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OPTIMIZATION METHODS OF OPTICAL SIGNALS BY THE CRITERION OF THE MINIMUM OF THE PRESENTED BASE

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МЕТОДИ ОПТИМІЗАЦІЇ ОПТИЧНИХ СИГНАЛІВ ЗА КРИТЕРІЄМ МІНІМУМУ ПРИВЕДЕНОЇ БАЗИ

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МЕТОДЫ ОПТИМИЗАЦИИ ОПТИЧЕСКИХ СИГНАЛОВ ПО КРИТЕРИЮ МИНИМУМА ПРИВЕДЕННОЙ БАЗЫ

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Abstract. The article is devoted to the development of methods for improving the spectral efficiency of fiber-optic transmission systems in terms of optimizing signals in a narrowband channel. The channel width is conventionally taken in the range of 50 – 100 GHz. The following conditions are also considered to be fulfilled: the signal at the input has approximately circular polarization; the instantaneous power of the signal is relatively small, which makes it possible to neglect nonlinear effects such as four-wave mixing or phase self-modulation; the fiber profile in the refractive index is strictly stepped; fiber operates in single mode; the core material is an isotropic medium; the dependence of the attenuation coefficient in the frequency in the considered range can be considered a constant function. Thus, the considered model of signal transformations in a fiber is reduced to its deformation due to material dispersion. The leading optimization criterion is the minimum of the reduced base. This concept was introduced earlier in the works of the authors. This criterion requires minimizing the product of the effective spectral width on the transmission side by the effective signal duration on the receiving side. The solution of the problem in general form is given - in the formulation of the isoperimetric problem of the calculus of variations. It is shown that the solution of the optimization problem by the minimum criterion of the reduced base can be reduced to the problem of optimizing the signal base in the classical sense. A general solution of the isoperimetric problem is given as functions of a parabolic cylinder. Also presented are particular solutions of the optimization problem on parametric families from the Nyquist pulse class. The obtained solutions show that the optimal values of the variable parameters practically do not depend on the carrier frequency and on the length of the regeneration section. This allows the extension of the optimal solutions obtained for a single narrowband channel to the case of multichannel fiber-optic transmission systems using frequency (spectral) multiplexing.

Key words: reduced base, isoperimetric problem, optimal optical signals, spectral efficiency, numerical optimization methods, Gaussian pulse, Nyquist pulses.

Анотація. Стаття присвячена розвитку методів підвищення спектральної ефективності волоконно-оптичних систем передачі в частині оптимізації сигналів у вузькополосному каналі. Ширина каналу умовно приймається в межах 50…100 ГГц. Також вважаються виконаними умови: сигнал на вході має приблизно колову поляризацію; миттєва потужність сигналу відносно мала, що дозволяє знехтувати нелінійними ефектами типу чотирихвильового зміщення або фазової самомодуляції; профіль волокна за коефіцієнтом заломлення строго ступінчастий; волокно функціонує в одномодовому режимі; матеріал серцевини є ізотропним середовищем; залежність коефіцієнта загасання за частотою в розглянутому діапазоні можна вважати постійною функцією. Тим самим дана модель перетворень сигналу у волокні зводиться до його деформації внаслідок матеріальної дисперсії. Провідним критерієм оптимізації обрано мінімум приведеної бази. Дане поняття введено раніше в роботах авторів. Цей критерій вимагає мінімізувати добуток ефективної ширини спектра на стороні передачі на ефективну тривалість сигналу на стороні прийому. Надається рішення задачі в загальному вигляді - в постановці ізопериметричної задачі варіаційного обчислення. Показано, що рішення задачі оптимізації за критерієм мінімуму приведеної бази можна звести до задачі оптимізації бази сигналу в класичному сенсі. Надано загальне рішення ізопериметричної задачі у вигляді функцій параболічного циліндра. Також надаються окремі рішення задачі оптимізації на параметричних сімействах з класу імпульсів Найквіста. Отримані рішення показують, що оптимальні значення змінних параметрів практично не залежать від несучої частоти і від довжини регенераційної ділянки. Це дозволяє поширити оптимальні рішення, отримані для одного вузькосмугового каналу на випадок багатоканальних волоконно-оптичних систем передачі з використанням частотного (спектрального) мультиплексування.

Ключові слова: приведена база, ізопериметрична задача, оптимальні оптичні сигнали, спектральна ефективність, чисельні методи оптимізації, гауссів імпульс, імпульси Найквіста.

Аннотация. Статья посвящена развитию методов повышения спектральной эффективности волоконно-оптических систем передачи в части оптимизации сигналов в узкополосном канале. Ширина канала условно принимается в пределах 50…100 ГГц. Также считаются выполненными условия: сигнал на входе имеет приблизительно круговую поляризацию; мгновенная мощность сигнала относительно мала, что позволяет пренебречь нелинейными эффектами типа четырехволнового смешения или фазовой самомодуляции; профиль волокна по коэффициенту преломления строго ступенчатый; волокно функционирует в одномодовом режиме; материал сердцевины является изотропной средой; зависимость коэффициента затухания по частоте в рассматриваемом диапазоне можно считать постоянной функцией. Тем самым рассматриваемая модель преобразований сигнала в волокне сводится к его деформации вследствие материальной дисперсии. Ведущим критерием оптимизации выбран минимум приведенной базы. Данное понятие введено ранее в работах авторов. Этот критерий требует минимизировать произведение эффективной ширины спектра на стороне передачи на эффективную длительность сигнала на стороне приема. Дается решение задачи в общем виде – в постановке изопериметрической задачи вариационного исчисления. Показано, что решение задачи оптимизации по критерию минимума приведенной базы можно свести к задаче оптимизации базы сигнала в классическом смысле. Дано общее решение изопериметрической задачи в виде функций параболического цилиндра. Также приводятся частные решения задачи оптимизации на параметрических семействах из класса импульсов Найквиста. Полученные решения показывают, что оптимальные значения варьируемых параметров практически не зависят от несущей частоты и от длины регенерационного участка. Это позволяет распространить оптимальные решения, полученные для одного узкополосного канала на случай многоканальных волоконно-оптических систем передачи с использованием частотного (спектрального) мультиплексирования.

Ключевые слова: приведенная база, изопериметрическая задача, оптимальные оптические сигналы, спектральная эффективность, численные методы оптимизации, гауссов импульс, импульсы Найквиста.

The urgency of the problem of increasing the spectral efficiency of fiber-optic transmission systems (FOTS) was considered in the works of the authors [1…3]. As one of the methods for increasing the bandwidth of FOTS, we considered a group of solutions called signaling methods [4]. The essence of these methods is reduced to the formation of alphabets of signals, providing the possibility of transferring more than one bit of information in one pulse. The possibility of generating short optical signals of a given structure was considered in works [5, 6]. At the same time, the problem of optimizing optical signals by various criteria has been little studied. In narrowband bandwidth FOTS channel ΔF transmission speed will be limited by the pulse duration ΔT. Obviously, the impulse with a minimum base will be optimal: $B_0 = \Delta T \cdot \Delta F \to \min$ (further – initial base). The task of optimizing signals by this criterion was solved in the classical theory of signals, for example, in [7]. Certain specifics of solving optimization problems as applied to FOTS are caused by active interaction of the signal with the transmission medium. In this case, the optical pulse (OP) is distorted due to the nonuniformity of the dependence of the attenuation coefficient in frequency, as well as due to dispersion. The consequence of these causes is the elongation of the OP as it propagates through the optical fiber (OF). Shown in [8], that for the narrowband channel of

FOTS (width of about 50…100 GHz), the energy spectrum is an invariant characteristic of the OP: the deformation of this function can be neglected. Therefore, the concept of the minimum of the reduced base is introduced [3]:

$$B(L) = \Delta T(z = L) \cdot \Delta F(z = 0) \to \min\,, \qquad (1)$$

where ΔT – OP duration characteristic; z – the distance from the point of entry OP in OF; L – the length of the regeneration section (RS); ΔF – OP spectrum width characteristic.

The purpose of this article is to develop methods for solving optimization problems of OP by the criterion (1) – the minimum of the reduced base.

For the narrowband channel of the FOTS, we suppose that: the signal at the input of an optical fiber has approximately circular polarization; the instantaneous power of the OP is relatively small, which allows one to neglect nonlinear effects such as four-wave mixing or phase self-modulation; the OF profile in refractive index is strictly stepped; the OF operates in single mode; the core material of the OF is an isotropic medium; and the dependence of the attenuation coefficient in frequency in the considered range can be considered a constant function.

In this case, the effects of the interaction of the OP with the medium are reduced to the material dispersion in the core of the OF [9]. Then the model of OP distortion in the medium of OF is represented as a dispersion of group velocities [10]:

$$U(t,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G_0(\Omega)\exp\left(j\Omega t - jz\frac{1}{2}k_{\omega 0}^{[2]}\Omega^2 \right) d\Omega \;, \qquad (2)$$

where $U(t,z)$ – low frequency component of the signal (LFC) at distance z from the entry point in the OF; $G_0(\Omega)$ – spectral density of OP at the time of entry into the OF; Ω – frequency deviation (within 50 – 100 GHz) relative to carrier frequency ω_0 (which has an order of 160…375 THz); $k_{\omega 0}^{[2]}$ – second derivative of the wave number at point ω_0, which is expressed through the values of the derivatives of the refractive index:

$$k_{\omega 0}^{[2]} = \frac{1}{c}\left[2\frac{dn(\omega)}{d\omega}\Big|_{\omega_0} + \omega_0 \frac{d^2 n(\omega)}{d\omega^2}\Big|_{\omega_0} \right]. \qquad (3)$$

Next, we replace the designation of the parameter frequency deviation Ω by ω, realizing that the carrier frequency is moved to a point $\omega_0 = 0$ (however, the numeric values of the parameter $k_{\omega 0}^{[2]}$ taken at the real value of the carrier frequency of the order of 160 – 375 THz). We will also use the notation for expression (3): $k_{\omega 0}^{[2]} = k_0$.

Dependence (2) shows that the LFC OP at distance z is expressed as the inverse Fourier transform of the spectral density in the form:

$$G(z,\omega) = G_0(\omega)\exp\left(-j\frac{1}{2}zk_0\omega^2 \right) = G_0(\omega)H(\omega)\,, \qquad (4)$$

where $H(\omega) = \exp\left(-j\frac{1}{2}zk_0\omega^2 \right)$ – frequency transmission coefficient (FTC) of OF.

In the following, we will use the following two positions.

First, the solution of optimization problems for the initial and reduced base (1) is equivalent to the solution of the optimization problem for the functional [3]:

$$\alpha B^{\beta} = \alpha \Delta T^{\beta} \Delta F^{\beta} \to \min\,, \qquad (5)$$

where α and β – arbitrary positive numbers.

Second: the effective duration of the OP in the time domain is expressed through the derivative of its spectral density [11]:

$$\Delta T^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty}[G'(\omega)]^2 d\omega . \quad (6)$$

As characteristics of the OP, we consider three main functions, which are expressed in terms of spectral density:

$$E(G) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G^2(\omega) d\omega , \quad (7)$$

$$\Delta\Omega^2(G) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \omega^2 G^2(\omega) d\omega , \quad (8)$$

$$\Delta T^2(G) = \frac{1}{2\pi}\int_{-\infty}^{\infty}[G'(\omega)]^2 d\omega , \quad (9)$$

where $E(G)$ – energy of OP; $\Delta\Omega^2(G)$ – effective spectral width of OP; $\Delta T^2(G)$ – effective duration of OP.

Taking into account position (5), the dimensional coefficient $1/2\pi$ can be neglected. Next, we fix the energy of the OP, assuming its constant $E(G) = E = \mathrm{const}(G)$. You can also fix any of the parameters $\Delta\Omega^2(G)$ or $\Delta T^2(G)$, and optimization is performed on the free parameter. So, assuming equal to the effective duration of the OP (9) $\Delta T^2(G) = \Delta T = \mathrm{const}(G)$ we come to the formulation of the variational problem:

$$\Delta\Omega^2(G) \to \min(G), \quad E(G) = E = \mathrm{const}(G), \quad \Delta T^2(G) = \Delta T = \mathrm{const}(G), \quad (10)$$

where are expressions for functionals are given by formulas (7…9). The problem in formulation (10) is reduced to the classical isoperimetric problem of the calculus of variations. Its statement is given in the monograph [7]. There's also this problem is reduced to a functional optimization

$$\int_{-\infty}^{\infty}\Phi(\omega, G_0, G_0')d\omega = \int_{-\infty}^{\infty}\left[\omega^2 G_0{}^2(\omega) + \lambda_1 G_0^2(\omega) + \lambda_2 \, | G_0'(\omega)|^2\right] d\omega \to \min! , \quad (11)$$

where λ_1 and λ_2 – free parameters determined by isoperimetric conditions (7) and (9). After determining the first variation of the functional (11), the solution of the problem is reduced to the solution of the Euler differential equation:

$$\frac{\partial^2 G_0(\omega)}{\partial\omega^2} - \frac{(\omega^2 + \lambda_1)}{\lambda_2} G_0(\omega) = 0 . \quad (12)$$

It is interesting that in [7] the final solution of the problem in the form of a concrete optimal function was not given (the author limited himself to obtaining a theoretical numerical lower bound for the values of the optimal base). With reference to the applied problem under study, the solution of equation (12) is obtained in a closed form [3]:

$$G_0(\omega) = C_1 D_{\frac{1}{2}(-a\sqrt{b}-1)}\left(\omega\sqrt{2}\sqrt[4]{b}\right) + C_2 D_{\frac{1}{2}(a\sqrt{b}-1)}\left(j\omega\sqrt{2}\sqrt[4]{b}\right), \quad (13)$$

where D – parabolic cylinder functions (for example, [12, с. 1078]); $a = \lambda_1$, $b = 1/\lambda_2$.

The spectral density in the form of (13) is difficult to analyze, and it is even more difficult to reproduce a real signal with such a frequency response. At the same time, the function in the form of the spectral density of a Gaussian ("bell") pulse gives an obvious particular solution to equation (12)

$$G_0(\omega) = \alpha \exp(-\beta\omega^2), \quad \alpha = \lambda_1^2 / \lambda_2 , \quad \beta = -1 / \lambda_1 .$$

The above dependencies relate to the case of optimizing the initial base of the OP. The solution of the optimization problem by the criterion of the minimum of the reduced base (1) or, equivalently, by the criterion (5), requires certain clarifications. Taking into account the expression (6) for the effective duration of the OP, formula (9) can be represented in the form:

$$\Delta T^2(G,z)=\frac{1}{2\pi}\int_{-\infty}^{\infty}H^2(\omega)[G_0'(\omega)]^2d\omega+\frac{1}{2\pi}\int_{-\infty}^{\infty}[H'(\omega)]^2G_0^2(\omega)d\omega=I_1+I_2. \quad (14)$$

Where are the integrals I_1 and I_2 taking into account the dependence of FTC (4) can be represented as

$$I_1=\frac{1}{2\pi}\int_{-\infty}^{\infty}[G_0'(\omega)]^2d\omega,\quad I_2=z^2k_0^2\frac{1}{2\pi}\int_{-\infty}^{\infty}\omega^2G_0^2(\omega)d\omega. \quad (15)$$

From expressions (15) the following are clear: the integral I_1 – the value of the effective duration of the OP at the time of entry into the OF, and the integral I_2 – the increment of the effective duration of the OP at a distance from the point of entry into the OF.

The optimization problem by the criterion of the reduced base is, generally speaking, more difficult from an analytical point of view than the optimization problem by the criterion of the initial base. In some cases, the solution of the first of them allows us to simplify the following theorem.

Let there be some function $G(\omega)$, delivering the minimum of the isoperimetric problem (10) in the sense of minimizing the initial base. Then, up to constant coefficients, the same function delivers the minimum of the solution of the isoperimetric problem in the sense of minimizing the reduced base:

$$\Delta T^2(G,z)\to\min(G),\quad E(G)=E=\text{const}(G),\quad \Delta\Omega^2(G)=\Delta\Omega=\text{const}(G). \quad (16)$$

To prove the theorem, the optimized functional (11), taking into account dependencies (14 – 15), is represented as:

$$\int_{-\infty}^{\infty}\Phi(\omega,G_0,G_0')d\omega=\int_{-\infty}^{\infty}\left\{\omega^2G_0{}^2(\omega)+\lambda_1G_0^2(\omega)+\lambda^*{}_2[z^2k_0^2\omega^2G_0{}^2(\omega)+G_0'(\omega)^2]\right\}d\omega\to\min! \quad (17)$$

In the general case, equating to zero the first variation of the functional, depending on the function under study and its first derivative, is given in the form of the equation [13]:

$$\frac{\partial\Phi}{\partial G_0}-\frac{\partial}{\partial\omega}\left(\frac{\partial\Phi}{\partial G_0'}\right)=0. \quad (18)$$

Taking into account the form of the integrand in the functional (17), the Euler equation (18) is reduced to the form:

$$\frac{d^2G_0(\omega)}{d\omega^2}-\frac{[\omega^2(1+z^2k_0^2\lambda_2^*)+\lambda_1]}{\lambda_2^*}G_0(\omega). \quad (19)$$

Denote $\lambda_2=\dfrac{\lambda_2^*}{1+z^2k_0^2\lambda_2^*}$; $\dfrac{\lambda_1}{\lambda_2}=\dfrac{\lambda_1(1+z^2k_0^2\lambda_2^*)}{\lambda_2^*}$, but then equation (19), obviously, in its form reduces to equation (12), which proves the theorem.

In fact, the solution in the form of a Gaussian pulse does not exhaust the possible solutions of equation (12). The proved theorem allows us to use solutions of the optimization problem by the criterion of the minimum of the initial base as initial approximations to the solution of the optimization problem by the criterion of the minimum of the reduced base.

In this case, the optimization problem is considered by criterion (1), when the optimized function is specified in an explicit parametric form: $G_0(\omega)=G_0(\omega,\alpha_{1,},\alpha_2,...,\alpha_M)$, where $\alpha_{1,},\alpha_2,...,\alpha_M$ – free (variable) model parameters. Functions of this type may not be solutions of the Euler equations (12) or (19) at all. However, they may have additional optimal properties. In particular, the Nyquist pulses [14] possess the property of high selectivity in the time domain. What is important in our case, the spectral density of such pulses can be expressed by functions finite in the frequency domain. That is, the Nyquist signals can be considered as optimal OP from the point of view of the selectivity property in the frequency domain. It is important that there are methods for the physical reproduction of Nyquist pulses in the optical range. At least, with sufficient accuracy it is possible to reproduce pulses with a rectangular spectral density [15]. Such signals are called in [14] Nyquist-Kotelnikov pulses. For parametric families, the optimization problem is simplified and reduces not to the optimization problem for functionals, but to the optimization problem for functions of many variables. The equations for the general solution of the optimization problem in this case can be represented as a system:

$$\frac{\partial}{\partial\alpha_m}\int_{-\infty}^{\infty}[G'(\omega,\alpha_1,\alpha_2,...,\alpha_M]^2 d\omega = 0, \quad m = 1,2,...,M-2, \tag{20}$$

$$E(G) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G^2(\omega,\alpha_1,\alpha_2,...,\alpha_M)d\omega; \qquad \Delta\Omega^2(G) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\omega^2 G^2(\omega,\alpha_1,\alpha_2,...,\alpha_M)d\omega. \tag{21}$$

Which of free parameters $\alpha_1,\alpha_2,...,\alpha_M$ include in the subsystem of equations (20), and which ones to be determined by equations (21) depends on the conditions of the problem being solved and on “mathematical convenience”: in this problem it is important that the integrals in the system of equations (20…21) are taken analytically or at least numerically at high stability of computational schemes.

Let us consider the solution of problem (20…21) using the example of one-parameter [14] Nyquist pulses with a trapezoidal spectral density

$$G_0(\omega) = \begin{cases} U, & |\omega| < \omega_A \\ \dfrac{U}{2}\left(\dfrac{\omega_B - |\omega|}{\alpha\omega_C}\right), & \omega_A \le |\omega| \le \omega_B \\ 0, & |\omega| > \omega_B \end{cases} \tag{22}$$

and with a spectral density in the form of a raised cosine

$$G_0(\omega) = \begin{cases} U, & |\omega| < \omega_A \\ \dfrac{U}{2}\left[1 + \cos\dfrac{\pi(|\omega| - \omega_A)}{2\alpha\omega_C}\right], & \omega_A \le |\omega| \le \omega_B, \\ 0, & |\omega| > \omega_B \end{cases} \tag{23}$$

where parameter α is called the roll-off factor (ROF) for function (23). For both functions (22) and (23) this parameter is defined in the same way:

$$\alpha = (\omega_C - \omega_A)/\omega_C = (\omega_B - \omega_C)/\omega_C = \frac{1}{2}(\omega_B - \omega_A)/\omega_C. \tag{24}$$

The graphs of functions (22) and (23) for different values of the parameter are given in Fig.1.

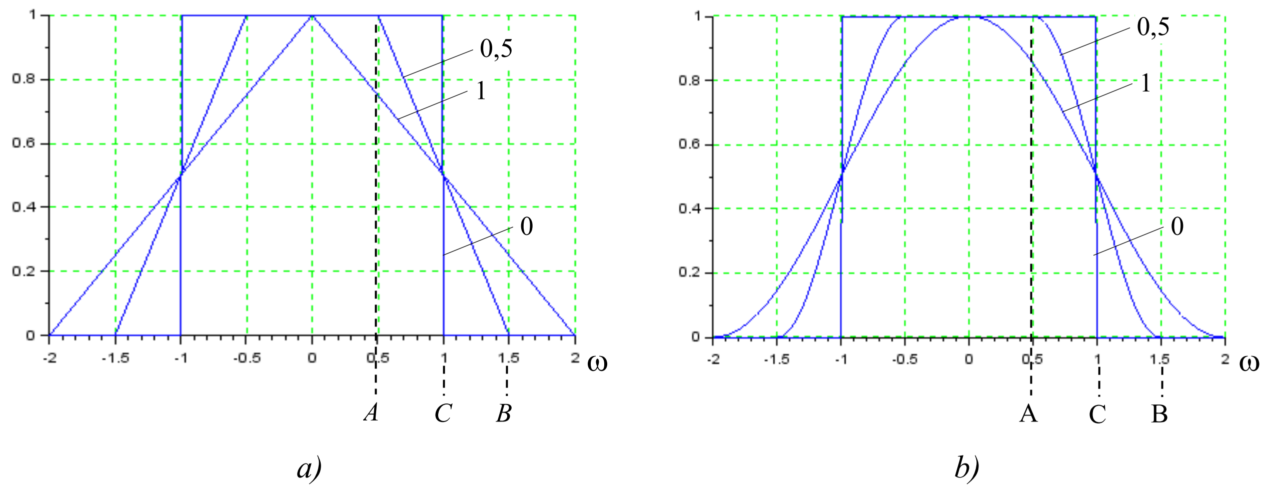


Figure 1 – spectral density graphs
a) spectral density graphs of (22); *b)* spectral density graphs of (23).

Points A, C, B correspond to the values of frequencies $\omega_A, \omega_C, \omega_B$. Figures are given for coefficient values $\alpha = 0; 0{,}5; 1$. In the figures conditionally $\omega_c = 1, U = 1$.

Both spectral densities (23, 24) in the transition region ($\omega_A \le \omega \le \omega_B$) possess the property of odd symmetry and, therefore, the corresponding signal functions satisfy the first Nyquist criterion [16].

Note that some of the variable parameters in the optimization problem (20…21) may be subject to additional conditions, as a rule, in the form of inequalities. Such conditions can follow from the physical (technical) essence of the problem being solved, as well as from the definition of functions $G_0(\omega, \alpha_1, \alpha_2, ..., \alpha_M)$. This class of conditions allows solving the problems of root selection of equations (20…21), if there are several such roots. For functions (23, 24), the restriction for ROF:

$$0 \le \alpha \le 1. \tag{25}$$

Taking into account the parity and finiteness of the spectrum (23), also taking into account the position (5), the integrals appearing in the conditions of the problem (20…21) can be calculated within $[0, \omega_B]$. Medium frequency parameter ω_C will be considered fixed for all OP. As a "standard" of energy, we use the energy of the Nyquist-Kotelnikov pulse (with a rectangular spectral density at $\alpha = 0$). The energy of such a signal is obviously $E_0 = U_0^2 \omega_C$. With an arbitrary value of the free parameter α expression for signal energy (taking into account the exemption from constant dimensional coefficients) can be written in the form:

$$E_\alpha = U_\alpha^2 \left[\int_0^{\omega_A} d\omega + \frac{1}{4\alpha^2 \omega_C^2} \int_{\omega_A}^{\omega_B} (\omega_B - \omega)^2 \, d\omega \right]. \tag{26}$$

Fair dependencies: $\omega_A = (1-\alpha)\omega_C$, $\omega_B = (1+\alpha)\omega_C$, which follow from definition (24). As a result of integration by formula (26) and the reduction of similar ones, we obtain the condition of constancy of energies:

$$E_\alpha = U_\alpha^2 \left(1 - \frac{1}{3}\alpha \right) \omega_C = U_0^2 \omega_C = \text{const}. \tag{27}$$

From condition (27) follows the dependence for the amplitude parameter, ensuring the constancy of the energies:

$$U_\alpha^2 = \frac{3U_0^2}{(3-\alpha)}. \tag{28}$$

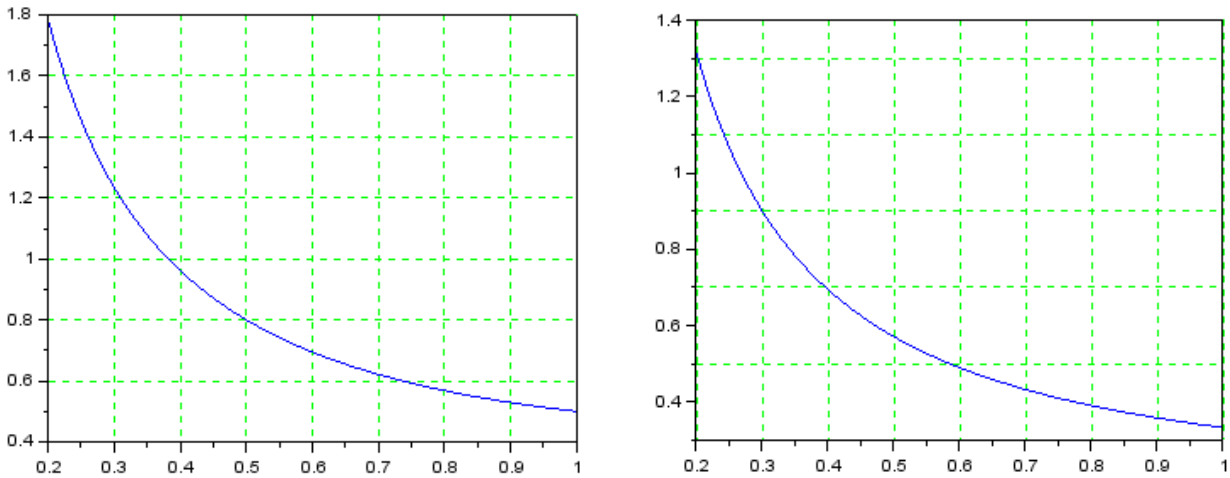
We take into account that on the interval $[0,\omega_A]$ the derivative of the function (22) is zero. Then the condition (20) for the parameter being optimized $\alpha = 0$ can be written in the form:

$$\frac{\partial}{\partial\alpha}\int_{\omega_A}^{\omega_B}[(G'(\omega)]^2 d\omega = \frac{\partial}{\partial\alpha}\frac{U_\alpha^2}{4\alpha^2\omega_C^2}\int_{\omega_A}^{\omega_B} d\omega = \frac{\partial}{\partial\alpha}\frac{9U_0^2(\omega_B-\omega_A)}{4\alpha^2\omega_C^2(3-\alpha)} = \frac{\partial}{\partial\alpha}\frac{9U_0^2\alpha\omega_C}{2\alpha^2\omega_C^2(3-\alpha)} = 0\,.$$

Having performed the differentiation in the last expression, we finally obtain the equation:

$$\frac{9U_0^2(2\alpha-3)}{2\alpha^2\omega_C(3-\alpha)^2} = 0\,,$$

which, with a nonzero denominator, has a unique solution $\alpha = 3/2$. As we see, the solution obtained goes beyond the permissible interval (25). But on this interval, the function of the form $1/(\alpha(3-\alpha))$ is monotonically decreasing (Fig. 2, *a*).



a) *b)*

Figure 2 – Optimized graphs of functions

a) graph of the function $1/(\alpha(3-\alpha))$; *b)* graph of the function $1/(\alpha(4-\alpha))$.

Thus, the optimal criterion for the minimum of the initial base in this case is an OP with a degenerate spectral density in the form of a triangular function with $\alpha = 1$ (Fig. 1.*a*, callout 1). This confirms the proposition that "good" OPs should have a unimodal spectrum [17].

Trapezoidal spectral density (22) is considered here as a theoretical example. The real reproduction of an OP with such a frequency response can hardly be carried out: the derivative of the function (22) is discontinuous at the points ω_A and ω_B. The function in the form of a raised cosine (23) is devoid of this disadvantage.

Omitting the intermediate calculations, we give the final results for this case. For the OP energy we get the expression:

$$E_\alpha = U_\alpha^2\left[\int_0^{\omega_A} d\omega + \frac{1}{4}\int_{\omega_A}^{\omega_B}\left[1+\cos\frac{\pi(|\omega|-\omega_A)}{2\alpha\omega_C}\right]^2 d\omega\right] = U_\alpha^2(1-\frac{1}{4}\alpha)\omega_C\,,$$

whence, by analogy with expression (28), we obtain the dependence of the amplitude coefficient equalizing the energy of the OP:

$$U_\alpha^2 = \frac{4U_0^2}{4-\alpha}.$$

The expression for the effective pulse duration is obtained in the form:

$$\int_{\omega_0}^{\omega_B}[(G'(\omega)]^2 d\omega = \frac{\pi^2 U_0^2}{4\alpha^2(4-\alpha)\omega_C^2}\int_{\omega_A}^{\omega_B}\left[\sin\left(\frac{\pi(\omega-\omega_A)}{2\alpha\omega_C}\right)\right]^2 d\omega = \frac{\pi^2 U_0^2}{4\alpha(4-\alpha)\omega_C}. \quad (29)$$

From the expression (29) it follows that, up to a constant coefficient of optimization, the function $1/(\alpha(4-\alpha))$ is subject to. Equating the first derivative of this function to zero gives the equation:

$$\frac{2(\alpha-2)}{\alpha^2(4-\alpha)^2} = 0,$$

which has a root $\alpha = 2$, that also goes beyond the interval (25). But as in the previous case, the function $1/(\alpha(4-\alpha))$ monotonically decreases on the interval $0 \le \alpha \le 1$, as shown in Fig. 2, *b*. So in this case, the value $\alpha = 1$ is also optimal (Fig. 1, *b*, callout 1).

The most effective and universal methods of theoretical studies, apparently, are numerical-analytical methods. They allow obtaining analytical expressions for relatively simple intermediate transformations, and obtaining the final result, using elements of computational mathematics. In this case, analytical transformations can significantly simplify computational algorithms.

In particular, for practical problems it is characteristic that the increment of the effective duration of an OP as it propagates along the OF significantly exceeds its duration at the moment it is entered into the OF. This means that in numerical schemes the value of the integral I_1 in formulas (15) can be neglected. Then the optimization problem is reduced to calculating an integral proportional to E_α and further calculating the integrals

$$Q_1(\alpha) = \int_{-B}^{B}\omega^2 G_0^2(\omega,\alpha)d\omega, \quad Q_2(\alpha) = Q_1(\alpha)/E_\alpha, \quad (30)$$

where the limits of integration $[-\mathrm{B},\mathrm{B}]$ are determined by the conditions of the particular problem being solved. For functions (22) and (23), these limits are defined as: $\pm B = \pm\omega_B = \pm(1+\alpha)\omega_C$. Note that in this case the function optimization $Q_1(\alpha)$ allows you to simultaneously optimize the OP by the criterion of the minimum of the effective duration, and by the criterion of the minimum of the effective width of the spectrum. You can also see that the resulting solutions will depend only on the type of the function $G_0(\omega,\alpha)$, but will not depend on the specific value of the dispersion coefficient k_0, not from the distance z. Also from the previous examples it is clear that the optimal value of the parameter α invariant to the choice of the amplitude parameter U_0 and on the dimension of the frequency scale.

Function optimization programs $Q_1(\alpha)$ and $Q_2(\alpha)$ are written in medium SciLab and is shown in listing 1. In calculations $U_0 = 1$, $\omega_C = 1$.

Listing 1 – (23) function optimization program

```
// Raised cosine function
function nyq=nyquist_cos(UT, alpha, omega_c, omega);
   w = abs(omega);
   wa = (1-alpha)*omega_c;
```

```
   wb = (1+alpha)*omega_c;
   delt_ba = wb - wa;
   if alpha < 1.0D-10 then
      if w < omega_c then
         nyq = UT;
      else
         nyq = 0
      end
   else
      if w < wa then
        nyq = UT;
      else
         if w > wb then
            nyq = 0;
         else
            nyq = (UT/2) * (1 + cos(%pi*(w - wa)/(2*alpha*omega_c)));
         end
      end
   end
endfunction
// Setting model parameters
UT = 1; om_c = 1;
// Preparation of the main cycle
alph = 0:0.01:1; m = size(alph,2);
Q1_arr = zeros(m); Q2_arr = Q1_arr; dw = w(2) - w(1);
// Main cycle
for k = 1:1:m
  E = 0; Q1 = 0;
  w = [-2:0.01:2]; n = size(w,2); nk = zeros(n);
  for i = 1:1:n
     nk(i)= nyquist_cos(UT,alph(k),om_c,w(i))
     E = E + dw * nk(i)^2;
     Q1 = Q1 + dw * w(i)^2 * nk(i)^2;
  end;
  Q2 = Q1 / E;
  Q1_arr(k) = Q1;
  Q2_arr(k) = Q2;
end;
// Calculation and output to the console screen of min Q1 and  arg min Q1
[Q1_min,k] = min(Q1_arr);
disp('Q1 min = ' + string(Q1)+' ; Alpha arg min = '+ string(alph(k)));
// Calculation and output to the console screen of  min Q2 and  arg min Q2
[Q2_min,k] = min(Q2_arr);
disp('Q2 min = ' + string(Q2)+' ; Alpha arg min = '+ string(alph(k)));
// Output function graphs
subplot(1,2,1);
plot2d(alph,Q1_arr,style = color('blue'));
xgrid(3,1,3)
subplot(1,2,2);
plot2d(alph,Q2_arr,style = color('blue'));
xgrid(3,1,3)
```

The results of the program, as well as a similar program for the trapezoidal function (24) are given in Fig. 3. Optimum function values $Q_1(\alpha)$ and $Q_2(\alpha)$ and variable parameter α are given in Table 1. Graphs of optimal functions (22) and (23) in the sense of the function $Q_2(\alpha)$ are given in Fig. 4.

Q_1 $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ Q_2

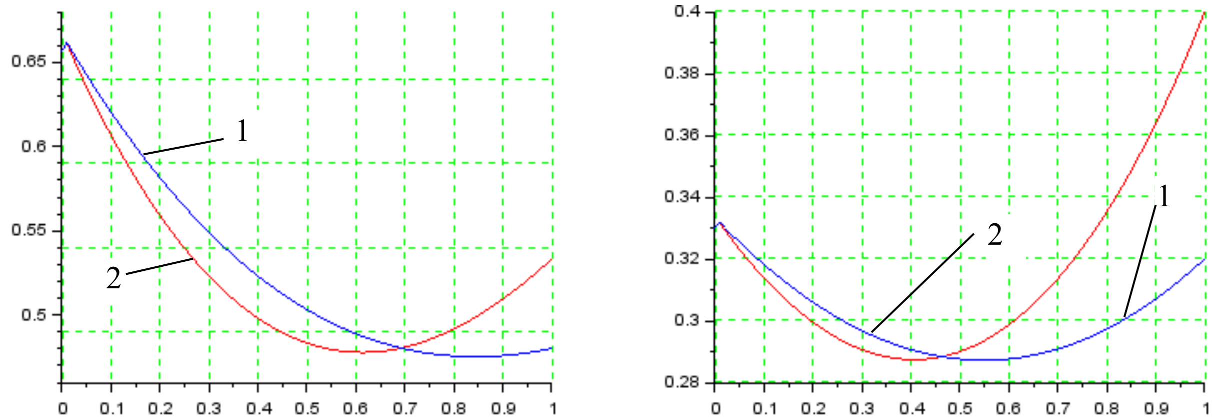


Figure 3 – Graphs of functions $Q_1(\alpha)$ and $Q_2(\alpha)$

1 – for spectral density in the for m of a raised cosine;
2 – for trapezoidal spectral density

Table 1 – Optimal values $Q_1(\alpha)$ and $Q_2(\alpha)$ and variable parameter α

Optimized function	$\alpha^* = \arg\min Q_1$	$\min Q_1$	$\alpha^* = \arg\min Q_1$	$\min Q_2$
Trapezius (22)	0,61	0,533333	0,41	0,39999
Raised cosine (23)	0,84	0,480182	0,55	0,32012

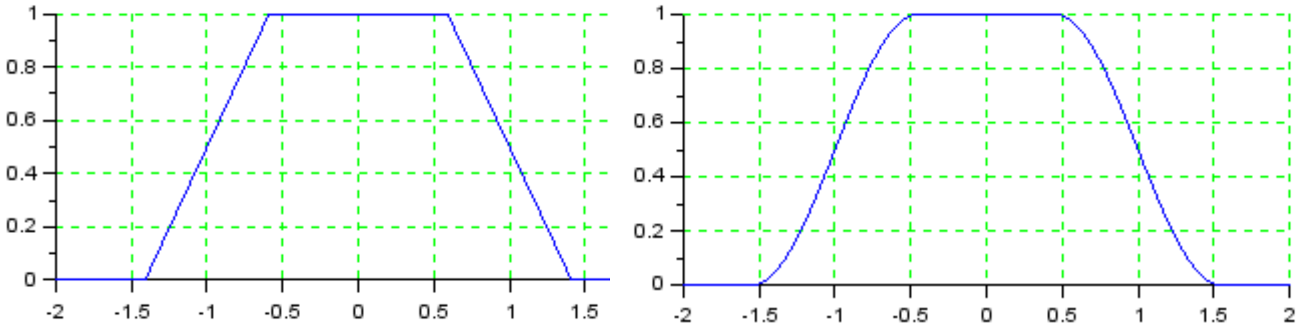


Figure 4 – Graphs of optimal functions (22, 23) by the criterion of the minimum of the reduced base

In this paper, the problem of optimizing optical signals by the criterion of the minimum of the reduced base is investigated. The problem is solved in general form using the methods of variation calculus. In particular cases, the task is reduced to the optimization of parametric families of functions. The most effective in this formulation are numerical-analytical optimization methods.

The problem was solved for a narrowband channel with a bandwidth of about 50…100 GHz. At the same time, the obtained results allow us to conclude that the optimal values of the parameters of optical signals are invariant to the length of the regeneration section and to the frequency scale used. This allows the extgension of the the solutions obtained for one channel to the case of multichannel transmission systems with frequency multiplexing.

In this article, the introduction and conclusion are written by the authors together. The material on the use of numerical methods prepared by D.H. Bahachuk. The rest of the material was prepared by N.A. Odegov.

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