \rho_i \Delta V_i \left(\bar{v}_o + \bar{v}_{oi}\right) = m\bar{v}_o + \bar{K}; \quad (1)$$

$$\bar{L}_1 = \sum_{i=1}^{N} \left(\bar{r}_o + \bar{r}_{oi}\right) \times \Delta m_i \bar{v}_i = \sum_{i=1}^{N} \left(\bar{r}_o + \bar{r}_{oi}\right) \times \rho_i \Delta V_i \bar{v}_i = \bar{L} + \bar{r}_o \times \bar{K}_1; \quad (2)$$

$$\bar{M}_1 = \sum_{i=1}^{N} \left(\bar{r}_o + \bar{r}_{oi}\right) \times \bar{R}_i = \bar{M} + \bar{r}_o \times \bar{R} = \bar{M} + \bar{r}_o \times \frac{d\bar{K}_1}{dt}. \quad (3)$$

Further, considering the main vector of external forces $\bar{R}$ and the main moment of external forces $\bar{M}$ are known, in accordance with the laws of mechanics, by analogy with the approach to describe the motion of a rigid body in an infinite incompressible fluid is contained in [1; 13; 14], we write the equation UWV movement in the classical formulation in a coordinate system $Ox'y'z'$:

$$\frac{d\bar{K}_1}{dt} = \bar{R}; \quad \frac{d\bar{L}_1}{dt} = \bar{M}_1, \quad (4)$$

where $\bar{R}$, $\bar{M}_1$ – under the main vectors of force and momentum. For the coupled system of coordinates $OXYZ$, equation (4) transform on the based of relations (1) – (3) and the properties of the module radius vector $\bar{r}_{oi}$ and angular velocity of any point i, that for a absolutely rigid body is the same vector, and the vector product of the velocity vector to vector velocity is zero, and the growth of any vector in the moving coordinate system is defined in addition to changing the origin, and even the vector product of the vector of angular speed on the vector alone

$$\frac{d\bar{K}}{dt}+\bar{\Omega}\times\bar{K}=\bar{R} \ ;$$

$$\frac{d\bar{L}}{dt}+\bar{\Omega}\times\bar{L}+\bar{v}_o\times\left(m\bar{v}_o+\bar{K}\right)+ +\bar{r}_o\times\frac{d\bar{K}_1}{dt}=\bar{M}+\bar{r}_o\times\bar{R} .$$

Finally, we are written:

$$\frac{d\bar{K}}{dt}+\bar{\Omega}\times\bar{K}=\bar{R} \ ;$$

$$\frac{d\bar{L}}{dt}+\bar{\Omega}\times\bar{L}+\bar{v}_o\times\bar{K}=\bar{M} \ . \quad (5)$$

We have been able to reduce the last two vectors' equations to six scalar, projecting the vector product of the elements of the first row – there orthogonal unity basis vectors in bounded coordinate system, followed in projections on the according axis [1; 13; 14]:

$$\frac{dK_x}{dt}+\omega_y K_z-\omega_z K_y=R_x;$$

$$\frac{dK_y}{dt}+\omega_z K_x-\omega_x K_z=R_y;$$

$$\frac{dK_z}{dt}+\omega_x K_y-\omega_y K_x=R_z; \quad (6)$$

$$\frac{dL_x}{dt}+\omega_y L_z-\omega_z L_y+v_y K_z-v_z K_y=M_x;$$

$$\frac{dL_y}{dt}+\omega_z L_x-\omega_x L_z+v_z K_x-v_x K_z=M_y;$$

$$\frac{dL_z}{dt}+\omega_x L_y-\omega_y L_x+v_x K_y-v_y K_x=M_z.$$

To further simplification the equations of motion will have been using of the substitution of the projection of the angular momentum of the momentum through the kinetic energy – W :

$$K_x=\frac{\partial W}{\partial v_x};K_y=\frac{\partial W}{\partial v_y};K_z=\frac{\partial W}{\partial v_z};$$

$$L_x=\frac{\partial W}{\partial \omega_x};L_y=\frac{\partial W}{\partial \omega_y};L_z=\frac{\partial W}{\partial \omega_z}. \quad (7)$$

These relations are the result of fundamental definitions: the concept of momentum; angular momentum; kinetic energy, due to of them system (6) is reduced to the system of differential equations by differentiating of expressions of the kinetic energy corresponding projections of the velocity vectors. This approach to the transformation of the equations of motion the UWV are realized, as it have been demonstrated in [1; 7; 13; 14]:

– the first represents the UWV as an integrated energy system consisting from UWV and fluid moves both outside and inside;

– in the second it allows to submit depending projection vector impulse and its moment via components of vectors of angular and linear velocities in explicit form, and under the condition of presence expressions of potentials to take into account the effect of surface waves.

Thus, for the kinetic energy of the system (UWV and fluid) is generalized

$$W=0.5[v^T\Omega^T](I_A+I_B^u)\begin{bmatrix} v \\ \Omega \end{bmatrix}, \quad (8)$$

where I_B^u – the matrix of inertia of the liquid due to the properties of the sum of matrices containing elements:

$$\lambda_{ik}=\rho\left(\int_{v_1}\lambda'_{ik}+\int_{v_2}\lambda'_{ik}\right). \quad (9)$$

This presentation explains the dependence of the coefficients added mass from flow conditions, speed of motion UWV, because they determine the field of velocity in surroundings fluid moves both outside and inside of UWV. Integral representation of the coefficients added mass allows them to obtain numerical values for any UWV at the design stage for a given geometry light hull and calculate the impact on them of parameters of surface regular waves [1; 17; 18].

Served on the basis of said kinetic energy expression similar to [1; 14]. With the inertia matrix, we can write the expression of the kinetic energy of the system fluid and the UWV, that in its moves subject to value of static moments of inertia

$$W=0.5\begin{bmatrix} (a_{11}v_x^2+a_{22}v_y^2+a_{33}v_z^2+ \\ +a_{44}\omega_x^2+a_{55}\omega_y^2+a_{66}\omega_z^2)+ \\ +v_x\left(a_{15}\omega_y+a_{16}\omega_z\right)+ \\ +v_y\left(a_{24}\omega_x+a_{26}\omega_z\right)+ \\ +v_z\left(a_{34}\omega_x+a_{35}\omega_y\right)+ \\ +\omega_x\begin{pmatrix} a_{42}v_y+a_{43}v_z+ \\ +a_{45}\omega_y+a_{46}\omega_z \end{pmatrix}+ \\ +\omega_y\begin{pmatrix} a_{51}v_x+a_{53}v_z \\ a_{54}\omega_x+a_{56}\omega_z \end{pmatrix}+ \\ +\omega_z\begin{pmatrix} a_{61}v_x+a_{62}v_y+ \\ +a_{64}\omega_x+a_{65}\omega_y \end{pmatrix}+ \end{bmatrix}. \quad (10)$$

Then after differentiation of the expression of the kinetic energy we obtain expressions projections impulse and angular momentum

$$K_x=a_{11}v_x+\frac{1}{2}\left[\begin{pmatrix} a_{15}+ \\ +a_{51} \end{pmatrix}\omega_y+\begin{pmatrix} a_{16}+ \\ +a_{61} \end{pmatrix}\omega_z\right];$$

$$K_y=a_{22}v_y+\frac{1}{2}\left[\begin{pmatrix} a_{24}+ \\ +a_{42} \end{pmatrix}\omega_x+\begin{pmatrix} a_{26}+ \\ +a_{62} \end{pmatrix}\omega_z\right];$$

$$K_z=a_{33}v_z+\frac{1}{2}\left[\begin{pmatrix} a_{34}+ \\ +a_{43} \end{pmatrix}\omega_x+\begin{pmatrix} a_{35}+ \\ +a_{53} \end{pmatrix}\omega_y\right]; \quad (11)$$

$$L_x = a_{44}\omega_x + \frac{1}{2}\left[\begin{matrix}\begin{pmatrix} a_{24}+ \\ +a_{42}\end{pmatrix} v_y + \begin{pmatrix} a_{34}+ \\ +a_{43}\end{pmatrix} v_z + \\ +\begin{pmatrix} a_{45}+ \\ +a_{54}\end{pmatrix}\omega_y + \begin{pmatrix} a_{46}+ \\ +a_{64}\end{pmatrix}\omega_z\end{matrix}\right];$$

$$L_y = a_{55}\omega_y + \frac{1}{2}\left[\begin{matrix}\begin{pmatrix} a_{15}+ \\ +a_{51}\end{pmatrix} v_x + \begin{pmatrix} a_{35}+ \\ +a_{53}\end{pmatrix} v_z + \\ +\begin{pmatrix} a_{45}+ \\ +a_{54}\end{pmatrix}\omega_x + \begin{pmatrix} a_{56}+ \\ +a_{65}\end{pmatrix}\omega_z\end{matrix}\right];$$

$$L_z = a_{66}\omega_z + 0.5\left[\begin{matrix}\begin{pmatrix} a_{16}+ \\ +a_{61}\end{pmatrix} v_x + \begin{pmatrix} a_{26}+ \\ +a_{62}\end{pmatrix} v_y + \\ +\begin{pmatrix} a_{46}+ \\ +a_{64}\end{pmatrix}\omega_x + \begin{pmatrix} a_{56}+ \\ +a_{65}\end{pmatrix}\omega_y\end{matrix}\right],$$

and in compliance with equation (6) is written with:

$$\frac{dv_x}{dt} = \frac{1}{a_{11}}\left\{R_x - G_{rx}(t)\right\};$$
$$\frac{dv_y}{dt} = \frac{1}{a_{22}}\left\{R_y - G_{ry}(t)\right\};$$
$$\frac{dv_z}{dt} = \frac{1}{a_{33}}\left\{R_z - G_{rz}(t)\right\};$$
$$\frac{d\omega_x}{dt} = \frac{1}{a_{44}}\left\{M_x - G_{mx}(t)\right\};$$
$$\frac{d\omega_y}{dt} = \frac{1}{a_{55}}\left\{M_y - G_{my}(t)\right\};$$
$$\frac{d\omega_z}{dt} = \frac{1}{a_{66}}\left\{M_z - G_{mz}(t)\right\}. \qquad (12)$$

The resulting system of equations (12) is nonlinear, since the projection of the main vector of external forces include frictional forces, which are proportional to the square of the projection vectors of linear and angular velocities. To reduce of size in recording system of equations (12) next functions are denoted:

$$G_{rx}(t) = \frac{1}{2}\frac{d}{dt}\left[\begin{pmatrix} a_{15}+ \\ +a_{51}\end{pmatrix}\omega_y + \begin{pmatrix} a_{16}+ \\ +a_{61}\end{pmatrix}\omega_z\right] +$$
$$+\omega_y\left\{a_{33}v_z + \frac{1}{2}\left[\begin{pmatrix} a_{34}+ \\ +a_{43}\end{pmatrix}\omega_x + \begin{pmatrix} a_{35}+ \\ +a_{53}\end{pmatrix}\omega_y\right]\right\} - ;$$
$$-\omega_z\left\{a_{22}v_y + \frac{1}{2}\left[\begin{pmatrix} a_{24}+ \\ +a_{42}\end{pmatrix}\omega_x + \begin{pmatrix} a_{26}+ \\ +a_{62}\end{pmatrix}\omega_z\right]\right\}$$

$$G_{ry}(t) = \frac{1}{2}\frac{d}{dt}\left[\begin{pmatrix} a_{24}+ \\ +a_{42}\end{pmatrix}\omega_x + \begin{pmatrix} a_{26}+ \\ +a_{62}\end{pmatrix}\omega_z\right] +$$
$$\omega_z\left\{a_{11}v_x + \frac{1}{2}\left[\begin{pmatrix} a_{15}+ \\ +a_{51}\end{pmatrix}\omega_y + \begin{pmatrix} a_{16}+ \\ +a_{61}\end{pmatrix}\omega_z\right]\right\} - ;$$
$$-\omega_x\left\{a_{33}v_z + \frac{1}{2}\left[\begin{pmatrix} a_{34}+ \\ +a_{43}\end{pmatrix}\omega_x + \begin{pmatrix} a_{35}+ \\ +a_{53}\end{pmatrix}\omega_y\right]\right\}$$

$$G_{rz}(t) = \frac{1}{2}\frac{d}{dt}\left[\begin{pmatrix} a_{34}+ \\ +a_{43}\end{pmatrix}\omega_x + \begin{pmatrix} a_{35}+ \\ +a_{53}\end{pmatrix}\omega_y\right] +$$
$$\omega_x\left\{a_{22}v_y + \frac{1}{2}\left[\begin{pmatrix} a_{24}+ \\ +a_{42}\end{pmatrix}\omega_x + \begin{pmatrix} a_{26}+ \\ +a_{62}\end{pmatrix}\omega_z\right]\right\} - ;$$
$$-\omega_y\left\{a_{11}v_x + \frac{1}{2}\left[\begin{pmatrix} a_{15}+ \\ +a_{51}\end{pmatrix}\omega_y + \begin{pmatrix} a_{16}+ \\ +a_{61}\end{pmatrix}\omega_z\right]\right\}$$

$$G_{mx}(t) = \left\{\begin{matrix}\dfrac{d}{2dt}\left[\begin{matrix}\begin{pmatrix} a_{24}+ \\ +a_{42}\end{pmatrix} v_y + \begin{pmatrix} a_{34}+ \\ +a_{43}\end{pmatrix} v_z + \\ +\begin{pmatrix} a_{45}+ \\ +a_{54}\end{pmatrix}\omega_y + \begin{pmatrix} a_{46}+ \\ +a_{64}\end{pmatrix}\omega_z\end{matrix}\right] + \\ +\omega_y L_z - \omega_z L_y + v_y K_z - v_z K_y\end{matrix}\right\};$$

$$G_{my}(t) = \left\{\begin{matrix}\dfrac{d}{2dt}\left[\begin{matrix}\begin{pmatrix} a_{15}+ \\ +a_{51}\end{pmatrix} v_x + \begin{pmatrix} a_{35}+ \\ +a_{53}\end{pmatrix} v_z + \\ +\begin{pmatrix} a_{45}+ \\ +a_{54}\end{pmatrix}\omega_x + \begin{pmatrix} a_{56}+ \\ +a_{65}\end{pmatrix}\omega_z\end{matrix}\right] + \\ \omega_z L_x - \omega_x L_z + v_z K_x - v_x K_z\end{matrix}\right\};$$

$$G_{mz}(t) = \left\{\begin{matrix}\dfrac{d}{2dt}\left[\begin{matrix}\begin{pmatrix} a_{16}+ \\ +a_{61}\end{pmatrix} v_x + \begin{pmatrix} a_{26}+ \\ +a_{62}\end{pmatrix} v_y + \\ +\begin{pmatrix} a_{46}+ \\ +a_{64}\end{pmatrix}\omega_x + \begin{pmatrix} a_{56}+ \\ +a_{65}\end{pmatrix}\omega_y\end{matrix}\right] + \\ \omega_x L_y - \omega_y L_x + v_x K_y - v_y K_x\end{matrix}\right\} \qquad (13)$$

Solution of motion equations of UWV

We apply to solution of the system the method of recurrent approximations, with limited linear approximation scheme [15]. The idea can be explain and make it feasible, if we consider presentation of conversion on the example of the first of equations:

$$\frac{dv_{x,n+1}}{dt} = \frac{1}{a_{11}}\left\{\begin{matrix} R_{xn} - \\ -G_{rxn}(t) + \begin{pmatrix} v_{x,n+1} - \\ -v_{x,n}\end{pmatrix} \times \\ \times\left[\dfrac{\partial R_x}{\partial v_x} - \dfrac{\partial G_{rx}}{\partial v_x}\right]\Bigg|_{v_x = v_{xn}}\end{matrix}\right\}. \quad (14)$$

We introduce the integrating factor

$$\exp\left(\frac{1}{a_{11}}\int_0^t \left(\frac{\partial G_{rx}}{\partial v_x} - \frac{\partial R_x}{\partial v_x}\right)\Bigg|_{v_x = v_{xn}} d\tau\right),$$

while taking into account the initial conditions we are wrote recursive solution (14) in the form:

$$v_{x,n+1} = \exp\left(\frac{1}{a_{11}}\int_0^t \left(\frac{\partial R_x}{\partial v_x} - \frac{\partial G_{rx}}{\partial v_x}\right)\Bigg|_{v_x = v_{xn}} d\tau\right) \times$$

$$\times\left\langle \begin{array}{l} v_{xo} + \\ + \dfrac{1}{a_{11}}\displaystyle\int_0^t \left\{\begin{bmatrix} R_{xn} - \\ -G_{rxn}(t) - v_{x,n}\left(\dfrac{\partial R_x}{\partial v_x} - \dfrac{\partial G_{rx}}{\partial v_x}\right)\Bigg|_{v_x = v_{xn}} \end{bmatrix} \\ \exp\left(\dfrac{1}{a_{11}}\displaystyle\int_0^s \left(\dfrac{\partial G_{rx}}{\partial v_x} - \dfrac{\partial R_x}{\partial v_x}\right)\Bigg|_{v_x = v_{xn}} d\tau\right) \end{array}\right\} ds \right\rangle .$$

The first approximation we will obtain, taking into account the initial conditions for the projection speed, which denote the second subscript zero. Denote as the number in the lower index number approach of components defined for the corresponding approximation speed, then to a first approximation we will write:

$$v_{x1} = v_{x0} + \frac{1}{a_{11}}\int_0^t \left[R_{x0}(\tau) - G_{rx0}(\tau)\right] d\tau ;$$

$$v_{y1} = v_{y0} + \frac{1}{a_{22}}\int_0^t \left[R_{y0}(\tau) - G_{ry0}(\tau)\right] d\tau ;$$

$$v_{z1} = v_{z0} + \frac{1}{a_{33}}\int_0^t \left[R_{z0}(\tau) - G_{rz0}(\tau)\right] d\tau ;$$

$$v_1 = \sqrt{v_{x1}^2 + v_{y1}^2 + v_{z1}^2} ;$$

$$\omega_{x1} = \omega_{x0} + \frac{1}{a_{44}}\int_0^t \left[M_{x0}(\tau) - G_{mx0}(\tau)\right] d\tau ;$$

$$\omega_{y1} = \omega_{y0} + \frac{1}{a_{55}}\int_0^t \left[M_{y0}(\tau) - G_{my0}(\tau)\right] d\tau ;$$

$$\omega_{z1} = \omega_{z0} + \frac{1}{a_{66}}\int_0^t \left[M_{z0}(\tau) - G_{mz0}(\tau)\right] d\tau ;$$

$$\omega_1 = \sqrt{\omega_{x1}^2 + \omega_{y1}^2 + \omega_{z1}^2} ; \quad (15)$$

$$x_1 = x_0 + v_{x0}t + \frac{1}{a_{11}}\int_0^t d\tau \int_0^\tau \begin{bmatrix} R_{x0}(q) - \\ -G_{rx0}(q) \end{bmatrix} dq ;$$

$$y_1 = y_0 + v_{y0}t + \frac{1}{a_{22}}\int_0^t d\tau \int_0^\tau \begin{bmatrix} R_{y0}(q) - \\ -G_{ry0}(q) \end{bmatrix} dq ;$$

$$z_1 = z_0 + v_{z0}t + \frac{1}{a_{33}}\int_0^t d\tau \int_0^\tau \begin{bmatrix} R_{z0}(q) - \\ -G_{rz0}(q) \end{bmatrix} dq ;$$

$$\theta_1 = \theta_0 + \omega_{x0}t + \frac{1}{a_{44}}\int_0^t d\tau \int_0^\tau \begin{bmatrix} M_{x0}(q) - \\ -G_{mx0}(q) \end{bmatrix} dq ;$$

$$\varphi_1 = \varphi_0 + \omega_{x0}t + \frac{1}{a_{55}}\int_0^t d\tau \int_0^\tau \begin{bmatrix} M_{y0}(q) - \\ -G_{my0}(q) \end{bmatrix} dq ;$$

$$\psi_1 = \psi_0 + \omega_{x0}t + \frac{1}{a_{66}}\int_0^t d\tau \int_0^\tau \begin{bmatrix} M_{z0}(q) - \\ -G_{mz0}(q) \end{bmatrix} dq .$$

Derivatives from the projection of force, which are expressions of these approximations based on the expression of module and the direction of the friction force, after differentiation and algebraic reduction, we can write:

$$\frac{\partial R_x}{\partial v_x} = -\frac{1}{2}\rho S C_x v\left(\frac{v_x^2}{v^2} + 1\right);$$

$$\frac{\partial R_y}{\partial v_y} = -\frac{1}{2}\rho S C_y v\left(\frac{v_y^2}{v^2} + 1\right);$$

$$\frac{\partial R_z}{\partial v_z} = -\frac{1}{2}\rho S C_z v\left(\frac{v_z^2}{v^2} + 1\right).$$

The second approximation we obtain, taking into account the first and so on for the recursive algorithm:

$$v_{x,2} = \frac{v_{xo}}{a_{x,1}} + \frac{1}{a_{11}a_{x,1}}\int_0^t \begin{bmatrix} R_{x1} - G_{rx1}(q) + \\ +v_{x,1}\dfrac{\rho S C_x}{2} v_1\left(\dfrac{v_{x,1}^2}{v_1^2} + 1\right) \end{bmatrix} a_{x,1} dq;$$

$$a_{x,1} = \exp\left[\frac{\rho S C_x}{2a_{11}}\int_0^t v_1\left(\frac{v_{x,1}^2}{v_1^2} + 1\right) dq\right];$$

$$v_{y,2} = \frac{v_{yo}}{a_{y,1}} + \frac{1}{a_{22}}\int_0^t \begin{bmatrix} R_{y1} - G_{ry1}(q) + \\ +v_{y,1}\dfrac{\rho S C_y}{2} v_1\left(\dfrac{v_{y,1}^2}{v_1^2} + 1\right) \end{bmatrix} a_{y,1} dq;$$

$$a_{y,1} = \exp\left(\frac{\rho S C_y}{2a_{22}}\int_0^t v_1\left(\frac{v_{y,1}^2}{v_1^2} + 1\right) dq\right);$$

$$v_{z,2} = \frac{v_{zo}}{a_{z,1}} + \frac{1}{a_{33}}\int_0^t \begin{bmatrix} R_{z1} - G_{rz1}(q) + \\ +v_{z,1}\dfrac{\rho S C_z}{2} v_1\left(\dfrac{v_{z,1}^2}{v_1^2} + 1\right) \end{bmatrix} a_{z,1} dq;$$

$$a_{z,1} = \exp\left(\frac{\rho S C_z}{2a_{33}}\int_0^t v_1\left(\frac{v_{z,1}^2}{v_1^2} + 1\right) dq\right);$$

$$\omega_{x,2} = \frac{\omega_{xo}}{b_{x,1}} + \\ + \frac{1}{a_{44} b_{x,1}} \int_0^t \left[\begin{matrix} M_{x1} - G_{mx1}(q) + \\ + v_{x,1} \dfrac{\rho SLC_{mx}}{2a_{44}\omega_1} v_1^2(q) \end{matrix} \right] b_{x,1} dq;$$

$$b_{x,1} = \exp\left(\frac{\rho SLC_{mx}}{2a_{44}\omega_1} \int_0^t v_1^2(q)dq \right);$$

$$\omega_{y,2} = \frac{\omega_{yo}}{b_{y,1}} + \\ + \frac{1}{a_{44} b_{y,1}} \int_0^t \left[\begin{matrix} M_{y1} - G_{my1}(q) + \\ + v_{y,1} \dfrac{\rho SLC_{my}}{2a_{55}\omega_1} v_1^2(q) \end{matrix} \right] b_{y,1} dq;$$

$$b_{y,1} = \exp\left(\frac{\rho SLC_{my}}{2a_{55}\omega_1} \int_0^t v_1^2(q)dq \right);$$

$$\omega_{z,2} = \\ \frac{\omega_{zo}}{b_{z,1}} + \frac{1}{a_{66} b_{z,1}} \int_0^t \left[\begin{matrix} M_{z1} - G_{mz1}(q) + \\ + v_{z,1} \dfrac{\rho SLC_{my}}{2a_{66}\omega_1} v_1^2(q) \end{matrix} \right] b_{z,1} dq;$$

$$b_{z,1} = \exp\left(\frac{\rho SLC_{mz}}{2a_{66}\omega_1} \int_0^t v_1^2(q)dq \right);$$

$$\begin{aligned} x_2 &= x_0 + v_{x0}t + \int_0^t v_{x2}(\tau)d\tau; \\ y_2 &= y_0 + v_{y0}t + \int_0^t v_{y2}(\tau)d\tau; \\ z_2 &= z_0 + v_{z0}t + \int_0^t v_{z2}(\tau)d\tau; \quad (16) \\ \theta_2 &= \theta_0 + \omega_{x0}t + \int_0^t \omega_{x2}(\tau)d\tau; \\ \varphi_2 &= \varphi_0 + \omega_{x0}t + \int_0^t \omega_{y2}(\tau)d\tau; \\ \psi_2 &= \psi_0 + \omega_{x0}t + \int_0^t \omega_{z2}(\tau)d\tau. \end{aligned}$$

The algorithm is recurrent, forming a sequence of decisions, determine the nature and rate of convergence

$$v_{x,n+1} = \frac{v_{xo}}{a_{x,n}} + \\ + \frac{1}{a_{11} a_{x,n}} \int_0^t \left[\begin{matrix} R_{xn} - G_{rxn}(q) + \\ + v_{x,n} \dfrac{\rho SC_x}{2} v_n \left(\dfrac{v_{x,n}^2}{v_n^2} + 1 \right) \end{matrix} \right] a_{x,n} dq;$$

$$a_{x,n} = \exp\left(\frac{\rho SC_x}{2a_{11}} \int_0^t v_n \left(\frac{v_{x,n}^2}{v_n^2} + 1 \right) dq \right);$$

$$v_{y,n+1} = \frac{v_{yo}}{a_{y,n}} + \\ + \frac{1}{a_{22} a_{y,n}} \int_0^t \left[\begin{matrix} R_{yn} - G_{ryn}(q) + \\ + v_{y,n} \dfrac{\rho SC_y}{2} v_n \left(\dfrac{v_{y,n}^2}{v_n^2} + 1 \right) \end{matrix} \right] a_{y,n} dq;$$

$$a_{y,n} = \exp\left(-\frac{\rho SC_y}{2a_{22}} \int_0^t v_n \left(\frac{v_{y,n}^2}{v_n^2} + 1 \right) dq \right);$$

$$v_{z,n+1} = \frac{v_{zo}}{a_{z,n}} + \\ + \frac{1}{a_{33} a_{z,n}} \int_0^t \left[\begin{matrix} R_{zn} - G_{rzn}(q) + \\ + v_{z,n} \dfrac{\rho SC_z}{2} v_n \left(\dfrac{v_{z,n}^2}{v_n^2} + 1 \right) \end{matrix} \right] a_{z,n} dq;$$

$$a_{z,n} = \exp\left(\frac{\rho SC_z}{2a_{33}} \int_0^t v_n \left(\frac{v_{z,n}^2}{v_n^2} + 1 \right) dq \right)$$

$$\omega_{x,n+1} = \frac{\omega_{xo}}{b_{x,n}} + \\ + \frac{1}{a_{44} b_{x,n}} \int_0^t \left[\begin{matrix} M_{xn} - G_{mxn}(q) + \\ + \omega_{x,n} \dfrac{\rho SLC_{mx}}{2a_{44}\omega_n} v_n^2(q) \end{matrix} \right] b_{x,n} dq;$$

$$b_{x,n} = \exp\left(\frac{\rho SLC_{mx}}{2a_{44}\omega_n} \int_0^t v_n^2(q)dq \right);$$

$$\omega_{y,n+1} = \frac{\omega_{yo}}{b_{y,n}} + \\ + \frac{1}{a_{44} b_{y,n}} \int_0^t \left[\begin{matrix} M_{yn} - G_{myn}(q) + \\ + \omega_{y,n} \dfrac{\rho SLC_{my}}{2a_{55}\omega_n} v_n^2(q) \end{matrix} \right] b_{y,n} dq;$$

$$b_{y,n} = \exp\left(\frac{\rho SLC_{my}}{2a_{55}\omega_n} \int_0^t v_n^2(q)dq \right)$$

$$\omega_{z,n+1} = \frac{\omega_{zo}}{b_{z,n}} + \\ + \frac{1}{a_{66} b_{z,n}} \int_0^t \left[\begin{array}{l} M_{zn} - G_{mzn}(q) + \\ + \omega_{z,n} \frac{\rho SLC_{my}}{2a_{66}\omega_n} v_n^2(q) \end{array} \right] b_{z,n} dq;$$

$$b_{z,n} = \exp\left(\frac{\rho SLC_{mz}}{2a_{66}\omega_n} \int_0^t v_n^2(q) dq \right);$$

Direct calculation of three successive approximations, provided the linear approximation scheme and for three terms in the schedule, can record for pairwise difference of two successive approximations according to (14) and the theorem of the mean:

$$\frac{d\left(v_{x,n+1} - v_{x,n}\right)}{dt} = \\ \frac{1}{a_{11}} \left\{ \begin{array}{l} R_{xn}\left(v_{x,n}\right) - R_{xn-1}\left(v_{x,n-1}\right) - \\ -\left(v_{x,n} - v_{x,n-1}\right) \left.\frac{\partial R_x}{\partial v_x}\right|_{v_x = v_{xn-1}} - \\ -G_{rx}\left(v_{x,n}, t\right) + G_{rx}\left(v_{x,n-1}, t\right) \\ +\left(v_{x,n} - v_{x,n-1}\right) \left.\frac{\partial G_{rx}}{\partial v_x}\right|_{v_x = v_{xn-1}} + \\ +\left(v_{x,n+1} - v_{x,n}\right) \left.\frac{\partial R_x}{\partial v_x}\right|_{v_x = v_{xn}} - \\ -\left(v_{x,n+1} - v_{x,n}\right) \left.\frac{\partial G_{rx}}{\partial v_x}\right|_{v_x = v_{xn}} \end{array} \right\} =$$

$$= \frac{1}{a_{11}} \left\{ \begin{array}{l} \frac{\left(v_{x,n} - v_{x,n-1}\right)^2}{2} \left. \left[\frac{\partial^2 R_X}{\partial v_X^2} - \frac{\partial^2 G_{rx}}{\partial v_X^2} \right] \right|_{v_x = v_{xn-1}} + \\ + \left(v_{x,n+1} - v_{x,n}\right) \left. \left[\frac{\partial R_X}{\partial v_X} - \frac{\partial G_{rx}}{\partial v_X} \right] \right|_{v_x = v_{xn-1}} \end{array} \right\}.$$

After the solution of this differential equation we obtain

$$\left(v_{x,n+1} - v_{x,n}\right) I_n = \\ = \frac{1}{a_{11}} \int_0^t \left\{ \frac{\left(v_{x,n} - v_{x,n-1}\right)^2}{2} \left. \left[\begin{array}{l} \frac{\partial^2 R_X}{\partial v_X^2} - \\ -\frac{\partial^2 G_{rx}}{\partial v_X^2} \end{array} \right] \right|_{v_x = v_{xn-1}} \right\} I_n d\tau;$$

$$I_n = \exp\left(\frac{1}{a_{11}} \int_0^t \left. \left(\frac{\partial G_{rx}}{\partial v_X} - \frac{\partial R_X}{\partial v_X} \right) \right|_{v_x = v_{xn}} d\tau \right);$$

Suppose that all the functions that are included in these expressions are those, what satisfy the requirement of integration with the square and enter the norm:

$$\|N\| = \sqrt{\int_0^t N^2(\tau) d\tau}.$$

Applying the rule to the last equation, is easy to check, that convergence is two-sided and quadratic convergence conditions determine the second derivatives of functions projection forces and impulses

$$\left\|v_{x,n+1} - v_{x,n}\right\| \le \\ \le \left. \left\| \left[\frac{\partial^2 R_X}{\partial v_X^2} - \frac{\partial^2 G_{rx}}{\partial v_X^2} \right] \right\|_{v_x = v_{xn-1}} \right|_{\max} \times. \quad (17) \\ \times \frac{\left\| \left(v_{x,n} - v_{x,n-1}\right)^2 \right\| t}{2a_{11}}.$$

Thus, the account of the impact of factors added mass formally changes the equations of motion, provided that the same factors are calculated using the algorithms [1; 14; 16; 17] consider the impact of infiltration processes and surface waves. This assessment can be obtained by analogy to other projections of the velocity vector.

Simulation results and discussion. An example for modeling the process of moving selected UWV, under the operator of automated control system in operating modes for underwater vehicle control and diagnostics of underwater pipelines. The calculations put: the initial velocity was chosen in the range (0-0.1) m/s; allowable fluctuation distance is 0.3 meters; other geometric, weighting, overall characteristics are chosen such, that matched for autonomous underwater vehicle [10; 15]. Results of simulation are presented in Table 1.

Table 1

The ratio of errors of two serial iterations

$\frac{\Delta v_{n+1}}{\Delta v_n}$	$R = 1000N, v_{x0} = 0m/c$			$R = 1000N, v_{x0} = 0,1m/c$		
n	$t = 0,5c$	$t = 1c$	$t = 2c$	$t = 0,5c$	$t = 1c$	$t = 2c$
1	0.001184	0.009631	0.087482	0.002128	0.013252	0.104405
2	0.001184	0.009635	0.089423	0.002128	0.013258	0.107133
3	0.001184	0.009635	0.089742	0.002128	0.013258	0.107625

The end of the table 1

4	0.001184	0.009635	0.089795	0.002128	0.013258	0.107713
5	0.001184	0.009635	0.089804	0.002128	0.013258	0.107729
6	0.001184	0.009635	0.089805	0.002128	0.013258	0.107732
7	0.001184	0.009635	0.089805	0.002128	0.013258	0.107733
8	0.001184	0.009635	0.089805	0.002128	0.013258	0.107733
9	0.001184	0.009635	0.089805	0.002128	0.013258	0.107733
10	0.001184	0.009635	0.089805	0.002128	0.013258	0.107733

Relative maximum of error for two consecutive iterations and fixed values R traction, initial velocity v_{x0}, time t are presented for ten iterations n. Follows from results of analysis (table 1), they are indicated, that the magnitude of maximum error and the number of iterations, starting from which the process coincides in the fourth significant digit for significant impact time. The latter allows to calculate requirements for speed control systems and actuators, as being necessarily had to be less than the period of time t, which is selected depending on the given error and needs to choose as step between two point of associative memory based on error and number of iterations.

Conclusions:

1. Asymmetry of hulls' shells of UWV significantly affects the system of equations describing its dynamic behavior, changes not only the matrix of inertia and hydrodynamic forces, but also the structure of the system of differential equations of motion.

2. Impact assessment of inertial characteristics of the UWV on its hull dynamics can be based on comparison of estimates of error solutions of equations according to expression (17), allowing to take into account the effect on the rate of convergence of solutions of the value initial velocity of translational and rotational motions, value over time and others.

3. The value of the time interval between states given in the associative memory can be selected based on the desired system performance of error control and restrictions on the amount of memory.

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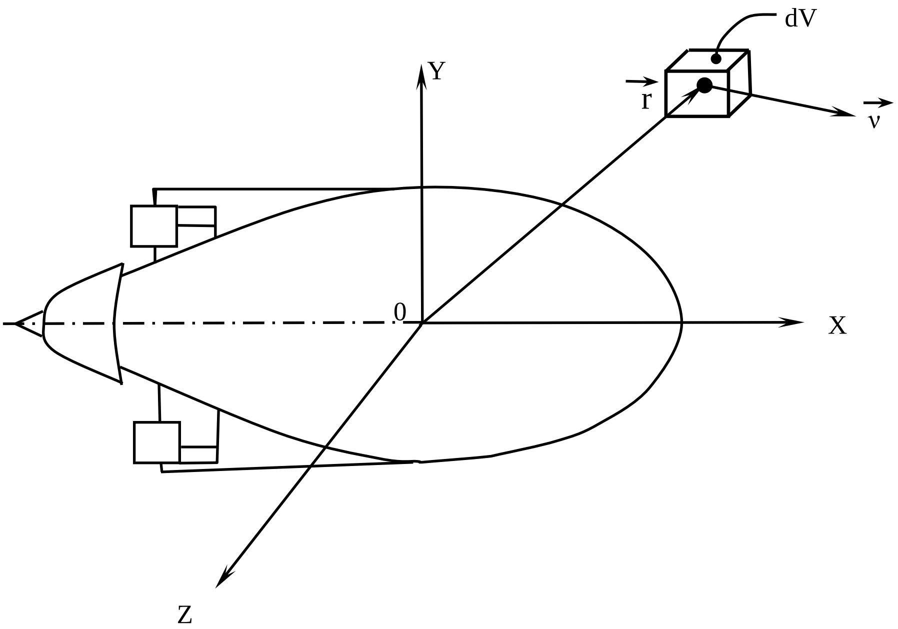

Fig. 1. Kinematic scheme of UWR

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