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LOCAL OPTIMAL RANK AFTER DEMODULATION ALGORITHM OF NOISE-TYPE SIGNAL DETECTION

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The synthesis of locally optimal distribution- free decision rule was considered. A new locally optimal rank algorithm that can detect the signals under the influence of different noise is proposed. The effectiveness of the proposed algorithm has been proved.

Keywords: signal noise situation, rank signal processing, function of ranks, detection characteristic.

Розглянуто синтез локально-оптимального вільного від розподілу вирішувального правила. Запропоновано новий локально-оптимальний ранговий алгоритм, який здатний виявляти сигнали при дії різних завад. Доведена ефективність запропонованого алгоритму.

Ключові слова: сигнально-завадова ситуація, рангова обробка сигналів, функція рангів, характеристика виявлення.

Introduction

One of the main problems in the theory and practice of signal processing is the synthesis of signal detection algorithms under prior uncertainty conditions of signal-interference situation (SIS), i.e. when the parameters or kind of signal distribution and noise mixture are unknown.

In the theoretical work on statistical signal processing P. S. Akimov, B. R. Levin, I. G. Prokopenko, E. A. Kornilyev, Y. G. Sosulin, as well as in the works on mathematical statistics J. Hayek, G. David, P. Huber there have been suggested the ways of efficient algorithms construction in prior uncertainty conditions.

In many problems of signals detection and processing kind the type of the initial observation distribution is unknown, or may change during the observation, that’s why the development, research and implementation of algorithms that stably operate in a wide variety of noise is the urgent task of radiolocation.

Problem formulation

SIS priory uncertainty is present in radio systems that operate in severe weather conditions, complex electromagnetic situation in the electronic counter conditions and so on.

During such systems designing it can not be limited by any one parametric model of signals and noise mixture distribution, or even a certain region of this model. The problem has non-parametric character and described in terms of nonparametric hypotheses.

To solve this problem it is necessary to find statistical procedures that could be applied to a wide class of distributions. These procedures are called distribution-free because their use does not depend on the form of the initial distribution.

The most distribution-free procedures were designed for nonparametric problems, such as hypothesis testing that two continuous distributions are identical. Such procedures are often named in the literature as nonparametric [1]. However, they can be applied for parametric problems solving. Of course they will have a slightly lower efficiency than the corresponding optimal or adaptive parametric procedures, but they provide probability stability of the first kind error during very weak restrictions due to noise probability distribution, such as continuity and independence of sample values.

The distribution-free procedures can solve a number of problems of signal detection during the noise effect with a priori uncertain characteristics. The main problem of signal detection in nonparametric formulation can be formulated as a problem of two samples comparison.

Hypothesis H_0 that is checked has the next content: two random samples $x_1, \ldots, x_n$ and $y_1, \ldots, y_m$ has the same probability distribution, that is they are generated by the noise action.

In case of hypothesis H_0 rejection the decision of the signal presence in one of the samples are accepted. (hypothesis H_1) [1]. Various agreement criteria (Wilcoxon, Kolmogorov, Smirnov, Mises, chi-square, etc.), on the basis of which solving rules of hypothesis H_0 testing is synthesized, provide the false alarm invariance regardless of the noise distribution. However, the signal detection effectiveness using such algorithms is low because they do not take into account the distribution law of a signal and noise mixture. In general case, the effectiveness of distribution-free algorithm depends on the alternative hypothesis. That’s why, if it is necessary to build an effective solving rule or signal detector, it is necessary to take into account the signal effect and formulate the corresponding alternative hypothesis.

Problem solving

Signal detection efficiency of nonparametric procedures can be increased by using of locally optimal (LO) approach due to detection optimization [2]. Herewith the energy signal parameter b is input in such way, that at $b = 0$ the signal disappears and the distribution of a signal and noise mixture transform to noise distribution.

In the basis of all distribution-free statistics criteria, lies a ranks vector (1) of signal sample readout $x_1, \ldots, x_n$ relatively to noise sample $y_1, \ldots, y_m$ [3].

$$\overline{R} = (R_1, R_2, \ldots, R_n), \quad (1)$$

where R_i — is calculated due to formula (2).

$$R_i = \sum_{k=1}^{m} \operatorname{sgn}(x_i - y_k). \quad (2)$$

Synthesis of robust LO distribution-free rank algorithms of signal detection is based on the research of the rank vector distribution in the case of alternative hypothesis (3), when the sample $x_1, \ldots, x_n$ contain the signal [1]:

$$w(\overline{R}|b \neq 0) \quad (3)$$

and construction of rank solving rule [1]:

$$S(\overline{R}) = \frac{\partial w(\overline{R}|b \neq 0)}{\partial b}\Big|_{b=0} \geq V_p. \quad (4)$$

For the distribution (3) construction it is necessary to know the distribution of signal samples for an alternative hypothesis. If $f(x, b)$ one-dimensional probability density function for H_1 hypothesis and $f(x, 0)$ — probability density function for the H_0 hypothesis, then the solving rule (4) will have the form (5) [4].

$$S(\overline{R}) = \sum_{i=1}^{n} a_m(R_i, f) \geq V_p. \quad (5)$$

The function meanings of ranks are calculated due to formula (6) [1].

$$a_m(R_i, f) = m \cdot C_{R_i-1}^{m-1} \int_{-\infty}^{\infty} \frac{\partial f(x,b)}{\partial b}\Big|_{b=0} \times$$
$$\times F(x)^{R_i-1}[1 - F(x)]^{m-R_i}\, dx. \quad (6)$$

where m — number of interference samples relatively to which the rank is calculated; R_i – x_i sample rank relatively to interference sample; $f(x, b)$ — probability density distribution during signal presence; $F(x)$ — integral function of the probability noise distribution.

Generalized block diagram of LO rank signal detector is shown on Fig. 1 [1].

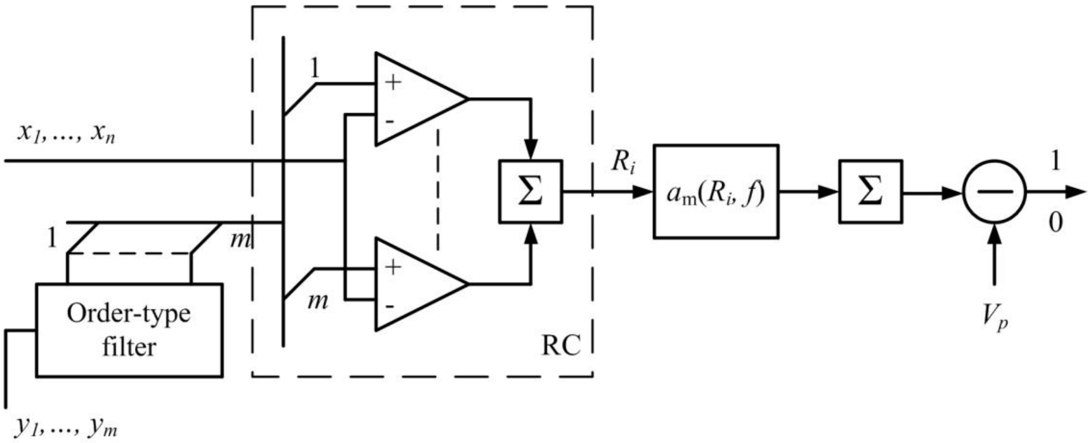

Fig. 1. Generalized block diagram of local-optimal rank signal detector

For the LO rank detector synthesis it is necessary to find an analytical expression for the rank function (6), the type of which depends on the distribution of signal and noise mixture. Let consider the situation when the sample $x_1, \ldots, x_n$ belong to a mixture of Gaussian noise signal with variance σ_s^2 and Gaussian noise with variance σ_i^2 and sample $y_1, \ldots, y_m$ is produced by the interference action with a variance σ_i^2. After the quadratic amplitude demodulation of signal its distribution is described by the exponential law. If the readout samples are independent, it is possible to restrict by one-dimensional probability density distribution:

$$f(x,b) = (1+b)\lambda e^{-(1+b)\lambda x},$$

where b — parameter that characterize the signal-to-noise ratio (signal parameter). Then the ranks function at $\lambda = 1$, based on the expression (6) has the form:

$$a_m(R_i, f) =$$
$$= mC_{R_i-1}^{m-1} \int_{-\infty}^{\infty} (1-x)e^{-x}[1-e^{-x}]^{R_i-1}[e^{-x}]^{m-R_i}\, dx.$$

In general case, at finite m and n it is difficult to calculate the statistical distribution of the criterion (5) for the LO rank after demodulation algorithm of noise-type signal.

We are being confined by the calculating of the first two moments for the null hypothesis.

Statistics mathematical expectation (5) as a result of the same values of ranks probability

$$m_1\{S\} = n\frac{1}{m+1}\sum_{i=1}^{m+1} a_m(i, f).$$

Due to the independence of signal samples ranks and correspondingly terms of the sum (5) the variance can be calculated by the formula

$$\mu_2\{S\} = n\left(\frac{1}{m+1}\sum_{i=1}^{m+1} a_m^2(i, f) - \left[\frac{1}{m+1}\sum_{i=1}^{m+1} a_m(i, f)\right]^2\right).$$

To calculate the detection threshold V_p, which provides the necessary probability of false alarm α, can be used the central limit theorem of probability theory, which allows in the case of large *n* and *m* to approximate distribution statistics (5) by the Gaussian law. Then the decision threshold is defined by the following expression:

$$V_p = m_1\{\lambda\} + A_{1-\alpha}\sqrt{\mu_2\{\lambda\}},$$

where $m_1\{\lambda\}$ – mathematical expectation of checked statistic; $A_{1-\alpha}$ – quantile of normalized Gaussian distribution of (1–α) level; $\mu_2\{\lambda\}$ – variance of checked statistic.

Research result

With the help of MATLAB environment was conducted computer modeling of LO rank after demodulation algorithm of noise-type signal detection. This modeling allowed us to obtain detection characteristics (Fig. 2). Using these characteristics the effectiveness of the algorithm can be analyzed.

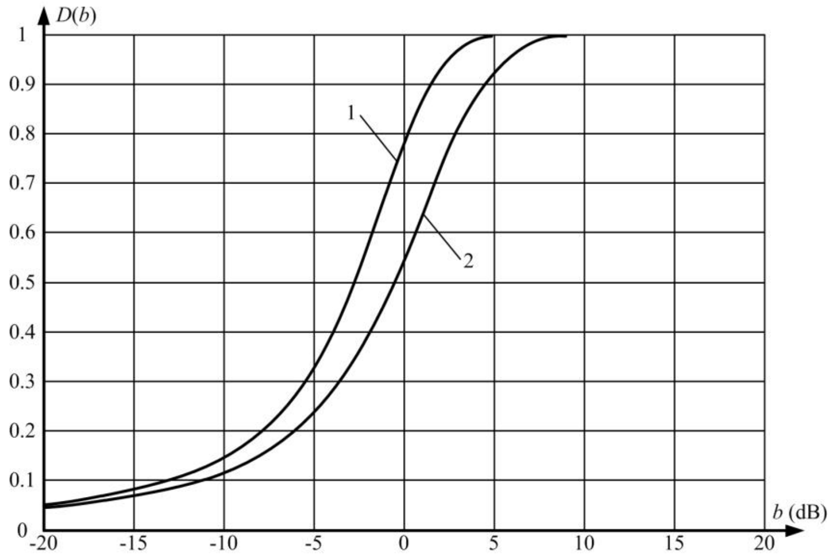

Fig. 2. Detection characteristics of LO rank after demodulation algorithm of noise-type signal detection:
1 — m = 16, n = 16; 2 — m = 8, n = 8

Simulations showed that with increasing *m* and *n* values the efficiency of LO rank after demodulation algorithm increase.

So for the parameters $m = 8$, $n = 8$ the meaning of threshold signal (signal-to-noise ratio under the probability of correct detection is 0.9) is 4.5 dB, and for $m = 16$, $n = 16$ this value is 1.8 dB.

It should be noted that *m* and *n* should be selected based on effectiveness indices, which is necessary to ensure during real systems design.

With increased number of samples the requirement of processing power systems which should provide sufficient detection rate is increased too.

Recommended minimum values are $m \geq 3$ and $n + m \geq 20$.

They must ensure the normality of checked distribution statistics.

During detection characteristics effectiveness investigation the developed algorithm was compared with the corresponding characteristics of the nonparametric Wilcoxon detector (Fig. 3–4).

Analysis of the obtained detection characteristics showed that LO rank algorithm is more effective than the Wilcoxon rank algorithm. Since the more effective is that algorithm which has the higher probability of right detection at a lower signal-to-noise ratio at stable level of false alarms.

Threshold signals values for the Wilcoxon rank algorithm is 13.8 dB for $m = 8$, $n = 8$ and 11.1 dB for $m = 16$, $n = 16$. That is, these values are on 9.3 dB worse than the corresponding values LO rank algorithm. Thus, the concept of local optimality usage due to detection optimizing permit to increase efficiency of nonparametric rank detection algorithm, while maintaining the main advantage of nonparametric algorithms — the independence of the false alarms probability from the shape of the noise distribution.

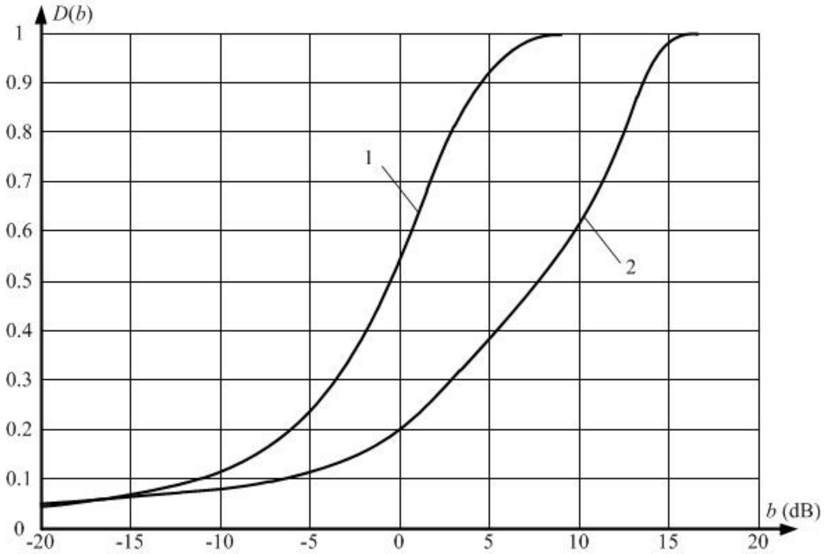

Fig. 3. Detection characteristics at $m = 8$, $n = 8$:
1 — LO rank after demodulation algorithm; *2* — rank Wilcoxon algorithm

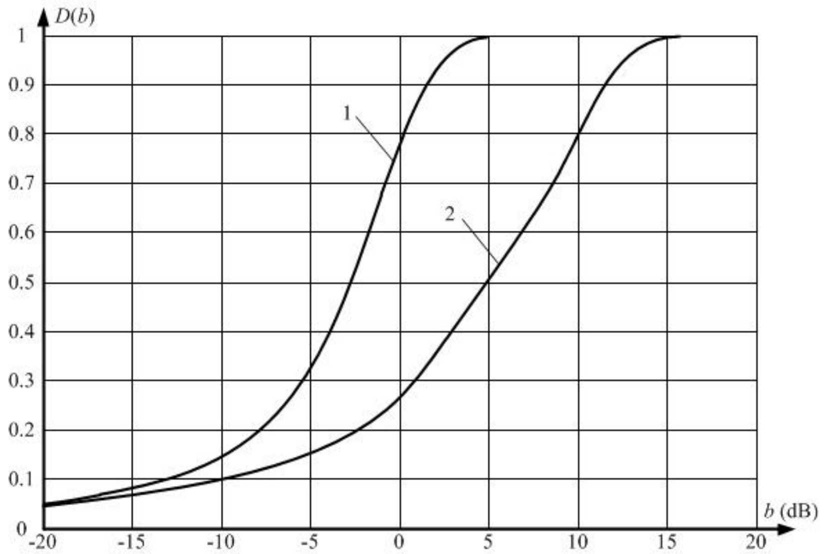

Fig. 4. Detection characteristics at $m = 16$, $n = 16$:
1 — LO rank after demodulation algorithm; *2* — rank Wilcoxon algorithm

Conclusions

1. Considered concept of local optimality permit to develop a new LO rank after demodulation algorithm of noise-type signal detection during interference action.

2. The conduct analysis of the obtained detection characteristics proved the effectiveness of the LO rank signal detection algorithm in comparison with Wilcoxon nonparametric algorithm.

3. Application in the developed algorithm signal detection exactly rank procedures allow to provides a given level of false alarms.

4. Application of the proposed algorithm will permit to increase the efficiency and quality of digital processing information systems in a prior uncertainty condition of signal-interference situation.

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