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MATHEMATICAL MODELING OF TRAJECTORY OF THE MAXIMUM SUBSIDENCE OF POINTS ON THE EARTH SURFACE CAUSED BY WORKING OUT OF COAL LAYERS

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МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ ТРАЄКТОРІЇ МАКСИМАЛЬНОГО ОСІДАННЯ ТОЧОК ЗЕМНОЇ ПОВЕРХНІ ПРИ ВІДПРАЦЮВАННІ ВУГІЛЬНИХ ПЛАСТІВ

Purpose. The aim of the research is using boundary values of parameters related to terrestrial surface displacement and clearing wastes of the rated scheme for forecasting a trajectory of the maximum subsidence points on the terrestrial surface at the stage of clearing works development.

Methodology. Empirical coefficients of mathematical dependences are determined in two ways: on the basis of processing experimental data by a method of least squares and according to the rated scheme using boundary values of parameters of terrestrial surface displacement and clearing wastes. The technique aims at establishment of the equivalence of these methods and the type of mathematical dependence, most accurately describing a trajectory of the maximum subsidence points on the terrestrial surface.

Findings. The exponential equation most precisely describes a trajectory of the maximum subsidence points on the terrestrial surface. The equivalence of ways of empirical coefficients determination has been established.

Originality. On the basis of the proximity of empirical coefficients determined by different ways, it has been proved that forecasting the trajectory of the maximum subsidence points on the terrestrial surface on the basis of boundary values of parameters of terrestrial surface displacement and clearing wastes is possible.

Practical value. Determination of empirical coefficients of the mathematical equations by the offered way will allow to further avoid labor- and time-consuming direct supervisions to receive experimental data and perform their processing by the method of least squares.

Keywords: *terrestrial surface, trajectory, empirical coefficients, modeling, maximum subsidence*

Statement of the problem. The solution of many topical mining production problems is associated with processes of used rocks and earth surface displacement. These include the choice of location and bearing capacity of the development workings lining, justification of roof control method in clearing faces, the forecast of gassing in goaf and the selection of ventilation schemes of mines and excavation sites, protection and safe working off water and other objects on the earth surface. The most promising forecast method for defining displacement parameters of the earth surface is mathematical modeling.

Analysis of recent research. Conventional mathematical models [1, 2] describe only a particular case of the earth surface subsidence after its complete undermining. The initial stage of surface subsidence and subsequent changes in parameters of displacement mould during the development of treatment works to complete undermining are not considered by known mathematical models.

The mathematical description of the stage of treatment works development is studied the least, except valid work [3], which shows the results of research only for conditions of Western Donbass mines, but there are no generalizing recommendations which could be used for other mining and geological conditions.

Segregation of unsolved problems. Determining the trajectory of maximum earth surface points' subsidence is necessary for mathematical description of processes parameters change in the displacement mould during the development of treatment works.

Currently, the solution of this problem is possible only on the basis of processing experimental point data, because direct and continuous determination of the maximum earth's surface subsidence is almost impossible given the potential of modern devices. For this reason, the most promising direction is mathematical modeling by computational schemes. In the original computational scheme, we laid scientific principles of formation parameters of the earth surface displacement mould during processing of shallow coal seams [4]. This scheme consists of two fundamentally different approaches to the calculation of parameters at different stages of treatment works.

The first stage is characterized by the beginning of the earth surface displacement to its complete undermining (formation of the mould flat bottom). The main influencing factor at this stage is the degree of second working development (one of the options is removal of the stope from the cut furnace). The maximum subsidence of the earth surface points is considered in relation to the boundaries of second working (goaf).

The second stage for a single face is associated only with the movement of stope after attaining the full earth surface undermining. Movement process parameters are considered in coordinates for this stage, taking into account the position of the stope projection relative to a point on the earth surface. Mathematical models of the process of earth surface shifting for this stepage are described in works [1, 2].

Considering the current state of this question, mathematical description of the trajectory of earth surface points maximum subsidence is the cutting-edge scientific and practical problem.

The purposes of the research are:

- to prove the possibility of using geometric dimensions of the stope and calculation scheme parameters for determination of empirical coefficients of mathematical functions;
- to establish the form of the equation, which describes the trajectory of the earth surface points maximum subsidence most accurately;
- to test the application of parameters of the earth surface subsidence mould and stope, calculated according to regulatory documents, to determine the empirical coefficients of equations.

The main material of the study. Surveying observations for the earth surface subsidence over second mining, represented in works of V.A. Nazarenko [3] (mines "Stepnaya", "Jubileynaya", "Pershotravneva"), G.A. Averin [6] (mine "M.V.Frunze"), A.F. Borzyh [7] (mine "P.L.Voykova"), M.A. Iofisa (mine "G.G.Kapustina") and V.G.Larchenko [9] (mine "Stepnaya"), indicates the presence of regularities of the subsidence mould formation.

On the basis of experimental data, it is proved that in some geological conditions with a constant power of formation's development (*m*), the depth of reference works (*H*) and strength properties of undermined rocks, the maximum subsidence of the earth surface points (η_m) is in close function with a change in one of geometric dimensions (*L*) of second mining (goaf).

According to the generalized calculation scheme [4], the trajectory of the earth surface points maximum subsidence can be forecast, using the parameters of the second mining and displacement mould at the surface (fig. 1).

Preliminary analysis of the known experimental data showed that the trajectory curve of maximum subsidence of earth surface points (1) can be described by a lognormal, exponential or hyperbolic curve.

The trajectory of maximum subsidence of the earth surface points for lognormal dependence is described by the equation

$$\eta_m = a \ln L + b . \qquad (1)$$

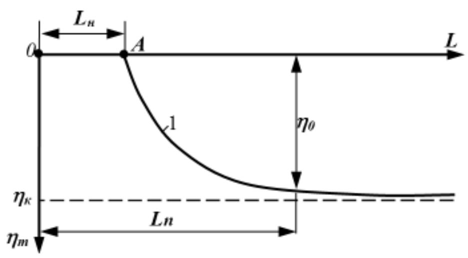

Fig. 1. The estimated trajectory (1) of the maximum subsidence of earth surface points η_m when resizing L of undermining space: A – point of the earth surface, where earth surface shifting begins; $L_н$ – size of developed space, corresponding to the beginning of earth surface shifting; L_n – size of the purification mining, characterizing complete earth surface undermining; η_0 – depth of the flat bottom of displacement mould; $\eta_к$ – final surface subsidence

Coefficients a and b according to the scheme (Fig. 1) are defined by parameters of the second mining $L_н$ and L_n. Value $L_н$ corresponds to the beginning of the displacement of point "A" on the earth surface. Its position is found from the condition $\eta_m = 0$, then from equation (1)

$$b = -a \ln L_н .$$

The depth of the flat bottom of the displacement mould (η_0) is equal approximately to the final subsidence of the earth surface ($\eta_к$). According to [1] $\eta_0 \approx (0{,}97 \div 0{,}99)\eta_к$, if it allows to use the equality

$$\eta_m \approx \eta_к \approx a \ln L_n + b$$

and to get a system of equations for determination of coefficients a and b

$$\begin{cases} b = -a \ln L_н \\ b = \eta_к - a \ln L_n \end{cases} .$$

Solving this system of equations, we find coefficients for equation (1)

$$a = \frac{\eta_к}{(\ln L_п - \ln L_н)}; \qquad (2)$$

$$b = \frac{-\eta_к \cdot \ln L_н}{(\ln L_п - \ln L_н)}. \qquad (3)$$

Similarly, we determine coefficients for the exponential dependence

$$\eta_m = a + b \cdot \exp(c \cdot L). \qquad (4)$$

Exponential function approaches asymptotically the maximum value of its coefficient a, which is equal numerically to the final subsidence of the earth surface $\eta_к$.

According to the condition of the initial displacement of the earth surface ($\eta_m = 0$), we determine the coefficient b

$$b = \frac{-a}{\exp(c \cdot L_н)} = \frac{-\eta_к}{\exp(c \cdot L_н)}.$$

The formation of displacement mould flat bottom ofcorresponds to the condition

$$\eta_m = \eta_0 = (0.97 \div 0.99)\eta_к = k \cdot \eta_к.$$

Substituting this expression in equation (4) and taking into account equality $a = \eta_к$, we find the coefficient b from the condition of the flat bottom formation

$$b = \frac{(k-1)\eta_к}{\exp(c \cdot L_п)}.$$

Considering all the above, we get the dependence of the coefficient c on the sought-for parameters

$$c = \frac{\ln(1-k)}{(L_п - L_н)}. \qquad (5)$$

The coefficient b value according to the diagram (fig. 1) and equation (5) will be equal in the final version

$$b = \frac{(k-1)\eta_к}{\exp(c \cdot L_п)} = -\frac{(1-k)\eta_к}{\exp\left[\ln(1-k) \cdot L_п / (L_п - L_н)\right]}. \qquad (6)$$

Coefficients a and b for hyperbolic dependence

$$\eta_m = \frac{a}{L} + b \qquad (7)$$

we determine from the conditions of the initial surface subsidence ($\eta_m = 0$) and the formation of displacement mould flat bottom ($\eta_m \approx \eta_к$)

$$b = \frac{\eta_к L_п}{(L_п - L_н)}; \qquad (8)$$

$$a = -\frac{\eta_к L_п L_н}{L_п - L_н}. \qquad (9)$$

To test the possibility of equations (1, 4, 7) practical use we calculate coefficients (a, b and c) for specific geological conditions, respectively by the method of least squares and equations (2, 3, 5, 6, 8, 9). Values of parameters ($L_н$, $L_п$, $\eta_к$, η_m), incoming into these equations, we identified previously in two ways.

In one case, they were determined visually on the graphs, based on experimental data, for specific geological and mining conditions.

In the other case, their calculation was made according to recommendations of the normative document [5]. This calculation has some conditional character due to the lack of determination specificity of unknown parameters. For example, it is recommended to determine the beginning of the earth surface subsidence from the condition $L_н = (0.1 \div 0.3)H$. The size of undermining space characterizing the condition of complete undermining and the end of displacement process, is calculated by the equation $L_п = (1.2 \div 1.4)H$. These dependences do not consider, except for the depth of treatment works, other mining and geological factors. The influence of H in defining $L_н$ and $L_п$ was taken as the averaging of recommended coefficients.

Maximum earth surface subsidence, with the full undermining, depends on the power of sloping layer $\eta_m = q_0 m$ [5]. Coefficient q_0 are chosen in the range $0.75 \div 0.85$, considering the type of coal.

Desired coefficients of equations (1, 4, 7), defined in three ways, are shown in tables 1 and 2 for these dependencies.

The most reliable and common method of equations' empirical coefficients definition is the processing of experimental data by the least squares method. Equations (1,4,7), coefficients of which are determined by least squares method, described practically functionally the trajectory of maximum subsidence of earth surface points for each specific object. Correlation ratios (*R*) were within the range $0.920 \div 0.998$ (tables 1 and 2). For this reasons, we can assume, that coefficients ($a_к, b_к, c_к$), determined by the least squares method, correspond, a high probability to specific conditions of experiments for each specific object. Comparing values $a_к$, $b_к$, $c_к$ with coefficients $a_ф$, $b_ф$, $c_ф$ и $a_н$, $b_н$, $c_н$, we can set the possibility of using the calculation scheme (fig. 1) and equations (1, 4, 7) for forecasting the trajectory of maximum subsidence of earth surface points.

Estimation of closeness of the connection between coefficients of equations, defined in many ways, was done by values of paired coefficients of correlation between them (table 3). The closest connection between coefficients is set, when they were calculated on the actual parameters $(\eta_m, \eta_к, L_н, L_к)$ of displacement mould and second mining (r = 0.867 ÷ 0.997). This indicates principled possibility of using the design scheme (fig. 1) and the equations (1, 4, 7) to simulate the trajectory of maximum subsidence of earth surface points. Lower paired correlation coefficients (r = 0.299 ÷ 0.791) were obtained during determining the input parameters (η_m, $\eta_к$, $L_н$, $L_к$), according to regulatory documents [5]. This indicates the insufficient accuracy of calculation of input parameters. For their defining, except *m* and *H*, it is necessary to take into account additional influencing factors. These factors can be the strength properties of rocks, the speed of stope movement, the angle of seam's incidence, the size of second mining and some others.

Table 1

The results of determining coefficients of the lognormal and the hyperbolic equations according to experimental data [3, 6–9]

Mine, bed (excavation), literary source	The method of coefficients determination													
	for a lognormal dependence							For a hyperbolic dependence						
	Least square method			According to the factual parameters		According to [5]		Least square method			according to the factual parameters		According to [5]	
	$a_к$	$b_к$	R	$a_ф$	$b_ф$	$a_н$	$b_н$	$a_к$	$b_к$	R	$a_ф$	$b_ф$	$a_н$	$b_н$
"Stepnaya", C_6' (excavation zone 715, 713), [3]	373	-1448	0.978	363	-1415	454	-1652	-34500	688	0.930	-37871	773	-38173	1005
"Stepnaya", C_6 (excavation zone 606) , [3]	606	-2078	0.982	646	-2333	477	-1516	-53000	1280	0.978	-45638	1233	-25329	1055
"Stepnaya", C_6 (excavation zone 604) , [3]	473	-1623	0.987	449	-1527	418	-1421	-35000	977	0.988	-30727	1024	-27726	924
"Pershotravneva", C_4' (excavation zone 302, 304) , [3]	187	-596	0.985	217	-681	286	-954	-14000	487	0.920	-12778	556	-17737	633
"Jubileynaya", C_1 (2-nd eastern. excavation zone), [3]	312	-1034	0.987	336	-1184	453	-1494	-26700	712	0.989	-24504	721	-27109	1004
"Jubileynaya", C_6' (excavation zone 605, 607) , [3]	396	-1605	0,977	375	-1501	323	-1262	-49600	820	0.968	-44669	812	-35691	714
"Jubileynaya", (excavation zone 530) C_6', [3]	551	-2026	0.983	522	-1854	454	-1545	-40600	958	0.931	-38914	1112	-30136	1005
M.V. Frunze, h_8, [6]	236	-1237	0.960	265	-1385	561	-2913	-105000	475	0.955	-98619	527	-223364	1241
P.L. Voykov, k_5^1, [7]	344	-1779	0.970	310	-1592	401	-1974	-104000	557	0.942	-103619	610	-122318	886
Polish mine, [8]	719	-3176	0.860	1188	-5879	705	-2669	-206000	1652	0.938	-255014	1809	-68640	1560
G.G. Kapustin, $m_3^н$	420	-1423	0.997	432	-1499	954	-3768	-57000	1186	0.951	-39844	1245	-109696	2110
"Stepnaya", C_6, [9]	343	-944	0.940	387	-1213	411	-1253	-26000	1000	0.989	-22032	958	-19176	913

Table 2

The results of determining coefficients of the exponential equation according to experimental data [3, 6–9]

Mine, bed (excavation zone), literary source	The method of coefficients determining									
	Least square method				according to the factual parameters			according to [5]		
	$a_к$	$b_к$	$c_к$	R	$a_ф$	$b_ф$	$c_ф$	$a_н$	$b_н$	$c_н$
"Stepnaya", C_6' (excavation zone 715, 713), [3]	648	-1350	-0.0150	0.970	640	-1442	-0.0166	850	-1731	-0.0187
"Stepnaya", C_6 (excavation zone 606) , [3]	980	-2450	-0.0250	0.994	950	-3053	-0.0316	893	-1819	-0.0296
"Stepnaya", C_6 (excavation zone 604) , [3]	899	-1450	-0.0160	0.991	880	-1671	-0.0214	782	-1593	-0.0237
"Pershotravneva", C_4' (excavation zone 302, 304), [3]	532	-665	-0.0095	0.997	500	-772	-0.0189	536	-1092	-0.0254
"Jubileynaya", C_1 (2-nd east. excavation zone), [3]	630	-1210	-0.0190	0.991	600	-1318	-0.0232	850	-1727	-0.0263
"Jubileynaya", C_6' (excavation zone 605, 607) , [3]	708	-1180	-0.0093	0.973	680	-1454	-0.0138	604	-1230	-0.0142
"Jubileynaya", (excavation zone 530) C_6', [3]	984	-1550	-0.0128	0.971	922	-2063	-0.0230	850	-1731	-0.0237
M.V. Frunze, h_8, [6]	428	-750	-0.0030	0.940	419	-1153	-0.0054	1050	-2138	-0.0040
P.L. Voykov, k_5^1, [7]	540	-900	-0.0030	0.967	480	-1379	-0.0062	750	-1527	-0.0052
Polish mine, [8]	1122	-9350	-0.0150	0.997	1080	-15122	-0.0187	1320	-2688	-0.0162
G.G. Kapustin, $m_3^н$	1178	-1500	-0.0075	0.989	1160	-1546	-0.0090	1785	-3635	-0.0137
"Stepnaya", C_6, [9]	853	-1550	-0.0260	0.998	852	-1386	-0.0212	774	-1564	-0.0335

Table 3

Determination results of paired correlation coefficients between coefficients of equations (1, 4, 7)

Coefficients of equations	Value of pair correlation coefficients (r)						
	for the exponential dependence			for the lognormal dependence		for the hyperbolic dependence	
	$a_к$	$b_к$	$c_к$	$a_к$	$b_к$	$a_к$	$b_к$
$a_ф$	0.997	—	—	0.899	—	0.873	—
$b_ф$	—	0.996	—	—	0.868	—	0.985
$c_ф$	—	—	0.867	—	—	—	—
$a_н$	0.621	—	—	0.363	—	0.499	—
$b_н$	—	0.405	—	—	0.299	—	0.552
$c_н$	—	—	0.791	—	—	—	—

Departing from the values of paired correlation coefficients, defined on the basis of actual and calculated values of input parameters, the most suitable function is an exponential equation to simulate the trajectory of the earth surface points maximum subsidence. More detailed examination of this equation coefficients (4) peculiarities features of their definition (fig. 2).

Ideally, calculated coefficients of the equation must be identical to the coefficients, determined by the least squares method, and straight averaging must be identical to the bisector of the grid. The determination of the coefficient $a_ф$ is very close to a similar value (fig. 2, *a*).

The averaging straight (1) almost coincides with the bisector of the grid (3). Some errors in the determination of $a_н$ are associated with inaccuracy of calculation of input parameters according to [5]. As a result, the averaging straight (2) is located slightly above the bisector (3) of the grid, and the points have a significant deviation from the approximating straight line.

Despite the high correlation coefficient (r = 0,996) of connection between the $b_ф$ and $b_к$, averaging straight (1) does not coincide with the bisector (3) of the grid (fig. 2, *b*). Also the averaging straight (2) of coefficient ($b_н$) does not coincide with the bisector. This indicates the necessity to adjust the coefficient $b_ф$ downwards, and change the coefficient $b_н$ to the increase.

Averaging straights (1) and (2) for coefficients $c_ф$ and $c_н$ are above the bisector (3) of the grid (fig. 2, *c*), and, therefore, to improve the prediction accuracy of the trajectory of the earth surface points maximum subsidence, it is necessary to justify making amendments for specific mining and geological conditions.

Despite some variations of coefficients $a_ф, b_ф$, $c_ф$ from $a_к$, $b_к$, $c_к$, exponential dependence describes quite accurately the trajectory of maximum subsidence of earth surface points (fig. 3).

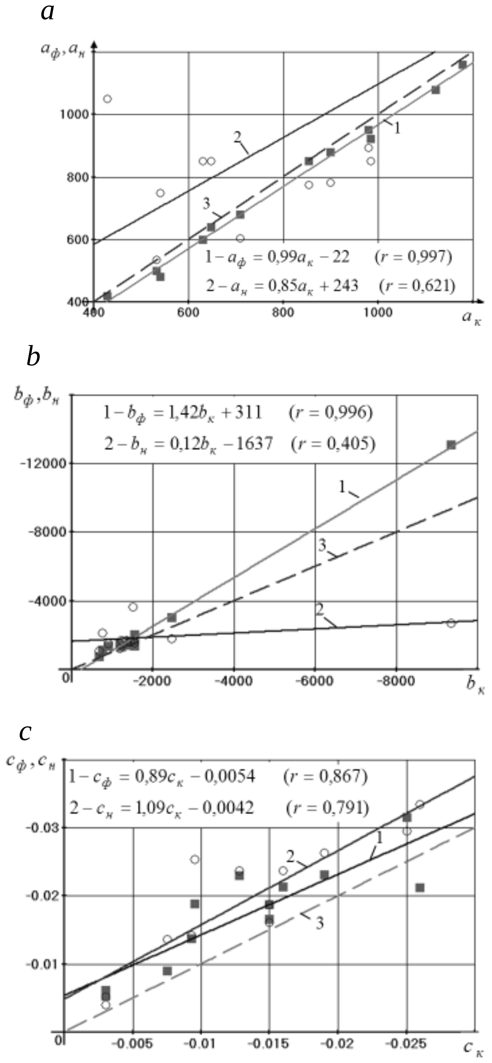

Fig. 2. Relationship between coefficients of the exponential equation (4), determined in different ways: 1– averaging straight of the connection between coefficients ($a_ф$, $b_ф$, $c_ф$), calculated from the actual parameters, with the least squares method ($a_к$, $b_к$, $c_к$); 2 – averaging straight of the connection between coefficients ($a_н$, $b_н$, $c_н$), calculated according to [5], with least squares method ($a_к$, $b_к$, $c_к$); 3 – bisectors of grids; ■, ○ – values of coefficients for specific geological conditions, determined according to the design scheme, respectively by actual parameters of displacement moulds and second minings and according to methods [5]

This confirms the possibility of using the design scheme (fig. 1) and the exponential equation (4) for mathematical modeling of maximum subsidence of earth surface points. The necessary condition for this is reliable determination of η_m, $\eta_к$, $L_н$, $L_к$. These parameters are studied enough for the Western Donbas mines, what is reflected in recommendations of the normative document [5].

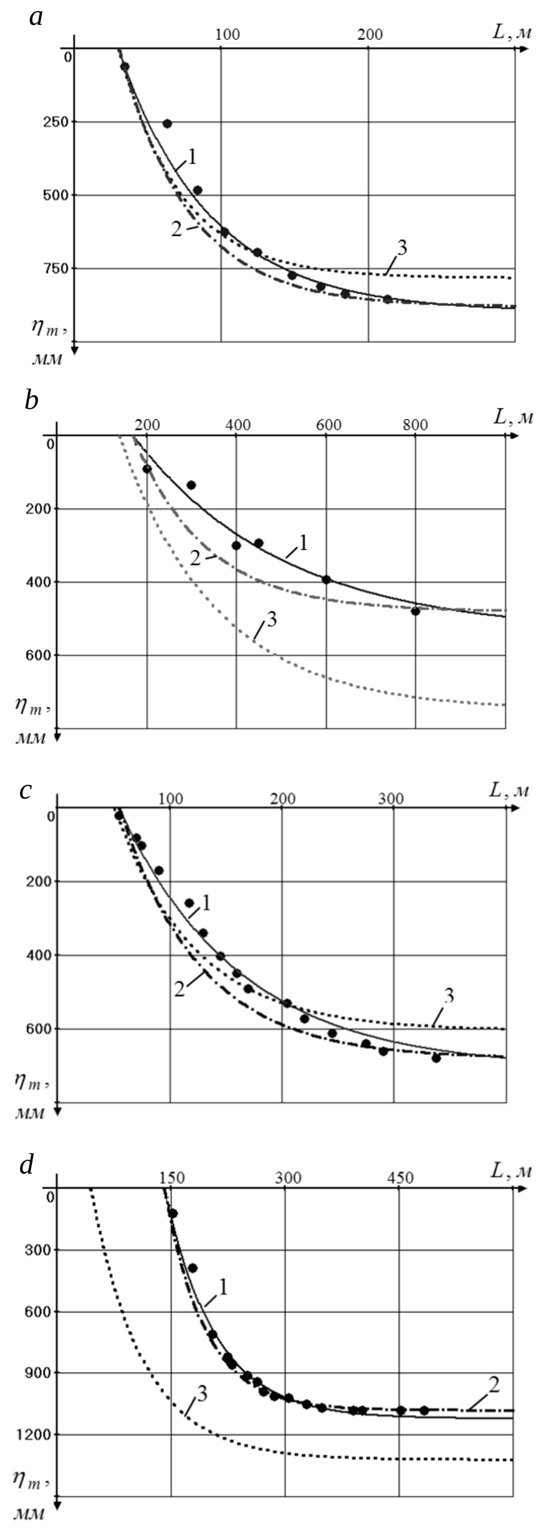

Fig. 3. The example of determining the trajectory of maximum subsidence of the earth surface points: a – mine "Stepnaya" [3]; b – mine "P.L. Voykov" [7]; c – mine "Jubileynaya" [3]; d – Polish mine [8]; 1– curves, defined by the least squares method; 2, 3 – trajectories, calculated according to the design scheme (fig. 1), with using respectively of actual parameters of displacement moulds and calculated according to the normative document [5]; ● – experimental data

This is confirmed by matching the actual trajectories of the earth surface points maximum subsidence with curves calculated by input parameters according to [5] for mines of this region (Fig. 3, *a* and *c*). Such compliance is not observed for other objects of mining-industrial areas. For example, for anthracite seams, according to [5], the maximum subsidence of the earth surface is significantly overestimated (Fig. 3, *b*), and for the Polish mine, a big uncertainty in determination of the parameter $L_н$ is added to the same unconformity (Fig. 3, *d*). These facts indicate the necessity for a more detailed study of parameters η_m, $\eta_к$, $L_н$, $L_к$, and for amendments in the normative document.

These studies led to the following **conclusions:**

- boundary values of displacement moulds parameters and second workings can be used to determine empirical coefficients of mathematical dependences of the trajectory of the earth surface points maximum subsidence;

- exponential equation describes most accurate by the trajectory of the earth surface points maximum subsidence, compared with other types of mathematical functions;

- parameters of displacement moulds and second workings, calculated according to the regulatory document, can be used to predict the trajectory of the earth surface points maximum subsidence only for Western Donbass' mines. More research is needed for other regions.

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Мета. Довести можливість використання граничних значень параметрів мульд зрушення земної поверхні та очисних виробок розрахункової схеми для прогнозування траєкторії максимального осідання точок земної поверхні на стадії розвитку очисних робіт.

Методика. Визначення емпіричних коефіцієнтів математичних залежностей двома способами: на підставі обробки експериментальних даних методом найменших квадратів і згідно з розрахунковою схемою з використанням граничних значень параметрів мульд зрушення та очисних виробок. Встановлення рівноцінності цих способів і виду математичної залежності, що найбільш точно описує траєкторію максимального осідання точок земної поверхні.

Результати. Експоненціальне рівняння найбільш точно описує траєкторію максимального осідання точок земної поверхні. Установлена рівноцінність способів визначення емпіричних коефіцієнтів.

Наукова новизна. На підставі близькості емпіричних коефіцієнтів, визначених різними способами, доведено можливість прогнозування траєкторії максимального осідання точок земної поверхні з використанням граничних значень параметрів мульд зрушення та очисних виробок.

Практична значимість. Визначення емпіричних коефіцієнтів математичних рівнянь запропонованим способом дозволить надалі уникнути трудомістких і тривалих безпосередніх спостережень для отримання експериментальних даних та їх обробки методом найменших квадратів.

Ключові слова: *земна поверхня, траєкторія, емпіричні коефіцієнти, моделювання, максимальне осідання*

Цель. Доказать возможность использования граничных значений параметров мульд сдвижения земной поверхности и очистных выработок расчетной схемы для прогнозирования траектории максимального оседания точек земной поверхности на стадии развития очистных работ.

Методика. Определение эмпирических коэффициентов математических зависимостей двумя способами: на основании обработки экспериментальных данных методом наименьших квадратов и в соответствии с расчетной схемой с использованием граничных значений параметров мульд сдвижения и очистных выработок. Установление равноценности этих способов и вида математической зависимости, наиболее точно описывающей траекторию максимального оседания точек земной поверхности.

Результаты. Экспоненциальное уравнение наиболее точно описывает траекторию максимального оседания точек земной поверхности. Установлена равноценность способов определения эмпирических коэффициентов.

Научная новизна. На основании близости эмпирических коэффициентов, определенных разными способами, доказана возможность прогнозирования траектории максимального оседания точек земной поверхности с использованием граничных значений параметров мульд сдвижения и очистных выработок.

Практическая значимость. Определение эмпирических коэффициентов математических уравнений предложенным способом позволит в дальнейшем избежать трудоемких и длительных непосредственных наблюдений для получения экспериментальных данных и их обработки методом наименьших квадратов.

Ключевые слова: *земная поверхность, траектория, эмпирические коэффициенты, моделирование, максимальное оседание*

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