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TO THE QUESTION OF FORMING BIHARMONIC OSCILLATIONS IN TWO-MASSES NONLINEAR VIBRATING MACHINES UNDER IDEAL HARMONIC EXCITATION

The principle scheme of two-masses vibrating machine with the cubic characteristics of elastic ties and dissipative force under ideal inertial excitation is considered. Its unidirectional oscillations are being studied. Mathematical model of the machine is described by the system of two ordinary differential equations of the second order. After representing it in the dimensionless form the complex form of harmonic balance method is used for analysis of its stationary motions. As a result the finding of periodic oscillations of the machine is reduced to the solving of system of polynomial equations with complex coefficients. The solving of this system under consequent changing of one of the parameters of the machine gives an opportunity to construct its bifurcation diagrams and is realized with the help of original software. The possible points of bifurcation in it are being found on the basis of the control of change of sign of the Jacobian of the system, stability of the motions in the first approximation is being analyzed with use of Floquet-Lyapunov theory. With the help of this software stationary motions of the machine in the frequency zone located between two natural ones are being investigated. For chosen parameters of the machine only the combinational resonances of the orders 3:1, 2:1 and 1:3 were discovered and the corresponding amplitude- and phase-frequency characteristics were constructed. Their analysis shows that from the practical point of view the most suitable of them is the resonance of the order 2:1 which is paid the main attention next. The influence of level of nonlinearity and asymmetry of elastic characteristics and dissipation factor upon the behavior of the system and the number of its regimes are marked. The possibility of forming of practically significant biharmonic oscillations is demonstrated, the number of diagrams of laws of displacements and accelerations for different values of the parameters are given. It is noted the opportunity of realization of superharmonic regimes in the considered vibromachine by choosing the parameters of elastic ties.

Keywords: Vibromachine, antiresonance, polyharmonic vibration, harmonic balance method, bifurcation diagram, numerical analysis.

Statement of the problem

In industry there are used vibrating machines with biharmonic oscillations of working organs. Traditionally such oscillations are realized with the help of two exciters having different frequencies [1]. But it is well known that in nonlinear systems the oscillations sometimes become quite complicated and may have pronounced polyharmonic character [2]. Usually these phenomena take place for certain parameters of the dynamical system when, the so-called, combination resonances take place. By this reason, the purpose of this article is to investigate principle possibility of forming practically important oscillations in two masses vibrating machine having polynomial elastic ties and harmonic excitation.

Principle scheme and mathematical model

The principle scheme of such vibrating machine is shown in Figure 1, where m_1 – mass of a frame, m_2 – mass of a working organ, m_0 – unbalanced mass, r – eccentricity of an exciter, ω – frequency of an vibroexciter, k_0 – stiffness coefficient and $\mu k_0'$ – coefficient of viscous resistance of shock absorbers, $k(x) = k_1 + k_2 x + k_3 x^2$ – stiffness coefficient and $\mu k'(x)$, where $k'(x) = k_1' + k_2' x + k_3' x^2$, – resistance coefficient of the elastic ties connecting frame and working organ, $F_e = k_1 x + k_2 x^2 + k_3 x^3$ – its elastic characteristic, $k_1,\ k_2,\ k_3$ – parameters of the elastic ties and $k_1',\ k_2',\ k_3'$, – of dissipation, μ – coefficient of inelastic resistance of absorbers and elastic ties, $P(t) = m_0 r \omega^2 \cos \omega t$ – constraining force of inertial vibroexciter.

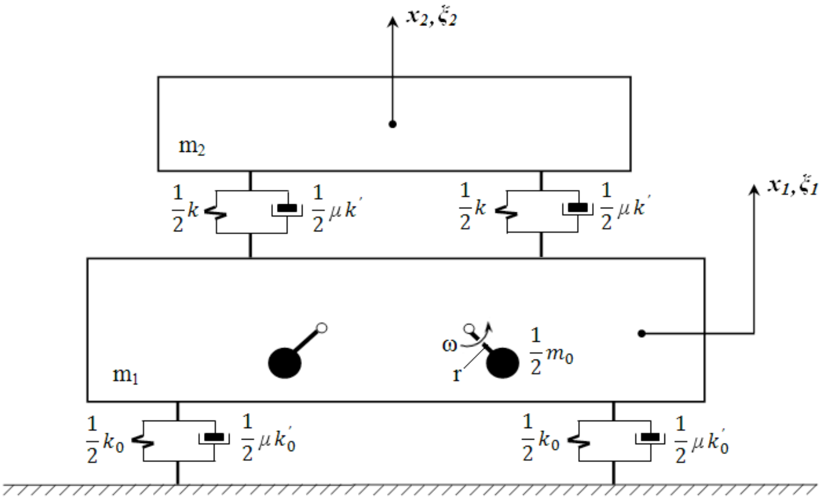

Figure 1 – The principal scheme of a vibrating machine

As generalized coordinates we take x_1, – displacement of the frame and x_2, – displacement of the working organ.

Using Lagrange equations [3] we get the equations of the motion

$$\begin{cases} m_1' \ddot{x}_1 + k_0 x_1 - k_1(x_2 - x_1) - k_2(x_2 - x_1)^2 - k_3(x_2 - x_1)^3 + \\ \qquad + \mu k_0' \dot{x}_1 - \mu\left(k_1' + k_2'(x_2 - x_1) + k_3'(x_2 - x_1)^2\right)(\dot{x}_2 - \dot{x}_1) = m_0 r \omega^2 \cos\omega t, \\ m_2 \ddot{x}_2 + k_1(x_2 - x_1) + k_2(x_2 - x_1)^2 + k_3(x_2 - x_1)^3 + \\ \qquad + \mu\left(k_1' + k_2'(x_2 - x_1) + k_3'(x_2 - x_1)^2\right)(\dot{x}_2 - \dot{x}_1) = 0. \end{cases}$$

Then subtracting one of them from the other one, denoting $x = x_2 - x_1$, turning to the variable $\tau = \omega_1 t$, where ω_1 – the first natural frequency of a vibromachine and introducing non-dimensional variables $\xi_1 = x_1/\Delta$, $\xi = x/\Delta$, where $\Delta = 10^{-3}$ m we represent mathematical model of the vibromachine in the form

$$\begin{cases} \dfrac{d^2\xi_1}{d\tau^2} + b_{10}\dfrac{d\xi_1}{d\tau} + b_{11}\dfrac{d\xi}{d\tau} + b_{12}\,\xi\dfrac{d\xi}{d\tau} + b_{13}\,\xi^2\dfrac{d\xi}{d\tau} + \\ \qquad + k_{10}\,\xi_1 + k_{11}\,\xi + k_{12}\,\xi^2 + k_{13}\,\xi^3 = P_1 \cos\eta\tau, \\ \dfrac{d^2\xi}{d\tau^2} + b_{20}\dfrac{d\xi_1}{d\tau} + b_{21}\dfrac{d\xi}{d\tau} + b_{22}\,\xi\dfrac{d\xi}{d\tau} + b_{23}\,\xi^2\dfrac{d\xi}{d\tau} + \\ \qquad + k_{20}\,\xi_1 + k_{21}\,\xi + k_{22}\,\xi^2 + k_{23}\,\xi^3 = P_2 \cos\eta\tau, \end{cases} \tag{1}$$

where $b_{10} = \dfrac{\mu k_0'}{m_1' \omega_1}$, $b_{11} = -\dfrac{\mu k_1'}{m_1' \omega_1}$, $b_{12} = -\dfrac{\mu k_2' \Delta}{m_1' \omega_1}$, $b_{13} = -\dfrac{\mu k_3' \Delta^2}{m_1' \omega_1}$,
$b_{20} = -\dfrac{\mu k_0'}{m_1' \omega_1}$, $b_{21} = \dfrac{\mu(m_1' + m_2)k_1'}{m_1' m_2 \omega_1}$, $b_{22} = \dfrac{\mu(m_1' + m_2)k_2' \Delta}{m_1' m_2 \omega_1}$,
$b_{23} = \dfrac{\mu(m_1' + m_2)k_3' \Delta^2}{m_1' m_2 \omega_1}$, $k_{10} = \dfrac{k_0}{m_1' \omega_1^2}$, $k_{11} = -\dfrac{k_1}{m_1' \omega_1^2}$, $k_{12} = -\dfrac{k_2 \Delta}{m_1' \omega_1^2}$,

$$k_{13} = -\frac{k_3 \Delta^2}{m_1' \omega_1^2},\ k_{20} = -\frac{k_0}{m_1' \omega_1^2},\ k_{21} = \frac{k_1 (m_1' + m_2)}{m_1' m_2 \omega_1^2},\ k_{22} = \frac{k_2 (m_1' + m_2)\Delta}{m_1' m_2 \omega_1^2},$$

$$k_{23} = \frac{k_3 (m_1' + m_2)\Delta^2}{m_1' m_2 \omega_1^2},\ P_1 = \frac{m_0 r}{m_1' \Delta}\eta^2,\ P_2 = -P_1,\ m_1' = m_0 + m_1,\ \eta = \omega/\omega_1 .$$

Values of the physical parameters of the vibromachine are $m_0 = 50$ kg, $m_1 = 700$ kg, $m_2 = 550$ kg, $k_0 = 0.12 \cdot 10^6$ N/m, $k_1 = 5.5 \cdot 10^6$ N/m, $r = 0.088$ m, $\mu = 0.0008$ s, $k_0' = k_0$, $k_1' = k_1$ and the working frequency of the engine $\omega = 100$ rad/s. We suppose below the possibility of changing η, k_2, k_3, k_2', k_3' only.

Method of investigation

The steady motions of the machine we find by the harmonic balance method [4]. According to it solutions of the system (1) we find in the form of finite complex expansions

$$\xi_1(\tau) = \sum_{n=-N}^{N} c_n^{(1)} e^{in\eta\tau},\ \xi(\tau) = \sum_{n=-N}^{N} c_n e^{in\eta\tau}, \qquad (2)$$

where N is a number of harmonics taken into consideration. It is supposed that the trigonometric view of the solutions of (1) is $\sum_{j=0}^{N} A_j \cos(j\eta\tau - \varphi_j)$, where the amplitude $A_j = 2\sqrt{c_j c_{-j}}$ and initial phase $\varphi_j \in [-\pi, \pi)$, i.e. $\varphi_j = \arccos\frac{c_j + c_{-j}}{2\sqrt{c_j c_{-j}}}$ and $\varphi_j = -\arccos\frac{c_j + c_{-j}}{2\sqrt{c_j c_{-j}}}$ if $(\Im c_{-j} = 0 \ \wedge\ \Re c_{-j} < 0) \vee \Im c_{-j} < 0$.

After substituting (2) into (1) and equating coefficients of equal powers of $e^{in\eta\tau}$ one may get the algebraic system of equations with respect to c_n

$$
\begin{cases}
(k_{10} + b_{10} i\eta n - \eta^2 n^2) c_n^{(1)} + (k_{11} + b_{11} i\eta n) c_n + \\
\quad + \sum\limits_{j=-N}^{N} c_j c_{n-j} (k_{12} + b_{12} i\eta (n-j)) + \\
\quad + \sum\limits_{j=-N}^{N} \sum\limits_{m=-N}^{N} c_j c_m c_{n-j-m} (k_{13} + b_{13} i\eta (n-j-m)) = \begin{cases} P_1/2, & n = \pm 1 \\ 0, & n \neq \pm 1 \end{cases}, \\
(k_{20} + b_{20} i\eta n) c_n^{(1)} + (k_{21} + b_{21} i\eta n - \eta^2 n^2) c_n + \\
\quad + \sum\limits_{j=-N}^{N} c_j c_{n-j} (k_{22} + b_{22} i\eta (n-j)) + \\
\quad + \sum\limits_{j=-N}^{N} \sum\limits_{m=-N}^{N} c_j c_m c_{n-j-m} (k_{23} + b_{23} i\eta (n-j-m)) = \begin{cases} P_2/2, & n = \pm 1 \\ 0, & n \neq \pm 1 \end{cases},
\end{cases} \quad (3)
$$

where $n, n-j, n-j-m \in [-N, N]$. Dimension of this system equals $2(2N+1)$. Then consequently changing one of the parameters of the system (1) and solving the system (3) one may find bifurcation diagrams of the system, in particular, the amplitude- (AFC) and phase-frequency characteristics (PFC). Computations are fulfilled for five harmonic components in (2) were taken into account, i.e. $N = 5$. The corresponding software are worked out as the toolbox of the program MATLAB and described in [5].

Results

Here we considered the frequency zone located between the natural ones (see Figure 2) and studied the pure resonances of lower order. The rectangular symbol in figure shows the present working frequency of the machine. One may mention that this is the frequency of antiresonance, – very small oscillations of the frame and quite sufficient motions of the working organ.

The main practical results are the following.

Using correlation $p\omega \approx |p_1|\omega_1 + |p_2|\omega_2$ [6], where $p, p_1, p_2 \in Z$ in this zone with the help of our software we succeeded to find only the resonances of $3:1$, $2:1$ and $1:3$. For linear dissipation ($k_2' = k_3' = 0$) the corresponding bifurcation diagrams (AFC and PFC) are shown in Figures 3–5 for $k_{13}/k_{11} = 1$, $k_{12}/k_{11} = 0$.

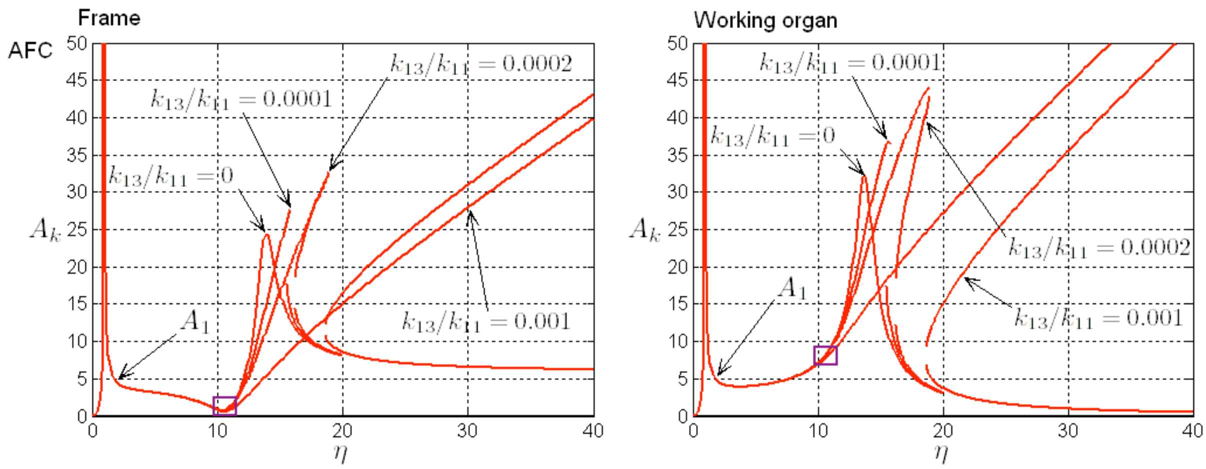

Figure 2 – AFC for different values of the degree of nonlinearity of the elastic characteristics

By our opinion the superharmonic resonance $2:1$ (see Figure 4) is the most interesting among it for practical purposes, it is rather intensive and gives an opportunity to form biharmonic oscillations which are necessary for practice. Really, one may compare, for example, motions that are recommended for concentration tables [1] and oscillations generated in superharmonic zone for $\eta = 20$ (see Figure 6). The relative value of the second harmonic A_2 / A_1 in

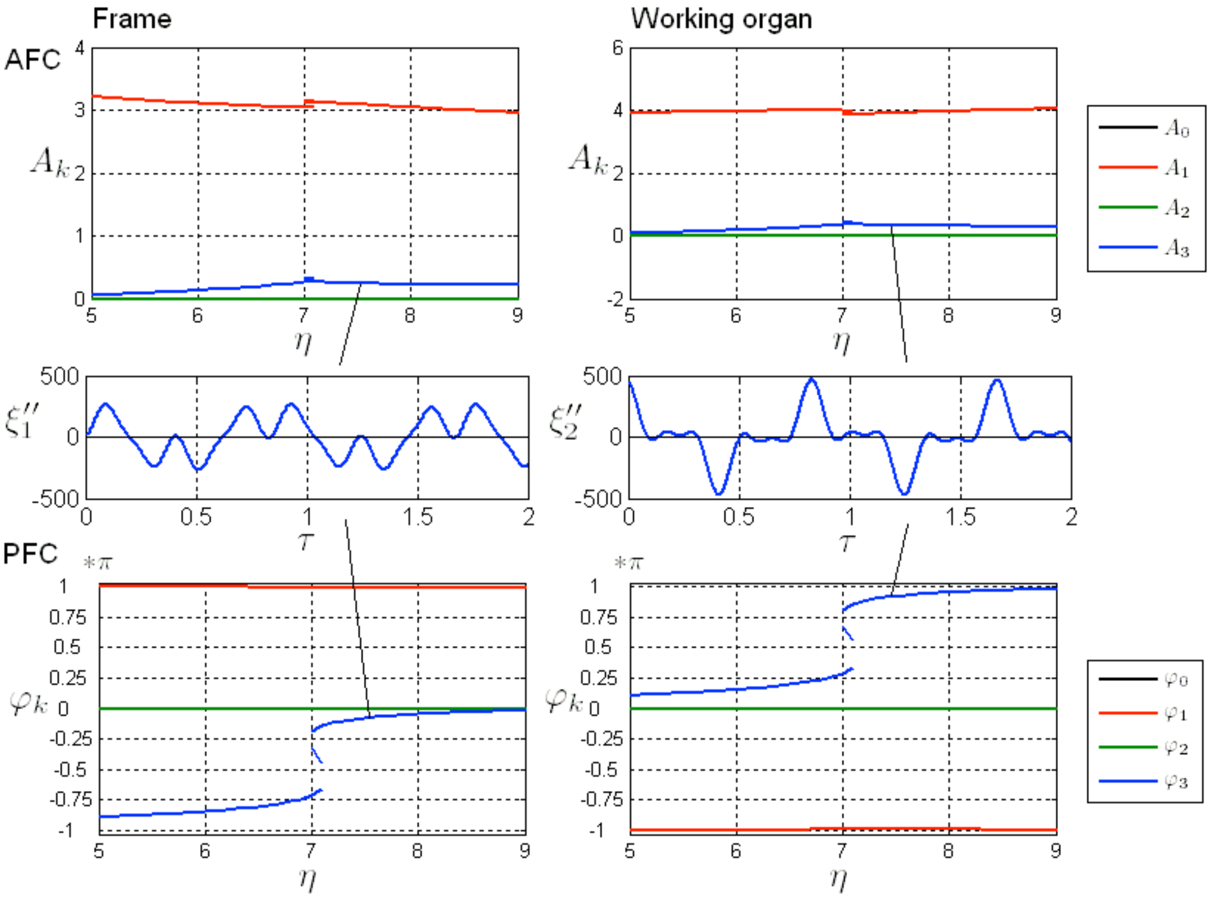

Figure 3 – Superharmonic resonance of order $3:1$

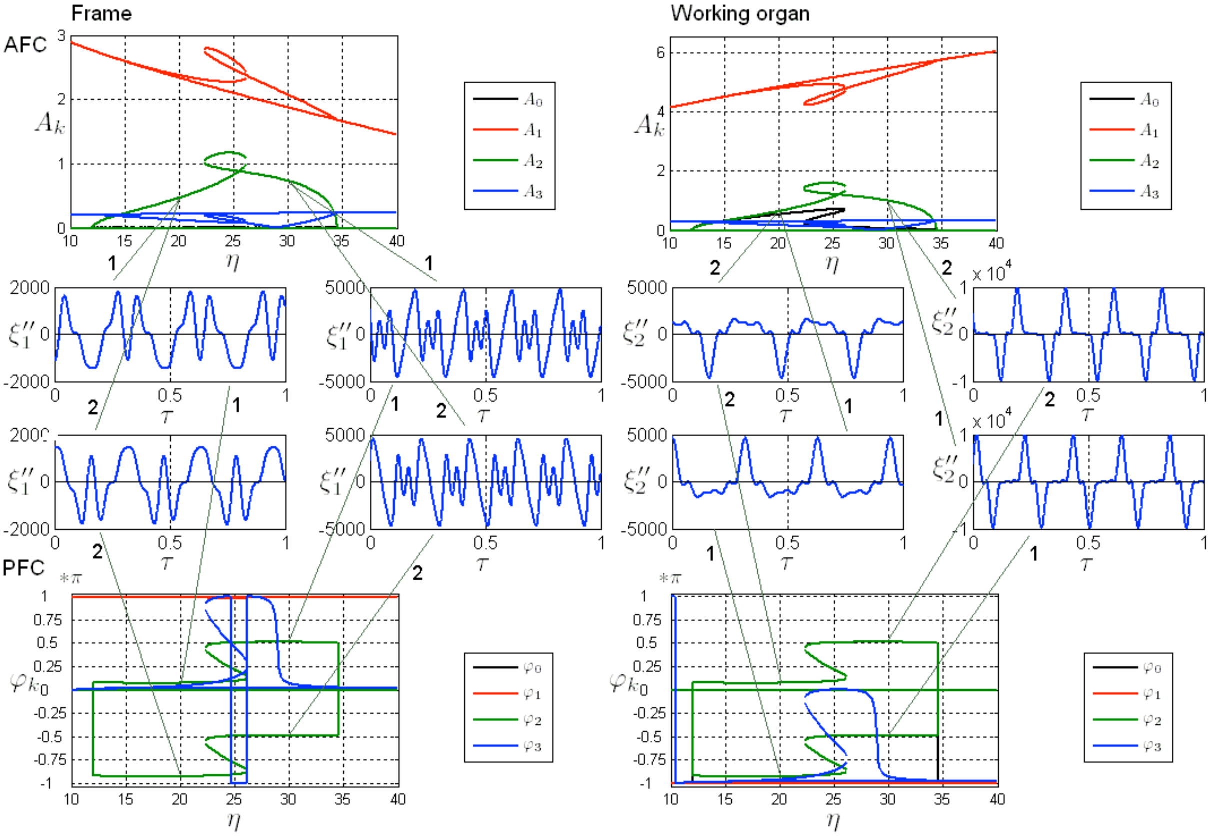

Figure 4 – Superharmonic resonance of order $2:1$

displacement of the working organ forms approximately $12 \div 15\,\%$ and skew of it approximately equals $2\varphi_2 - \varphi_1 \approx 0.08\pi$ (see Figure 4). But one of the essential faults of superharmonic resonances of even order is the existence of two opposite regimes (see regimes 1 and 2 in Figure 6), that may create certain difficulties when you start the engine.

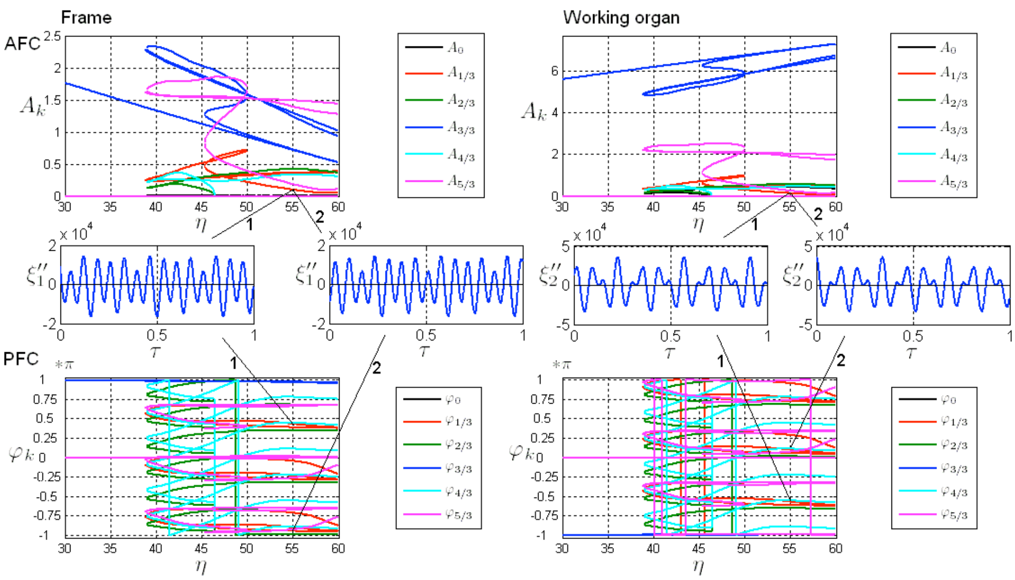

Figure 5 – Subharmonic resonance of order $1:3$

Trying to strengthen one of the superharmonic regimes we introduce asymmetry into elastic ties, this attempt is demonstrated in Figure 7. From the view of the PFC it may be stated that two opposite regimes continue to exist, but the practical results of this measure hardly may be interpreted surely without getting their basins of attraction.

It is also necessary to mention that the discovered resonance is quite stable to level of dissipation in the system. This fact is demonstrated in Figure 8 where the bifurcation diagrams (the amplitude and phase frequency response) are given for asymmetric elastic ties and nonlinear resistance. It is interesting that in this case the view of PFC already shows the existence of a single polyharmonic regime for $\eta = 17$, for example. But, truly saying, some certain changes also take place in the amplitude-phase correlations (see diagrams of accelerations in Figure 8). Thus, shift phase of the second harmonic with respect to the first one equals already $2\varphi_2 - \varphi_1 \approx 0.25\pi$.

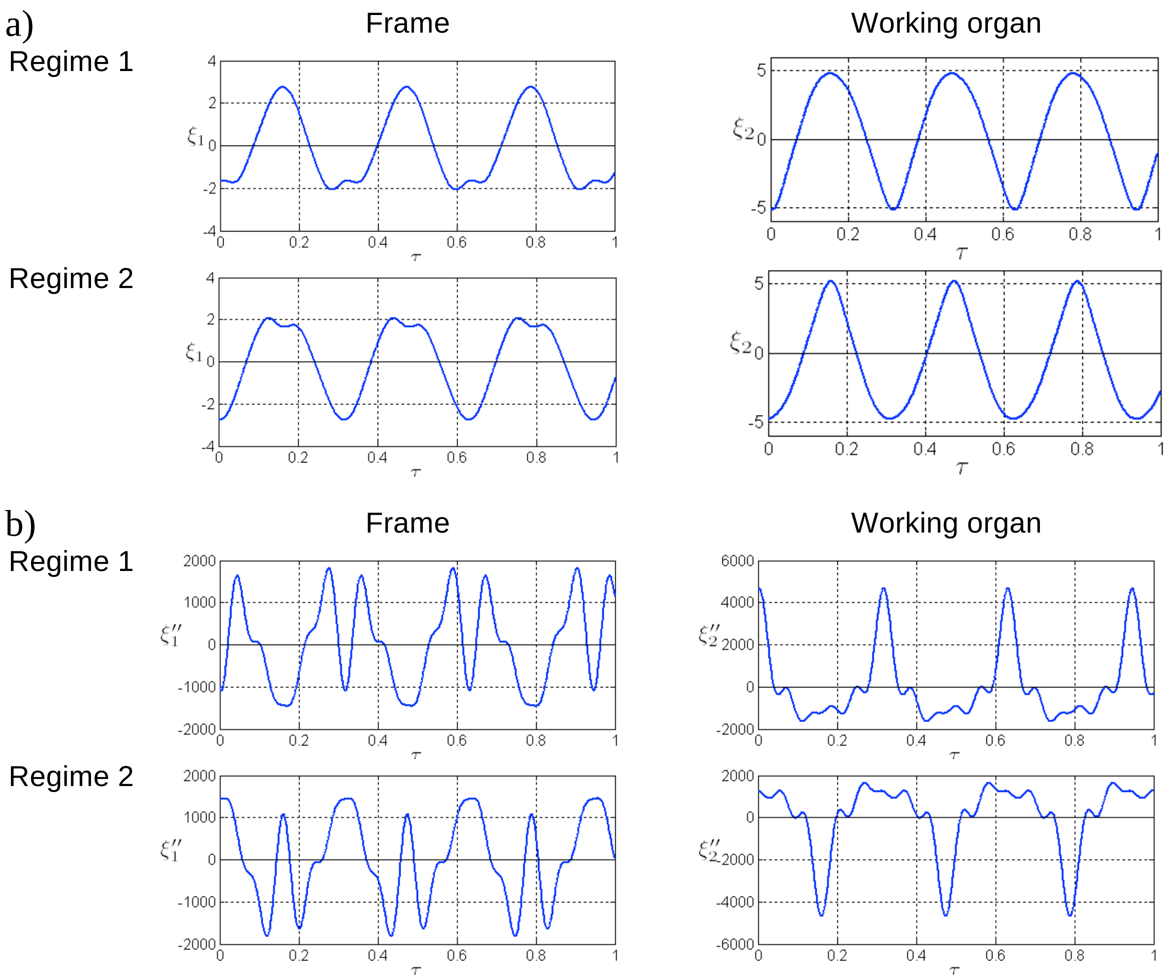

Figure 6 – Diagrams of displacements (a) and accelerations (b) of vibromachine for superharmonic resonance $2:1$ and $\eta = 20$, $k_{13} / k_{11} = 1$, $k_{12} / k_{11} = 0$ (see Figure 4)

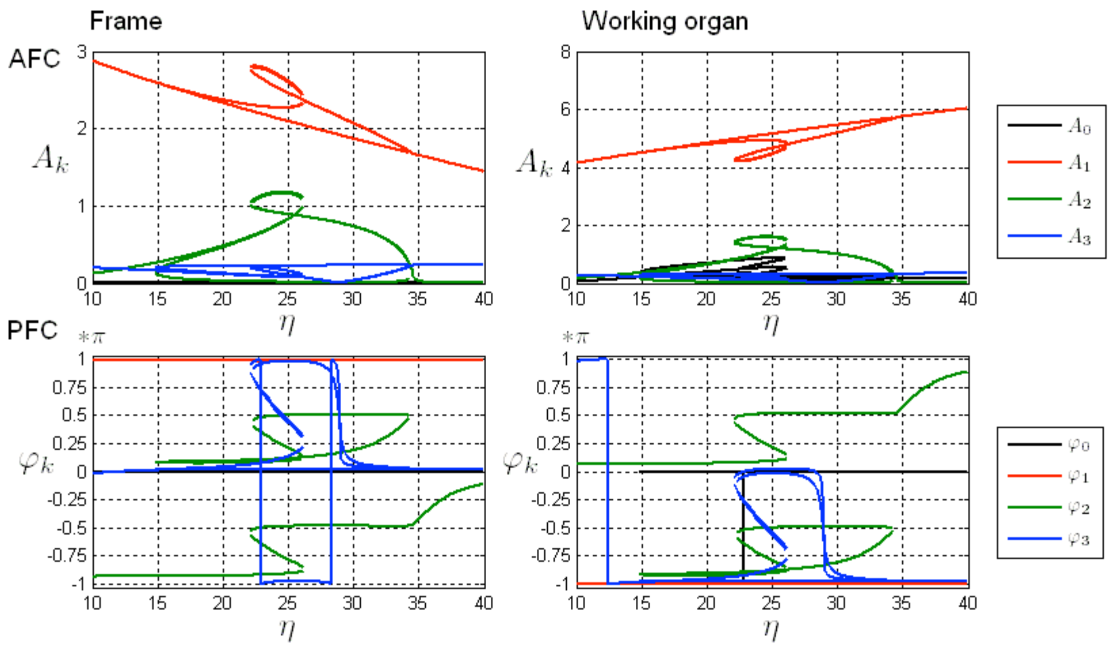

Figure 7 – Superharmonic resonance $2:1$ for linear dissipation and $k_{13}/k_{11}=1,\ k_{12}/k_{11}=-0.5$

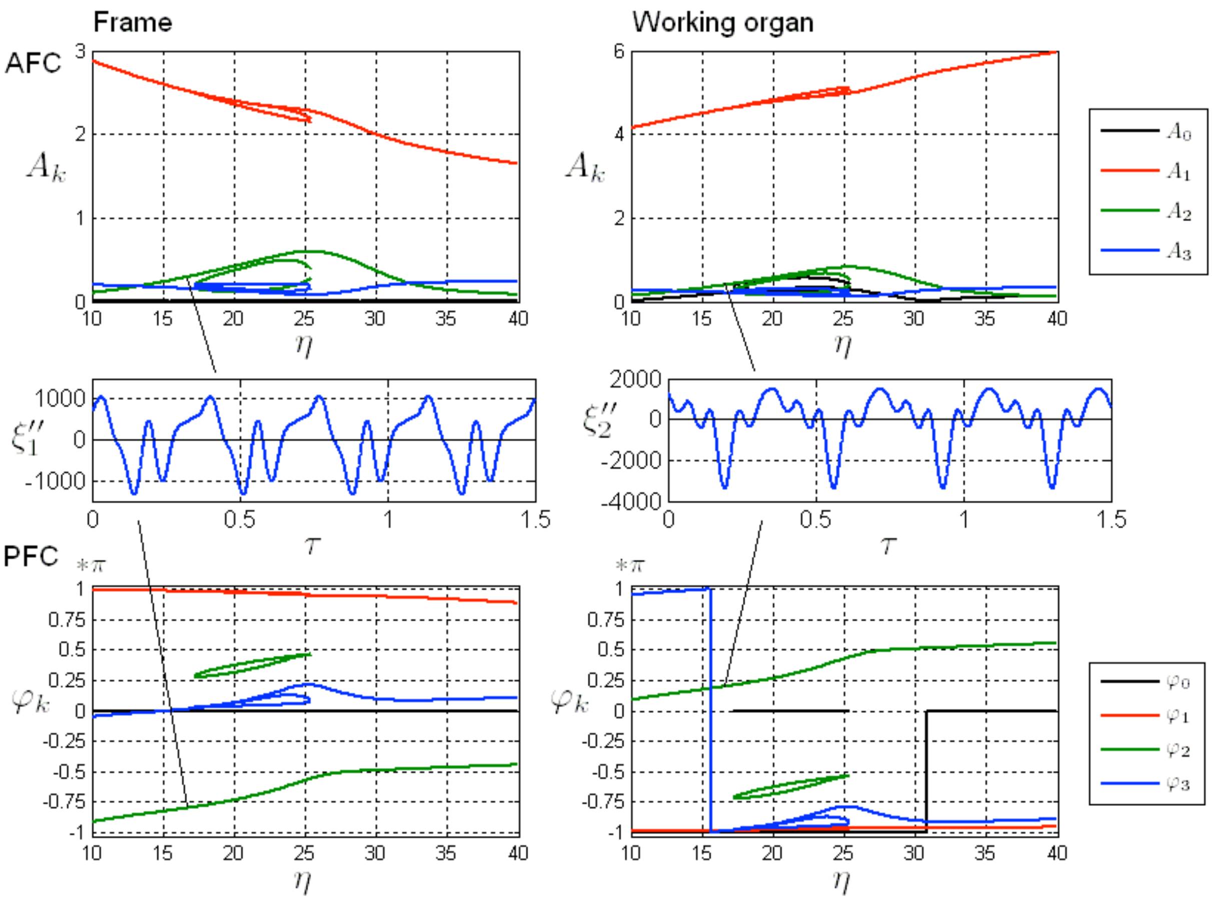

Figure 8 – Superharmonic resonance $2:1$ for nonlinear dissipation $(k_2'=k_2,\ k_3'=k_3)$ and $k_{13}/k_{11}=1,\ k_{12}/k_{11}=-0.5$

It is also important to notice that the resonance $2:1$ in this vibrating machine may be realized without essential changes in its construction and for the same angle velocity of the engine $\omega = 100$ rad/s by choosing only the parameters of the main elastic ties. Calculations which were performed on the base of correlations (1) and expression for the first natural frequency

$$\omega_1^2 = \frac{\omega^2}{\eta^2} = \frac{k_{10} + k_{21} - \sqrt{(k_{10} - k_{21})^2 + 4k_{11}k_{20}}}{2}$$

show that for getting the value of non-dimensional frequency $\eta = 17$ the stiffness of the linear part of the main elastic ties must be taken $k_1 = 0.36 \cdot 10^6$ N/m instead of its initial value equal $k_1 = 5.5 \cdot 10^6$ N/m.

Conclusion

So, the carried out investigations demonstrate the principal possibility of forming practically significant polyharmonic oscillations of the working organ of the vibrating machines by use of nonlinear elastic ties and realization of the superharmonic resonance of the second order. These oscillations exist for broad range of the parameters and quite stable to the level of dissipation. But one needs to keep in mind that the frame of the machine is also pulled into these motions and as a result quality of foundation insulation has become worse.

To the number of advantages of such way of forming polyharmonic vibrations one may include the presence only one harmonic exciter and the necessity to make just small changes in the machine construction. But some problems connected with the design of such ties still remain.

The limited capacity of the engine is also one of the interesting and important factors which can make adjustments upon the process of forming polyharmonic vibrations and, by this reason, must be taken into account too.

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К вопросу о формировании бигармонических колебаний в двухмассных нелинейных вибрационных машинах при идеальном гармоническом возбуждении. В работе рассматривается принципиальная схема двухмассной вибрационной машины с идеальным инерционным возбуждением и кубической характеристикой упругих и диссипативных связей. Изучаются ее однонаправленные колебания, которые описываются системой двух дифференциальных уравнений второго порядка. После перехода в системе к безразмерным параметрам для анализа стационарных движений вибромашины используется комплексная форма метода гармонического баланса. В результате этого задача нахождения периодических колебаний вибромашины сводится к решению системы полиномиальных уравнений с комплексными коэффициентами и осуществляется с использованием оригинального программного обеспечения. Решение указанной системы уравнений при последовательном изменении одного из параметров вибромашины позволяет строить бифуркационные диаграммы, возможные точки бифуркации устанавливаются путем контролирования смены знака якобиана системы, а анализ устойчивости решений по первому приближению осуществляется с использованием теории Флоке-Ляпунова. С использованием этого программного обеспечения анализируются стационарные колебания вибромашины в частотном диапазоне, расположенном между ее собственными частотами. Для выбранных параметров вибромашины устанавливается существование комбинационных резонансов порядков 3:1, 2:1 и 1:3, приводятся соответствующие амплитудно- и фазо-частотные характеристики колеблющихся масс. Их анализ показывает, что с практической точки зрения, наиболее значимыми является супергармонические колебания порядка 2:1, которым, далее, и уделено основное внимание. Отмечается влияние степени нелинейности и асимметрии упругой характеристики и уровня диссипации на поведение системы и число периодических режимов. Приводятся диаграммы законов движений и ускорений колеблющихся масс, демонстрируется возможность формирования практически значимых полигармоничеких вибраций. Отмечается возможность ввода рассматриваемой вибромашины в супергармонический резонанс путем подбора параметров упругих связей.
Ключевые слова: Вибромашина, антирезонанс, полигармоническая вибрация, метод гармонического баланса, бифуркационная диаграмма, численный анализ.

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До питання про формування бігармонійних коливань в двомасних нелінійних вібраційних машинах при ідеальному гармонійному збудженні. У роботі розглядається принципова схема двомасної вібраційної машини з ідеальним інерційним збудженням і кубічною характеристикою пружних і дисипативних зв'язків. Вивчаються її односпрямовані коливання, які описуються системою двох диференціальних рівнянь другого порядку. Після переходу в системі до безрозмірних параметрів для аналізу стаціонарних рухів вібромашини використовується комплексна форма методу гармонійного балансу. В результаті цього задача знаходження періодичних коливань вібромашини зводиться до вирішення системи поліноміальних рівнянь з комплексними коефіцієнтами і здійснюється з використанням оригінального програмного забезпечення. Рішення зазначеної системи рівнянь при послідовній зміні одного з параметрів вібромашини дозволяє будувати біфуркаційні діаграми, можливі точки біфуркації встановлюються шляхом контролювання зміни знака якобіана системи, а аналіз стійкості рішень по першому наближенню здійснюється з використанням теорії Флоке-Ляпунова. З використанням цього програмного забезпечення аналізуються стаціонарні коливання вібромашини в частотному діапазоні, розташованому між її власними частотами. Для обраних параметрів вібромашини встановлюється існування комбінаційних резонансів порядків 3:1, 2:1 і 1:3, приводяться відповідні амплітудно- і фазо-частотні характеристики коливних мас. Їх аналіз показує, що з практичної точки зору, найбільш значимими є супергармонійні коливання порядку 2:1, яким, далі, і приділена основна увага. Відзначається вплив ступеня нелінійності та асиметрії пружної характеристики і рівня дисипації на поведінку системи і число періодичних режимів. Приводяться діаграми законів рухів і прискорень коливних мас, демонструється можливість формування практично значимих полігармонійних вібрацій. Відзначається можливість введення даної вібромашини в супергармонійний резонанс шляхом підбору параметрів пружних зв'язків.

Ключові слова: Вібромашина, антірезонанс, полігармонійна вібрація, метод гармонійного балансу, біфуркаційна діаграма, чисельний аналіз.