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THE ANALYSIS OF QUASI-OPTIMAL TOPOLOGIES OF NETWORKS-ON-CHIP ON MEETING THE GLOBAL OPTIMUM

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Розглянуто основні підходи до синтезу мереж на кристалі на основі регулярних та спеціалізованих топологій. Визначено критерії оптимальності топологій мережі на кристалі та запропоновано новий клас квазіоптимальних топологій і методи їх синтезу. Визначено вимоги до квазіоптимальних топологій, за допомогою математичних методів оптимізації виконано аналіз отриманих квазіоптимальних топологій для кількості вузлів 25 і показано, що їхні характеристики наближені до теоретично можливих оптимальних топологій не менш ніж на 96,3 %.

Ключові слова: мережа на кристалі; квазіоптимальна топологія мережі на кристалі, критерій оптимальності мереж на кристалі.

The main approaches to the synthesis of networks-on-chip based on regular and specialized topologies are analyzed. The optimality criteria of the topologies of networks-on-chip and a new class of quasi-optimal topologies and methods for their synthesis are proposes. The requirements for quasi-optimal topologies are defined. By using the mathematical methods of optimization the analysis of the obtained quasi-optimal topologies for the number of nodes of 25 is performed. It is shown, that their properties are close to the theoretically possible optimal topologies to not less than 96.3 %.

Keywords: network-on-chip, quasi-optimal network-on-chip topology; networks-on-chip optimality criterion.

Introduction

Due to the rapid development of electronics networks-on-chip (NoC) are getting widespread [1]. NoC is a set of computing nodes united by a common communication subsystem, which consists of routers and short connections between them. NoC communication subsystem takes considerable resources of the chip and is energy-intensive [2]. It is primarily determined by the NoC topology which stipulates the necessity of finding of optimal solutions of its construction.

Problem statement and research objectives

Usually, the problem of the efficient NoC topology synthesis is resolved as following: it is selected either a specialized topology, in case when the information about the future computational tasks to be performed by the developed system is known, or it is used a regular or platform-dependent topology if synthesis is carried out on the basis of the multiprocessor chip [3].

The advantages of regular topologies are the deterministic routing algorithms and the unified structure of components and nodes placement. But such topologies have low scalability and non-optimal topological parameters. This explains the better characteristics of NoC, synthesized on the basis of specialized topologies, which, however also have some drawbacks: there is not always complete information about the task which will be performed at the future NoC and its amendment requires the reprocessing of the entire network, while for the regular NoC it is only needed to remap the characteristic graph of the problem on its topology [3, 4].

Therefore, it is reasonable to combine both approaches to the NoC synthesis based on specialized and regular topologies by applying the predefined optimal or quasi-optimal topologies that will reduce the backwards of both approaches by combining their advantages.

The criterion of NoC topologies efficiency

The most important parameters of NoC topologies are: degree of nodes, diameter, average distance and the number of connections between vertices [1, 4]. In this regard, one of the fundamental problems of graph theory is the synthesis of the optimal graphs of NoC topologies with minimal average distance and number of connections between the nodes with given diameter, degree and number of nodes.

There are following criteria of the NoC topologies optimality. The basic optimality criteria are: minimization of the average distance between nodes and minimization of number of nodes connections. Additionally, we can consider the criteria of minimum diameter of the graph and the minimum degree of vertices achievement for a given number of nodes. The proposed criteria are contradictory. Therefore, there is a need for determining of the integral criterion that will combine the single-factor criteria.

For this purpose the use of the principle of minimizing of the weighted sum of single-factor criteria is proposed. It will give an opportunity to formulate the additive integral criterion of NoC topologies efficiency:

$$optK = k_1 \cdot St_{\max\_norm} + k_2 \cdot D_{norm} + 1;$$
$$k_3 \cdot L_{av\_norm} + k_4 \cdot Ed_{norm},$$
$$optK \to \min, \quad (1)$$

where $\sum_{i=1}^{4} k_i = 1$ — criteria importance coefficients that are selected empirically on the basis of expert assessments;

$$St_{\max\_norm} = \frac{St_{\max}}{St_{\max\_reg}}, \quad D_{norm} = \frac{D}{D_{reg}},$$

$L_{av\_norm} = \frac{L_{av}}{L_{av\_reg}}$, $Ed_{norm} = \frac{Ed}{Ed_{reg}}$ — normalized values of average distance, the number of connections, diameter and maximum degree of vertices, resulted to the known characteristics of regular topologies (e.g., torus or mesh [4]).

This criterion is a universal one. By setting the coefficients of importance, it is possible to choose an optimization strategy and even remove some unimportant single-factor criteria referring them to the restrictions region.

Synthesis of optimal NoC topologies

Unfortunately, the formulation of nonlinear programming problem for synthesis of optimal topologies on the basis of the matrix describing of NoC topologies, their characteristics and limitations, is complicated because of the impossibility of a clear mathematical formulation of finding of the average distance between nodes. The average distance between nodes can be found from the shortest distances between all nodes of the graph, which are normally found by special algorithms (e.g. searching in width). Thus, the problem of synthesis of optimal NoC topologies is referred to “hard solved” ones [5], and the synthesis of optimal topological solutions is possible only by usage of zero order methods.

In [6] by applying the method of exhaustive search and its improvements by the methods of branches and bounds and concurrent computing it is synthesized a number of optimal topologies with the number of nodes limited by 9. Unfortunately, increasing of the number of nodes causes the growth of the computing volume in power function, which makes impossible the synthesis of NoC topologies with greater number of nodes by the methods of zero order.

Quasi-optimal NoC topologies

However, NoCs are not limited to the number of 9 nodes and may reach tens or even hundreds of nodes [7]. For the synthesis of efficient topologies for such networks there proposed a new quasi-optimal class of topologies. **Quasi-optimal** (suboptimal, pseudo-optimal) **NoC topology** — is a topology for a given number of vertices and limitations and close to the theoretically possible optimum topology according to certain criteria. The criteria of optimality are: reduction of the number of connections, decrease in the average distance between nodes, decrease of the diameter and the maximum degree of vertices according to the integral criterion (1), which is used for the synthesis of optimal topologies. But the methods by which the quasi-optimal topologies are synthesized enable finding of the local optimum and do not guarantee achieving of the most optimal results. They only give the results which are close to the optimal ones.

The example of such methods is the method of Monte-Carlo, according to which part of the compounds for certain vertices is fixed heuristically prior to the beginning of topology synthesis, whereby by combining this method with the method of parallelization of computations there synthesized quasi-optimal topologies with the number of nodes up to 16. Generally, on the full-testing field the methods of Monte-Carlo give an opportunity to find the global optimum. However, in case of heuristic approach and time limits in search it is only guaranteed the local optimum.

The evolutionary method is extended for synthesis of quasi-optimal NoC topologies and a genetic algorithm GeNoC in MatLab is developed. Using GeNoC it is synthesized a number of quasi-optimal topologies with the number of units up to 100. Combining this approach with parallel computing method makes a possibility of further acceleration of topologies synthesis and receive the quasi-optimal topologies with even more number of units.

Quasi-optimality criterion of NoC topologies

In the analysis of the synthesized quasi-optimal topologies there arises a problem of evaluation of the obtained topologies on how they meet the efficiency criterion (1) and how close they are to the optimum. In [8] there carried out a comparative analysis of the quasi-optimal and regular NoC topologies. It is shown that quasi-optimal topologies make it possible to increase the throughput and reduce resource costs for the NoC connection subsystem compared to the networks based on regular topologies. However, a comparison with regular topologies doesn’t make it impossible to determine for how much the synthesized quasi-optimal topologies are close to the optimum.

For this purpose it is formulated a criterion of quasi-optimality. The criterion of quasi-optimality is a measure of closeness of quasi-optimal topology to the optimum, which is a difference between the

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value of the objective function (1) of the found topology to the theoretically possible value of the objective function for the optimal topology. On the basis of expert assessments the threshold value for determining of quasi-optimality is set to 5%, at which the found topologies can be attributed to quasi-optimal ones, as the further procedure of topology synthesis process optimization requires excessive resource costs. Eventually, with the improvement of computing power, the threshold value can be improved by expert assessments.

Analysis of quasi-optimal topologies on meeting the global optimum

To assess quasi-optimal topologies, it is necessary to find the approximated characteristics of the theoretically possible optimal topologies for a given number of nodes and constraints which can be done by formulating of the linear programming problem.

In general, the problem of linear programming for minimization of function with *n* variables and *m* constraints is formulated as follows [9]:

$$CostF = C_1x_1 + C_2x_2 + \ldots + C_nx_n \to \min;$$

$$\sum_{j=1}^{n} a_{ij}x_i \geq \mathrm{cost}_i\,,\ i = 1\ldots m; \qquad (2)$$

$$x_j \geq 0,\ i = 1\ldots n,$$

where x_i — unknown variables values of which are approximated; C_i, a_{ij}, $Cost_i$ — constants.

To formulate the linear programing problem with the tool of GeNoC it is synthesized a NoC topologies field for the number of nodes of 25 with the different coefficients of importance $k_1,\ldots,k_4$ of the objective function (1). Among the elements of the field of the synthesized topologies it is selected the samples of topologies with the best characteristics which are listed in table 1.

As normalizing values according to (1) there selected characteristics of mesh topology for 25 nodes, which are also given in table 1.

There is an analysis of the obtained quasi-optimal topology on the proximity to the optimum (see the first row of table 1). If we take as a basis, that the synthesis is carried out with the coefficients $k_1 = k_2 = k_3 = k_4 = 0{,}25$, than the ratios $C_1\ldots C_4$ can be set in the system (2) as follows:

$$C_1 = \frac{0{,}25}{St_{\max\_mesh}},\ C_2 = \frac{0{,}25}{D_{mesh}},$$

$$C_3 = \frac{0{,}25}{L_{av\_mesh}},\ C_4 = \frac{0{,}25}{Ed_{mesh}}.$$

The variables $x_1\ldots x_4$ in the formula (1) are the unknown values of the degree of nodes, diameter, average distance and the number of connections between vertices, at which the objective function will be minimal.

The next three rows of the table 1, which set the other received quasi-optimal topologies, can be used for the formulation of linear programming problem constraints, by setting the coefficients a_{ij} as the ratio of relevant coefficients of importance to the characteristics of the mesh topology and cost_i — as values of the objective functions for the appropriate topologies. The conditions of $x_j \geq 0$ can also be refined by setting the following limitations: $x_1 \geq 4$ (degree of vertices), $x_2 \geq 3$ (diameter), $x_3 \geq 2$ (average distance), $x_4 \geq 20$ (number of connections).

Thus, the classical problem of linear programming has been formulated. It has been resolved in MatCad, as a result of which it has been determined that there should be an optimal topology with such characteristics: $St_{opt} = x_1 = 4$, $D_{opt} = x_2 = 3$, $L_{av\_opt} = x_3 = 2{,}42$, $Ed_{opt} = x_4 = 40$. The value of the objective function (1) in such topology with coefficients of importance $k_1 = k_2 = k_3 = k_4 = 0{,}25$ is equal to $CostF_{\min} = 0{,}775$. The quasi-optimal topology, synthesized under the same coefficients of importance has $CostF = 0{,}804$ (table 1), which is 3.7 % more than the minimal topology.

Similarly approximated optimal topologies characteristics and values of their objective functions for other sets of importance coefficients are given in Table 2.

Thus, the deviation from the optimum value of the objective function, which determines the efficiency of the synthesized quasi-optimal topologies, has amounted to no more than 3,7 % and is less than the prescribed limit of 5 % according to the quasi-optimality criterion.

This demonstrates the high efficiency of the GeNoC algorithm, which in combination with a significant acceleration of the synthesis of NoC topologies, compared to other methods, stipulates the possibility of usage of evolutionary computations method for the synthesis of NoC quasi-optimal topologies with the number of nodes amounting the hundreds.

The analysis of the obtained topologies with different coefficients of importance shows that there is the possibility of the synthesis of various topologies with a reduced number of connections or the average distance between nodes and diameter that can be used by developers when synthesizing of NoC topologies for the specific conditions in terms of limitation in hardware resources and minimal throughput.

Table 1

Characteristics of the quasi-optimal topologies with the number of nodes of 25

k_1	k_2	k_3	k_4	*St*	*D*	L_{av}	*Ed*	*CostF*
0,25	0,25	0,25	0,25	4	4	2,64	37	0,804
0,1	0,1	0,7	0,1	4	4	2,24	49	0,744
0,1	0,1	0,1	0,7	4	8	3,87	24	0,736
0,1	0,1	0,4	0,4	4	5	3,13	29	0,828
Mesh topology characteristics:				4	8	3,33	40	

Table 2

The approximated optimal characteristics of the topologies with the number of nodes of 25

k_1	k_2	k_3	k_4	St_{opt}	D_{opt}	L_{av_opt}	Ed_{opt}	$CostF_{\min}$	$CostF_{quasi}$	$\Delta CostF$
0,25	0,25	0,25	0,25	4	3	2,42	40	0,775	0,804	3,7%
0,1	0,1	0,7	0,1	4	4	2,20	44	0,723	0,744	3,0%
0,1	0,1	0,1	0,7	4	8	3,23	24	0,717	0,736	2,7%
0,1	0,1	0,4	0,4	4	5	2,80	30	0,799	0,828	3,6%

Conclusions

The application of irregular quasi-optimal topological solutions for the synthesis of NoC, which is a compromise between regular and specialized topologies is substantiated.

Single-factor criteria and integral criterion of NoC topologies efficiency are defined. The definition of quasi-optimal topologies and requirements of quasi-optimal topologies are specified.

By means of mathematical optimization methods there performed an analysis of quasi-optimal topologies for the number of nodes of 25 and is shown that they have only to 1,8–3,7 % worse performance characteristics, compared to the approximated optimal topologies. This demonstrates the high efficiency of the synthesized quasi-optimal topologies and the possibility of their use for the synthesis of NoC.

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