UDC 51-8

Determination of the optimal pure strategies subset as the latent predominance set in some matrix games

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Abstract

Romanuke V. V. Determination of the optimal pure strategies subset as the latent predominance set in some matrix games. *There has been defined the latent predominance set of a player in the matrix games with the nonempty set of the saddle points in pure strategies. Suggested the operations for aggregating the payoffs of a player, applying the optimal pure strategies, when the other player digresses from the optimality principle. Also displayed how and what aggregated advantage a player takes, applying a strategy from the latent predominance set.*

Preamble

Processes of making the optimal decisions certainly are the everyday events. If there is the possibility to formalize a decision making problem, then the result of the decision making process is logically substantiated. Frequently a problem of resolving the conflict between two competitors may be simulated with the antagonistic games mathematical apparatus [1]. This is really efficacious when the final mathematical model is the matrix game [2]. And there are the stably ever reliable methods for solving any matrix game [3, 4]. Certainly, the solution is preferable to be obtained in pure strategies, rather than in mixed strategies, as the optimal mixed strategy practice has the known deficiencies [3, 5]. If the matrix game is solved in pure strategies, then both either the competitors or players acquire at least the one logically founded version of the withdrawal from the conflict. However, in the paper [6] there was investigated the exampled $M \times N$-game with the payoff matrix $\mathbf{K} = \left(k_{ij}\right)_{M \times N}$ by the zero first column and line, and the nonnegative matrix elements k_{ij} $\forall\ i = \overline{1, M}$ and $\forall\ j = \overline{1, N}$, with the conditions

$$\exists\ i_j \in \left\{\overline{2, M}\right\},\ k_{i_j j} > 0\ \ \forall\ i = \overline{1, M}\,, \qquad (1)$$

$$\exists\ j_i \in \left\{\overline{2, N}\right\},\ k_{i j_i} > 0\ \ \forall\ j = \overline{1, N}\,, \qquad (2)$$

where the sets $X = \left\{x_i\right\}_{i=1}^{M}$ and $Y = \left\{y_j\right\}_{j=1}^{N}$ are the pure strategies sets respectively of the first and second players, corresponding to the numbered lines and columns. It was shown, in that game the second player has the single optimal strategy $y_{\text{opt}} = y_1$, while the first player may select any pure strategy from the set $X = \left\{x_i\right\}_{i=1}^{M}$ of its optimal strategies. The game value $V_{\text{opt}} = 0$ here is clear. The paper [6] was directed towards some specific properties of the assigned game solutions. It is easy to see, that even with the simplest example $\mathbf{K} = \begin{bmatrix} 0 & 0 \\ 0 & k_{22} \end{bmatrix}$ by the element $k_{22} > 0$ the first player, having by the classical optimality principle the two optimal pure strategies x_1 and x_2, would have the explicit advantage if applying only strategy x_2 by the second player just swerved from its single optimal strategy y_1. And the magnitude of this conditioned advantage depends on the element k_{22}. Clearly, that by the sufficiently great k_{22} it would be obviously irrational for the first player to select the pure strategy x_1 as permanently there is a nonzero probability of the improper or fallacious decision, that may be made by the player, that is here by the second player. Certainly, that if such game is played for a great number of times, and simulates a real conflict event, then the probability of the second player fallacious decision is increasing. And this noteworthy detail prompted to investigate the matrix games solutions with the matrix $\mathbf{K} = \left(k_{ij}\right)_{M \times N}$ by the stated above conditions (1) and (2), though the specified location of the zero line and column is not of principle.

The important question, arising here, is how to evaluate the conditioned advantage for the first player, having more than one optimal pure strategy, corresponding to the cases when the second player digresses from applying its optimal pure strategy, which, speaking generally, may be non-unique. And this paper intent is to investigated the class of the matrix games, that generalizes the stated above games, including the duality, and then to form the criterion for optimizing the conditioned advantage for the player.

Latent predominance definition and use

Hereinafter, may $\mathbf{K} = \left(k_{ij}\right)_{M \times N}$ be the matrix of the antagonistic game, that is solved in pure strategies, where the sets $X_{\text{opt}} \subset X = \left\{x_i\right\}_{i=1}^{M}$ and $Y_{\text{opt}} \subset Y = \left\{y_j\right\}_{j=1}^{N}$ are the optimal pure strategies sets respectively of the first and second players, and V_{opt} is assigned as the game value. First of all, it matters to mark that the game with the single saddle point in pure strategies is not the game, where the conditioned advantage takes place, as $\left|X_{\text{opt}}\right| = 1$ and $\left|Y_{\text{opt}}\right| = 1$, whence it is impossible to get the payoff $V > V_{\text{opt}}$ for the first player on its another optimal strategy by the second swerves from the optimality principle, or to pay $V < V_{\text{opt}}$ for the second player on its another optimal strategy by the first swerves from the optimality principle.

The said above conditioned advantage of the player, naturally, may be reached on some subset of the optimal pure strategies set of the player. Consequently, this subset possesses the dominance over the whole optimal pure strategies set. But as this dominance is realized just by the other player swerves from the optimality principle, then the conditioned advantage subset should be called the latent predominance set.

For further stating, may the set $\widehat{X}_{\text{opt}} \subset X_{\text{opt}}$ be the latent predominance set of the first player, and the set $\breve{Y}_{\text{opt}} \subset Y_{\text{opt}}$ be the latent predominance set of the second. For the exact definition of the sets $\widehat{X}_{\text{opt}}$ and $\breve{Y}_{\text{opt}}$ it is suitable primarily to separate off those subsets of the sets X_{opt} and Y_{opt}, that certainly are not included into $\widehat{X}_{\text{opt}}$ and $\breve{Y}_{\text{opt}}$. The case with $\left|X_{\text{opt}}\right| = 1$ and $\left|Y_{\text{opt}}\right| = 1$ has been considered in the starting paragraph of this section. When the matrix game has more than one saddle point in pure strategies, but a player possesses the single optimal pure strategy, then again this player has the empty latent predominance set. Summarizing it over, if $\left|X_{\text{opt}}\right| = 1$ then $\widehat{X}_{\text{opt}} = \varnothing$, and if $\left|Y_{\text{opt}}\right| = 1$ then the set $\breve{Y}_{\text{opt}} = \varnothing$.

Nevertheless, the latent predominance takes place for the first player, if there are at least two optimal pure strategies $x_{\text{opt}}^{\langle 1 \rangle} \in X_{\text{opt}}$ and $x_{\text{opt}}^{\langle 2 \rangle} \in X_{\text{opt}}$ that have different results of their application by the second player swerves from the optimality principle. At that those results should be meant in the integrated implication, inasmuch as the set $Y \setminus Y_{\text{opt}}$, speaking generally, is not the one-element set. The same argumentation, but only dual, is for the second player. For instance, in the game with the matrix $\mathbf{K} = \begin{bmatrix} 2 & 2 & 3 \\ 2 & 4 & 3 \end{bmatrix}$ each element from the optimal set $X_{\text{opt}} = \left\{x_1, x_2\right\}$ has its own consequence results after the second player selects the strategy $y \notin Y_{\text{opt}} = \left\{y_1\right\}$, although there still is the need to formulate the rule for aggregating these consequence results.

If to assign with the symbol $R\left(x_{\text{opt}}, Y \setminus Y_{\text{opt}}\right)$ the operation for aggregating the payoffs of the first player, selecting its optimal pure strategy x_{opt} by the second player selects a strategy from its nonoptimal set $Y \setminus Y_{\text{opt}}$, then there is the following definition for the latent predominance set of the first player. In the matrix game with the nonempty set of the saddle points in pure strategies the set $\widehat{X}_{\text{opt}} \neq \varnothing$ if there are two optimal pure strategies $x_{\text{opt}}^{\langle 1 \rangle} \in X_{\text{opt}}$ and $x_{\text{opt}}^{\langle 2 \rangle} \in X_{\text{opt}}$ such as that

$$R\left(x_{\text{opt}}^{\langle 1 \rangle}, Y \setminus Y_{\text{opt}}\right) \neq R\left(x_{\text{opt}}^{\langle 2 \rangle}, Y \setminus Y_{\text{opt}}\right), \tag{3}$$

and the latent predominance set of the first player

$$\widehat{X}_{\text{opt}} = \left\{\arg\max_{x \in X_{\text{opt}}} R\left(x, Y \setminus Y_{\text{opt}}\right)\right\} \subset X_{\text{opt}}. \tag{4}$$

If the symbol $R\left(y_{\text{opt}}, X \setminus X_{\text{opt}}\right)$ means the operation for aggregating the payoffs of the second player, selecting its optimal pure strategy y_{opt} by the first player selects a strategy from its nonoptimal set $X \setminus X_{\text{opt}}$, then there is the following definition for the latent predominance set of the second player. In the matrix game with the nonempty set of the saddle points in pure strategies the set $\breve{Y}_{\text{opt}} \neq \varnothing$ if there are two optimal pure strategies $y_{\text{opt}}^{\langle 1 \rangle} \in Y_{\text{opt}}$ and $y_{\text{opt}}^{\langle 2 \rangle} \in Y_{\text{opt}}$ such as that

$$R\left(y_{\text{opt}}^{\langle 1 \rangle}, X \setminus X_{\text{opt}}\right) \neq R\left(y_{\text{opt}}^{\langle 2 \rangle}, X \setminus X_{\text{opt}}\right), \tag{5}$$

and the latent predominance set of the second player

$$\breve{Y}_{\text{opt}} = \left\{\arg\min_{y \in Y_{\text{opt}}} R\left(y, X \setminus X_{\text{opt}}\right)\right\} \subset Y_{\text{opt}}. \tag{6}$$

Naturally, these given definitions allow to determine the latent predominance set of a player only implicitly. Besides, even in the game with the matrix $\mathbf{K} = \begin{bmatrix} 2 & 2 & 3 \\ 2 & 4 & 3 \end{bmatrix}$ those definitions do not help in ascertaining which of the strategies x_1 and x_2 belongs to the latent predominance set of the first player. Moreover, it is obscure whether here the set $\widehat{X}_{\text{opt}} \neq \varnothing$ or not.

However, in the game with the matrix $\mathbf{K}=\begin{bmatrix}2 & 2 & 3\\ 2 & 3 & 2\end{bmatrix}$ or the matrix $\mathbf{K}=\begin{bmatrix}2 & 2 & 3\\ 2 & 2 & 3\end{bmatrix}$, by having the same solution as the previously exampled matrix game, it is clear that $\widehat{X}_{\text{opt}}=\varnothing$, because the aggregated result of applying the pure strategy x_1 or x_2 is the same. Then, increasing the last matrix element k_{22} from 2 up to 4, the optimal pure strategy x_2 will become predominating the pure strategy x_1 in the spoken above latency. It clearly follows from that when the second player selects either the pure strategy y_2 or y_3, the mathematical expectation of the first player payoff is greater for the optimal pure strategy x_2. Consequently, as the operation $R\left(x_{\text{opt}}, Y\setminus Y_{\text{opt}}\right)$ there may be applied the mathematical expectation. But inasmuch as there is the probabilistic indefiniteness of the distribution of the elements of the set $Y\setminus Y_{\text{opt}}$, then this mathematical expectation lies actually in averaging out. Naturally, that this claim is the dually same for the operation $R\left(y_{\text{opt}}, X\setminus X_{\text{opt}}\right)$.

Now the formula (4) by the true condition (3) may be stated as

$$\widehat{X}_{\text{opt}}=\left\{\arg\max_{\substack{i=\overline{1,M}\\ x_i\in X_{\text{opt}}}}\sum_{\substack{j=1\\ y_j\notin Y_{\text{opt}}}}^{N}k_{ij}\right\}\subset X_{\text{opt}}. \qquad (7)$$

And the formula (6) by the true condition (5) may be stated as

$$\breve{Y}_{\text{opt}}=\left\{\arg\min_{\substack{j=\overline{1,N}\\ y_j\in Y_{\text{opt}}}}\sum_{\substack{i=1\\ x_i\notin X_{\text{opt}}}}^{M}k_{ij}\right\}\subset Y_{\text{opt}}. \qquad (8)$$

For instance, may consider the game with the matrix

$$\mathbf{K}=\begin{bmatrix}2 & 4 & 0 & 1 & 0 & 0 & 3\\ 0 & 1 & -2 & 1 & -3 & 0 & -1\\ 2 & 2 & 0 & 0 & 0 & 0 & 1\end{bmatrix}. \qquad (9)$$

Here the game value $V_{\text{opt}}=0$, $X_{\text{opt}}=\{x_1, x_3\}$ and $Y_{\text{opt}}=\{y_3, y_5, y_6\}$. By the criterion (7), that actually is the Bayes — Laplace criterion within the total indeterminancy conditions, the first player has the latent predominance set

$$\widehat{X}_{\text{opt}}=\arg\max_{\substack{i=\overline{1,3}\\ x_i\in\{x_1, x_3\}}}\sum_{\substack{j=\overline{1,7}\\ y_j\notin\{y_3, y_5, y_6\}}}k_{ij}=$$

$$=\arg\max_{\substack{i=\overline{1,3}\\ x_i\in\{x_1, x_3\}}}\{2+4+1+3,\ 2+2+0+1\}=$$

$$=\arg\max_{\substack{i=\overline{1,3}\\ x_i\in\{x_1, x_3\}}}\{10, 5\}=\{x_1\}. \qquad (10)$$

By the analogous criterion (8) the second player has the latent predominance set

$$\breve{Y}_{\text{opt}}=\arg\min_{\substack{j=\overline{1,7}\\ y_j\in\{y_3, y_5, y_6\}}}\sum_{\substack{i=\overline{1,3}\\ x_i\notin\{x_1, x_3\}}}k_{ij}=$$

$$=\arg\min_{\substack{j=\overline{1,7}\\ y_j\in\{y_3, y_5, y_6\}}}\{-2, -3, 0\}=\{y_5\}. \qquad (11)$$

And now this is clear that in the game with the matrix (9) the first player should use the strategy x_1 as it will allow to get on average the twice greater payoff, than using the strategy x_3. As for the second player, that it pays certainly lesser than the game value $V_{\text{opt}}=0$ by applying the strategy y_5, when the first player selects the nonoptimal strategy x_2. However, in the single play with the digressed second player, the first player on its latent predominance set gets the payoff, being equal to 2, 4, 1 or 3, that is assuredly the greater payoff, than the game value $V_{\text{opt}}=0$. But if in the matrix (9) to change the element $k_{12}=4$ into $k_{12}=5$, and the element $k_{14}=1$ into $k_{14}=0$, then the sets (10) and (11) will remain, though then there will be no guarantee for the greater payoff, than the game value $V_{\text{opt}}=0$. Such disadvantageous game situation is $\{x_1, y_4\}$. Even by using the latent dominating strategy x_1, the first player gets only the game value payoff, when the second player swerves from its optimal strategies set on the strategy y_4.

The method of determination of the optimal pure strategies subset as the latent predominance set in the matrix game, applying the Bayes — Laplace criterion by the total indeterminancy conditions, has been programmed within the powered software environment MATLAB. The program module or function, named “lps” (Latent Predominance Set), directly has the one input, which is a payoff matrix, what is displayed by typing “help lps” in the MATLAB Command Window (figure 1).

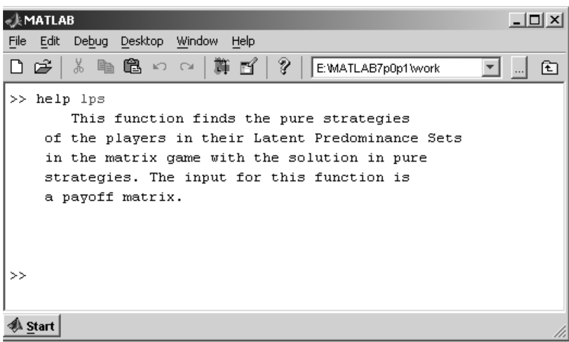


Figure 1 — The help hint in the MATLAB Command Window for the function “lps”

The program code head of the function “lps” (figure 2) checks whether the game with the input matrix is solved in pure strategies or not. This

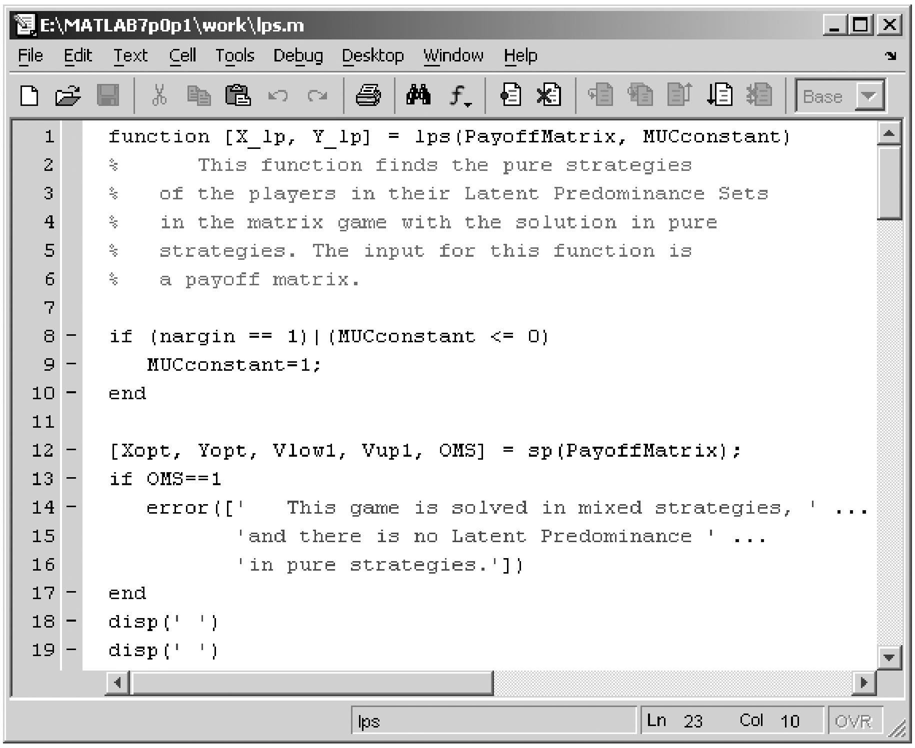


Figure 2 — Program code head of the function “lps”

checking is realized with the function “sp”, determining the matrix game solution for any input matrix (figure 3). If the input matrix game is solved in mixed strategies, then there is generated the message on the error. For instance, the game with the payoff matrix $\mathbf{K} = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$ is solved in mixed strategies, so usage of the function “lps” is erroneous (figure 4), as there are no latent predominance sets.

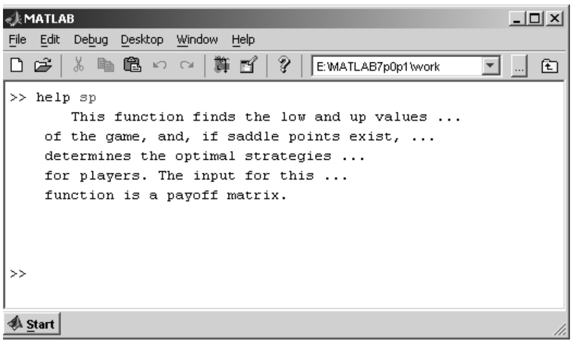


Figure 3 — The help hint in the MATLAB Command Window for the function “sp”, applied as a subfunction in the module “lps”

The first main part of the “lps” code produces the sets $\hat{X}_{\text{opt}}$ and $\breve{Y}_{\text{opt}}$ by the criterions (7) and (8) respectively (figure 5). Stepping back to the example with the matrix game (9), the latent predominance sets of both the players now may be determined with the elaborated software, just by typing and entering the corresponding payoff matrix in the MATLAB Command Window (figure 6). Naturally, that the results of the program module “lps” is the same as in the formulas (10) and (11).

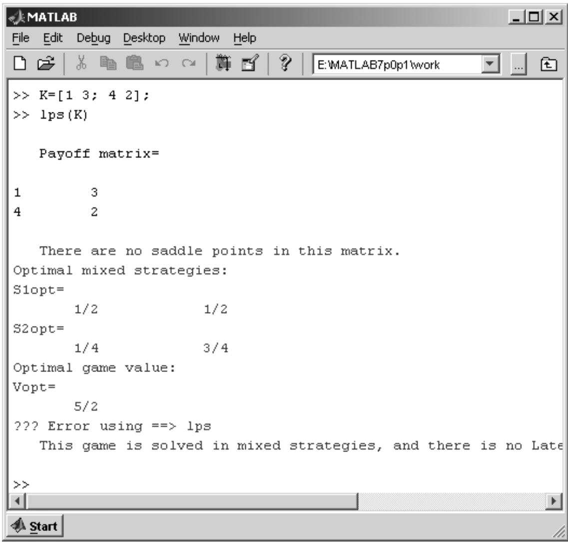
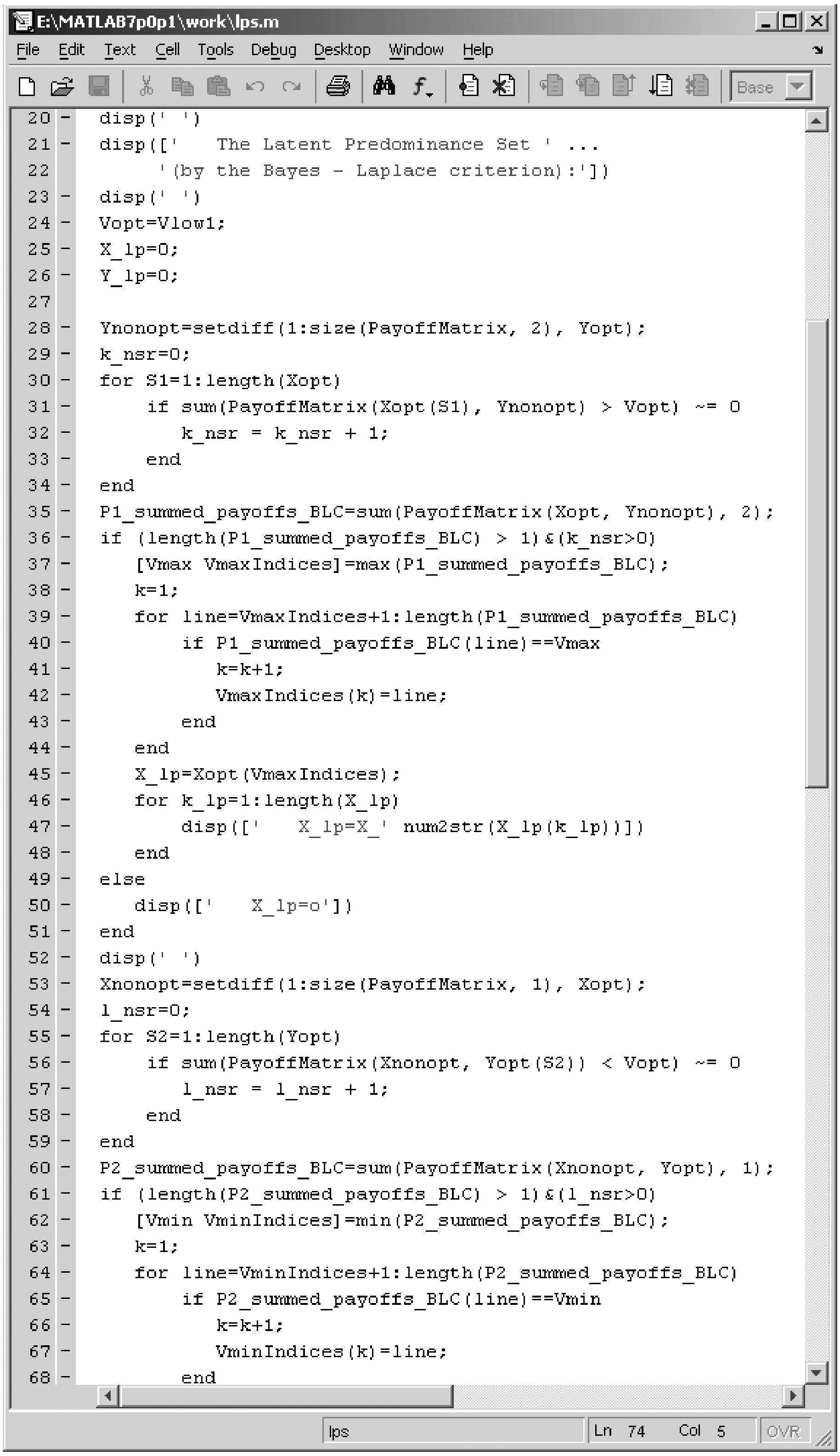


Figure 4 — Erroneous occurrence with the input

Though the given definitions for the latent predominance sets as the subsets of the optimal pure strategies sets of the players, defined numerically as the arguments (7) and (8), do produce the definite unequivocal result, they may be spread out on the



E:\MATLAB7p0p1\work\lps.m

```
disp(' ')
disp(['   The Latent Predominance Set ' ...
      '(by the Bayes - Laplace criterion):'])
disp(' ')
Vopt=Vlow1;
X_lp=0;
Y_lp=0;

Ynonopt=setdiff(1:size(PayoffMatrix, 2), Yopt);
k_nsr=0;
for S1=1:length(Xopt)
    if sum(PayoffMatrix(Xopt(S1), Ynonopt) > Vopt) ~= 0
       k_nsr = k_nsr + 1;
    end
end
P1_summed_payoffs_BLC=sum(PayoffMatrix(Xopt, Ynonopt), 2);
if (length(P1_summed_payoffs_BLC) > 1)&(k_nsr>0)
   [Vmax VmaxIndices]=max(P1_summed_payoffs_BLC);
   k=1;
   for line=VmaxIndices+1:length(P1_summed_payoffs_BLC)
       if P1_summed_payoffs_BLC(line)==Vmax
          k=k+1;
          VmaxIndices(k)=line;
       end
   end
   X_lp=Xopt(VmaxIndices);
   for k_lp=1:length(X_lp)
       disp(['   X_lp=X_' num2str(X_lp(k_lp))])
   end
else
   disp(['   X_lp=o'])
end
disp(' ')
Xnonopt=setdiff(1:size(PayoffMatrix, 1), Xopt);
l_nsr=0;
for S2=1:length(Yopt)
    if sum(PayoffMatrix(Xnonopt, Yopt(S2)) < Vopt) ~= 0
       l_nsr = l_nsr + 1;
    end
end
P2_summed_payoffs_BLC=sum(PayoffMatrix(Xnonopt, Yopt), 1);
if (length(P2_summed_payoffs_BLC) > 1)&(l_nsr>0)
   [Vmin VminIndices]=min(P2_summed_payoffs_BLC);
   k=1;
   for line=VminIndices+1:length(P2_summed_payoffs_BLC)
       if P2_summed_payoffs_BLC(line)==Vmin
          k=k+1;
          VminIndices(k)=line;
       end
```

Figure 5 — Code part for the Bayes — Laplace criterion by the total indeterminancy conditions

whole sense of the inserted symbols $R\left(x_{\text{opt}}, Y \setminus Y_{\text{opt}}\right)$ and $R\left(y_{\text{opt}}, X \setminus X_{\text{opt}}\right)$. It is known [7], that the operation for aggregating the payoffs of a player, selecting its pure strategy, may be also expressed with the multiplication criterion [7, 8], where the necessary condition is the positiveness $\mathbf{K} > 0$.

If the condition $\mathbf{K} > 0$ is not true, then there

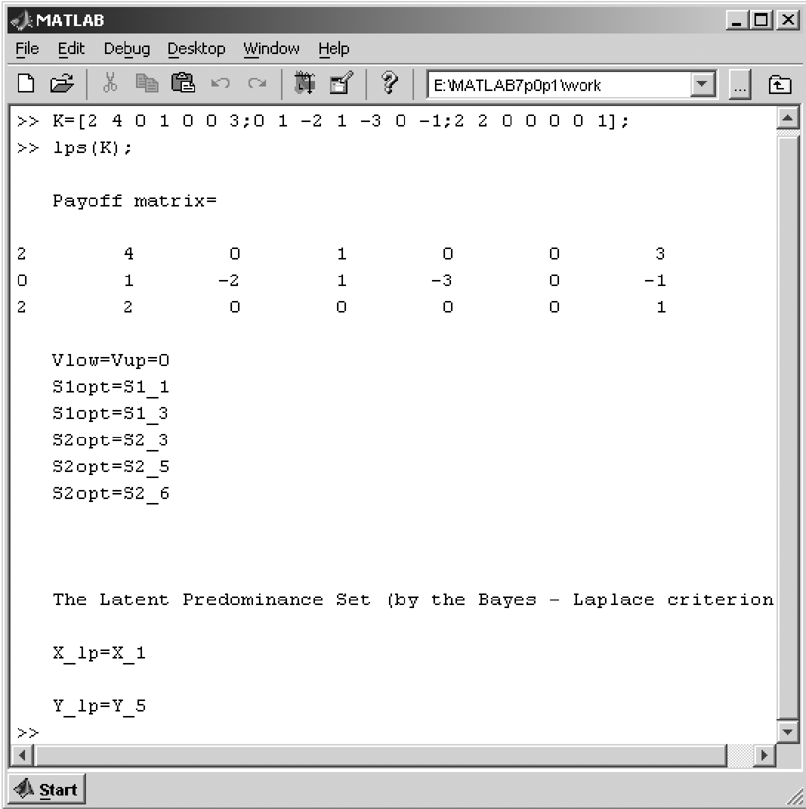


Figure 6 — The latent predominance sets for the players in the game with the matrix (9)

should be produced the matrix

$$\mathbf{K}_{\lambda} = \lambda - \min_{i=\overline{1, M}} \min_{j=\overline{1, N}} k_{ij} \qquad (12)$$

for some $\lambda > 0$ that may be selected at will. And hence by the multiplication criterion the first player latent predominance set (4) is

$$\hat{X}_{\text{opt}}(\lambda) = \left\{ \arg \max_{\substack{i=\overline{1, M} \\ x_i \in X_{\text{opt}}}} \prod_{\substack{j=1 \\ y_j \notin Y_{\text{opt}}}}^{N} k_{ij}(\lambda) \right\} \subset X_{\text{opt}}, \qquad (13)$$

where the value $k_{ij}(\lambda)$ is the element of the matrix $\mathbf{K}_{\lambda} = \left[k_{ij}(\lambda) \right]_{M \times N}$ been produced by the shift (12) for the false condition $\mathbf{K} > 0$. Analogously the formula (6) by the true condition (5) is stated as

$$\breve{Y}_{\text{opt}}(\lambda) = \left\{ \arg \min_{\substack{j=\overline{1, N} \\ y_j \in Y_{\text{opt}}}} \prod_{\substack{i=1 \\ x_i \notin X_{\text{opt}}}}^{M} k_{ij}(\lambda) \right\} \subset Y_{\text{opt}}. \qquad (14)$$

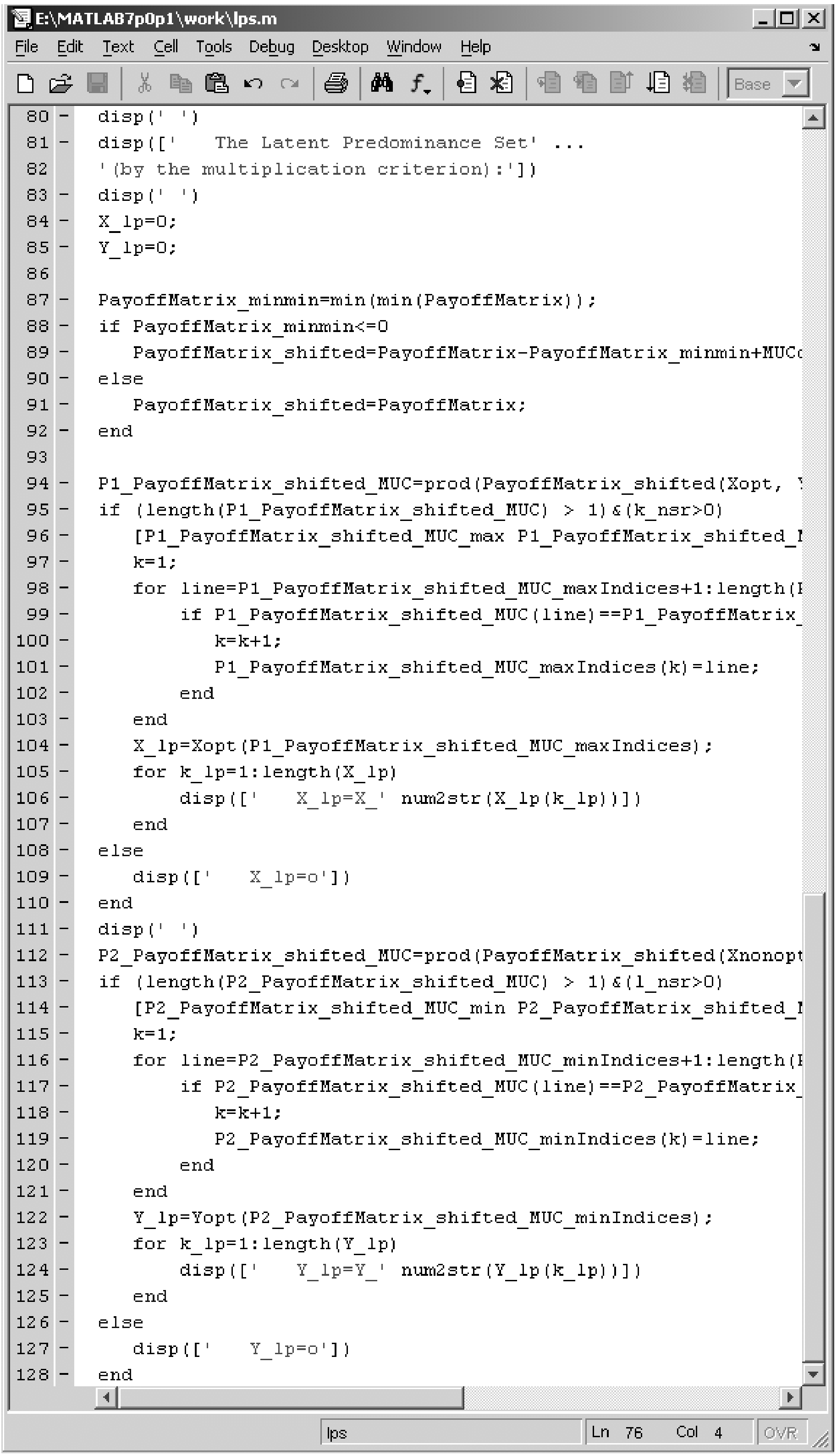
But surely, that if the condition $\mathbf{K} > 0$ is true then there is the identity $k_{ij}(\lambda) = k_{ij}$ $\forall\, i = \overline{1, M}$ and $\forall\, j = \overline{1, N}$ in the formulas (13) and (14).

The partially visualized above program module “lps” contains also the just stated method for determining the latent predominance set (figure 7). The influential constant $\lambda > 0$ may not be typed in the MATLAB Command Window, and then there is tolerated the value $\lambda = 1$. For the game with the matrix (9) the determined latent predominance sets of both the players are the same as the sets, determined by the criterions (7) and (8), what is displayed on the figure (8). But speaking generally, the criterions by the formulas (7) and (13), (8) and (14) may produce different sets [8].

For generalizing the sum-based criterions by the formulas (7) and (8), there should be laid down the rule of the nonlinear stretch of the matrix elements. This may be accomplished by raising each element k_{ij} within the sums (7) and (8) to some positive power. As there may be the case when $k_{ij} < 0$ then each element k_{ij} must be taken by its absolute value. But only it is fundamental, that this power would be put at so, that the sign of the powered element could be saved. Being driven by this convention, the third operating method to aggregate the payoffs of the first player by the formula (4) for the true condition (3) generates the latent predominance set

$$\hat{X}_{\text{opt}}(p) = \left\{ \arg \max_{\substack{i=\overline{1, M} \\ x_i \in X_{\text{opt}}}} \sum_{\substack{j=1 \\ y_j \notin Y_{\text{opt}}}}^{N} \left[\operatorname{sign}(k_{ij}) \right] \cdot \left| k_{ij} \right|^{p} \right\}, \qquad (15)$$

where $p > 0$. For the second player this method outline generates the latent predominance set




```
disp(' ')
disp(['   The Latent Predominance Set' ...
'(by the multiplication criterion):'])
disp(' ')
X_lp=0;
Y_lp=0;

PayoffMatrix_minmin=min(min(PayoffMatrix));
if PayoffMatrix_minmin<=0
   PayoffMatrix_shifted=PayoffMatrix-PayoffMatrix_minmin+MUCc
else
   PayoffMatrix_shifted=PayoffMatrix;
end

P1_PayoffMatrix_shifted_MUC=prod(PayoffMatrix_shifted(Xopt, Y
if (length(P1_PayoffMatrix_shifted_MUC) > 1)&(k_nsr>0)
   [P1_PayoffMatrix_shifted_MUC_max P1_PayoffMatrix_shifted_M
   k=1;
   for line=P1_PayoffMatrix_shifted_MUC_maxIndices+1:length(P
       if P1_PayoffMatrix_shifted_MUC(line)==P1_PayoffMatrix_
          k=k+1;
          P1_PayoffMatrix_shifted_MUC_maxIndices(k)=line;
       end
   end
   X_lp=Xopt(P1_PayoffMatrix_shifted_MUC_maxIndices);
   for k_lp=1:length(X_lp)
       disp(['   X_lp=X_' num2str(X_lp(k_lp))])
   end
else
   disp(['   X_lp=o'])
end
disp(' ')
P2_PayoffMatrix_shifted_MUC=prod(PayoffMatrix_shifted(Xnonopt
if (length(P2_PayoffMatrix_shifted_MUC) > 1)&(l_nsr>0)
   [P2_PayoffMatrix_shifted_MUC_min P2_PayoffMatrix_shifted_M
   k=1;
   for line=P2_PayoffMatrix_shifted_MUC_minIndices+1:length(P
       if P2_PayoffMatrix_shifted_MUC(line)==P2_PayoffMatrix_
          k=k+1;
          P2_PayoffMatrix_shifted_MUC_minIndices(k)=line;
       end
   end
   Y_lp=Yopt(P2_PayoffMatrix_shifted_MUC_minIndices);
   for k_lp=1:length(Y_lp)
       disp(['   Y_lp=Y_' num2str(Y_lp(k_lp))])
   end
else
   disp(['   Y_lp=o'])
end
```

Figure 7 — Code part for the multiplication criterion

$$\breve{Y}_{\mathrm{opt}}(p) = \left\{ \arg \min_{\substack{j=\overline{1,N} \\ y_j \in Y_{\mathrm{opt}}}} \sum_{\substack{i=1 \\ x_i \notin X_{\mathrm{opt}}}}^{M} \left[\mathrm{sign}\left(k_{ij}\right) \right] \cdot \left| k_{ij} \right|^{p} \right\}. \quad (16)$$

Thus the oddness or the parity of the power p does not influence on the final result. Particularly, if the value $p \in (0; 1)$ then the operations (15) and (16) will be implemented by the peculiar compression of the matrix elements: those elements k_{ij}, satisfying the condition $0 < k_{ij} < 1$, will be dragged up to the value 1, and those elements k_{ij}, that satisfying the condition $-1 < k_{ij} < 0$, will be dragged up to the value -1; the elements $k_{ij} > 1$ are drawn down to the value 1, and those elements k_{ij}, that satisfying the condition $k_{ij} < -1$, will be drawn down to the value -1. Otherwise, if the value $p > 1$ then the operations (15) and (16) will be implemented by the peculiar extension of the matrix elements, satisfying the condition $|k_{ij}| > 1$, though for $|k_{ij}| < 1$ there will be the reduction, converging to the zero point. Obviously, that the case $p = 1$ corresponds to the transference from the nonlinear stretch criterions by the formulas (15) and (16) towards the sum-based criterions by the formulas (7) and (8).

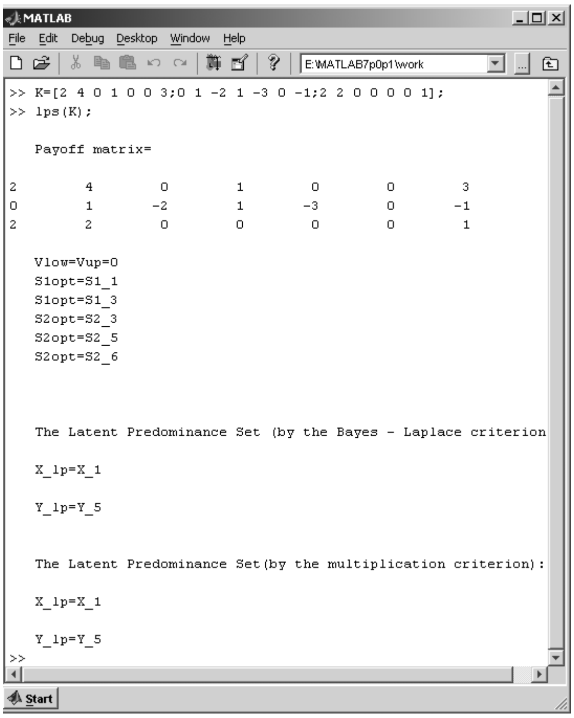


Figure 8 — Total coincidence in results by $\lambda = 1$

Conclusion

Being solved in pure strategies, the matrix game may have the specific subset of the optimal pure strategies set of a player, which latently predominates over the rest part of this player optimal strategies set. This latent predominance exposes in the advance sense of the mathematical expectation of the player payoff, when the other player digresses from the known optimality principle, selecting the nonoptimal strategies. The latent predominance set of the first player is determined by one of the statements (7), (13) and (15), and the second player latent predominance set is found by one of the dual statements (8), (14) and (16). The math environment MATLAB for numerical computations allowed to create the program function for producing the latent predominance sets of the players in any matrix game. Furthermore, the definitely applied criterions here, that is the Bayes — Laplace criterion and the multiplication criterion, may be expanded or even hybridized.

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