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ROLLING ELECTRICAL COMPLEX ON THE BASIS OF THE CRITERION OF MINIMIZING THE AREA UNDER THE CURVE OF MOTION

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КЕРУВАННЯ РУХОМИМ ЕЛЕКТРОТЕХНІЧНИМ КОМПЛЕКСОМ НА ОСНОВІ КРИТЕРІЮ МІНІМІЗАЦІЇ ПЛОЩІ ПІД КРИВОЮ РУХУ

Purpose. Generic description of the content of criterion of minimization of the area under the curve of the motion of the electrical complex.

Methodology. The methods of variational calculus in the combination with mechanics approaches were used for systematic studies of the properties of the mobile object. By the optimal control over the motion of the electrical complex was understood such a choice of the sequence of the traction drive control modes, which ensures minimization of the criterion of optimality allowing for all restrictions imposed on the parameters and conditions of operation. The quantitative measure that characterizes the decision, i.e. the choice of the electrical complex motion control, was understood to be the criterion of optimality.

Findings. We developed the analytical model of application of the criterion of minimization of the area under the curve of the motion of the electrical complex. It was proved that the minimization of the square, which is formed by the $\text{arc}\,S = S(t)$ during its rotation about the x-axis, also leads to a decrease of the length of the $\text{arc}\,S = S(t)$ and a corresponding decrease of traction.

Originality. For the first time, the use of the criterion of minimizing the area under the curve of the movement was proposed and justified to develop operating procedure of traction drives of the mobile electromechanical objects, which allows for the minimum value of traction. In contrast to existing criteria, the proposed one allows implementing the optimal dynamic properties of the electrical system when in motion.

Practical value. Based on the developed criterion the procedure of calculation of the parameters of the optimal motion curve was developed. It allows us to design systems of automatic regulation and control systems of traction drives, to perform testing, improvement, repair and diagnostics of systems of rolling electrical complex.

Keywords: *mobile complex, work, cost minimization, motion curve, electric transmission*

Statement of the problem. Electrical transmissions are used mainly in high power machines (rolling stock of Railways, tractors, combines, trucks, and buses, urban and suburban transport). Such transmissions (drivelines) have the following advantages over mechanical and hydromechanical transmissions [1]:

- the possibility of smooth, infinitely variable torque;
- a simplified mechanical part of the actuator;
- large environmental safety (machines with hydraulic transmissions and automatic transmissions frequently leak oil which gets into the soil posing significant risks to the environment);
- the drive motor (diesel or gasoline engine) runs at its optimum, almost constant, operating mode;
- the use of electrical methods of braking, which reduces the wear of mechanical parts of the braking system of the machine;
- lower weight of transmission per mass unit of the machine for the complexes with an engine capacity of over 700 kW.

Functional tasks assigned to the electric traction drive, the standards and considerations applicable to its techno-economic, environmental, ergonomic and other indicators (precision, performance, the range of permissible changes in the performance, electromagnetic compatibility with other components of energy systems, energy efficiency) lead to the development of the systems of traction drives. Apart from the main system-forming component of the electromechanical transducer, the systems should include a variety of energy converters, as well as control, master and protection devices. The basis of the information subsystems of modern traction drives typically comprise a microcontroller device that has some significant advantages compared with analogue control devices that implement typical arithmetic and logical functions, processing of arrays, the regulation of electromagnetic

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and mechanical variables, stabilization, correction and compensation of nonlinearities, observation, simulation of the control object and processing of operation laws. Such systems allow fulfilling precise mathematical control laws, which have been almost impossible to implement so far with limited resources and space on a movable electrical complex; this allows saving fuel and energy resources more efficiently [1].

The optimal traffic control of the electrotechnical complex is considered to be the choice of the sequence of control modes which ensures minimization of the criterion of optimality when all restrictions imposed on the parameters and conditions of the operation of the traction electric drive are met [2].

By the criterion of optimality a quantitative indicator is understood to feature the decision made, i.e. the choice of motion control of the electrical complex [2].

Analysis of the recent research and publications. Many fundamental works both in our country and abroad have been devoted to the motion optimization of autonomous mobile electrical systems depending on certain criteria [1–5]. Most of these works resolve into the minimum cost criterion regarding the movement in a certain area, or minimum travel time in this mine section. Moreover, the criterion of the minimum operating time almost always results in a significant excess of standard indicators of fuel and energy resources for the movement of electrical engineering complex, because it is oriented to overcome the schedule delay in time and does not allow for the possibility of a comprehensive solution to the elimination of delay in time with the possibility of providing the most efficient fuel consumption. In contrast to this criterion, in the case of using the criterion of minimum costs, in practice the situation quite often occurs which does not allow producing the maximum values for acceleration and jerk when driving electrical complex, and hence the optimal dynamic properties during movement.

Unsolved aspects of the problem. In this paper, we analyse the criterion of minimization of the area under the curve of the motion of the electrical engineering complex which is in fact reduced to the criterion of the lowest cost associated with the movement in a specific area with a possibility to obtain a set of dynamic indicators of movement, including the running time. In addition, its mathematical description is convenient while constructing the necessary motion trajectory based on Autonomous on-Board intelligent control systems, which are known to have limited resources.

Objectives of the study. To describe, in general terms, the content of the criterion of minimization of the area under the curve of the motion of the electrical engineering complex.

The main material. The essence of this criterion is to obtain the minimum area under the curve of motion of the electrotechnical complex, which defines the minimum value of the performed work required to ensure the transportation process. Let us explain the subject matter of this criterion.

The optimality criterion should meet the following requirements:

1. The optimality criterion must be expressed quantitatively.
2. The optimality criterion must be unique.
3. The value of the optimality criterion should vary monotonically (without gaps and jumps).
4. The optimality criterion should reflect the most essential part of the process.
5. It is desirable that the optimality criterion should have a clear physical or geometric meaning and is easily calculated.

However, for systems of automatic control the optimality criteria can provide a certain level of the deviation of the system from the desired or planned status. The choice of the optimization criterion represents a compromise between the desire to describe the specific purpose of optimization more accurately and the need to get as easy a solution for the desired task as possible. In this case, the optimization criterion does not necessarily need to have a clear physical or geometric meaning, but must meet other requirements which regard it.

Let a material point M, moving in a straight line affected by constant resultant force $\overline{F_p}$ travel a certain distance (value) which is expressed by the vector $\overline{r}$. The work A_F done by this force (hereafter, only the component of force required for traction is considered), is called the scalar product of the force $\overline{F_p}$ vector and the displacement vector $\overline{r}$ [6]

$$A_F = \overline{F_p} \cdot \overline{r}. \tag{1}$$

Considering the facts that in the general case of motion of the complex the resultant force $\overline{F_p}$ varies both in magnitude and direction, and that the move towards the curve $S = S(t)$ is not straight, to apply the (1) directly is impossible. Then let us use the method of finding the work done by a force along a curved trajectory [7].

We divide the curve AB arbitrary points $A = A_0(S_0;\, t_0)$, $A_1(S_1;\, t_1), \ldots, B = A_n(S_n;\, t_n)$, taken in the direction from A to B into n arcs (Fig. 1).

Thus, on grounds that for each point of the curve of motion the resultant force is defined which acts on the electrical complex, the magnitude and direction of which depend on the driving conditions and the position of the object, we can determine that the motion of the given point M occurs in a force field (F) (Fig. 1).

On each partial arc $\overset{\frown}{A_{k-1}A_k}$ let us choose arbitrarily points $M_k\left(S_k;t_k\right)$, $k = \overline{1,n}$ (Fig. 2).

On the partial arc $\overset{\frown}{A_{k-1}A_k}$, let us replace the approximately variable force $\overline{F_p}$ by the constant force $F_{pk}(S_k;\, t_k)$, which is equal to the modulus of the resultant force vector $\overline{F_p}$ at the point M_k. The motion of the material point along this arc will be replaced by its movement along the chord $A_{k-1}A_k$ of this arc.

Let us do all this for $\forall\ k = \overline{1,n}$. As a result of approximate substitutions we have the following:

- a particle moves along the polygon inscribed in the curve AB;
- a constant force acts on a material point on each broken link.

The work of force $\overline{F_{pk}}\left(S_k;t_k\right)$ on the chord $A_{k-1}A_k$ equals

$$A'_k = \overline{F_{pk}}\left(S_k;t_k\right) \cdot \overline{A_{k-1}A_k}. \tag{2}$$

Let us find the total work along the whole link for $k = 1, n$

$$A' = \sum_{k=1}^{n} A'_k = \sum_{k=1}^{n} \overline{F_{pk}}\left(S_k;t_k\right) \cdot \overline{A_{k-1}A_k}, \tag{3}$$

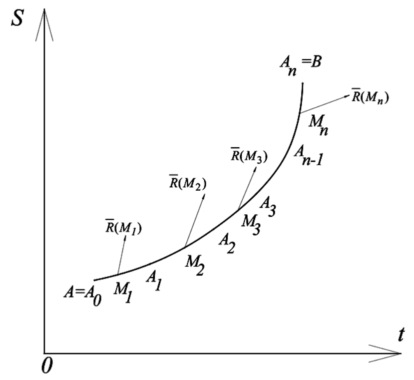

Fig. 1. The separation of the curve of the motion into parts

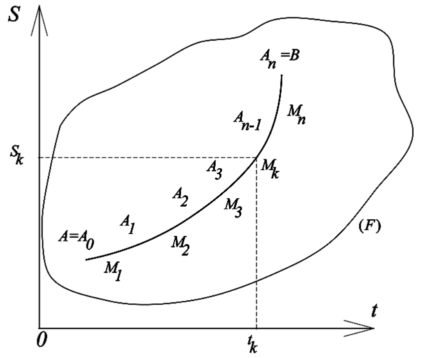

Fig. 2. Force field of the curve of the motion

where A' is the work speed of force during the motion of the point along the polyline $A_0A_1\ldots A_n$, inscribed in the curve AB.

The work, which is determined by the equation (3) is the approximate value of the desired work A_F of force $\overline{F_p}$ when the material point is moving along the curve AB

$$A' \approx A_F. \tag{4}$$

Let the resultant force $\overline{F_p}$ in the force field (F) be decomposed into components along the respective axes of the coordinate system which shows the force field according to the following equation

$$\overline{F_p} = P(S;t)\bar{i} + Q(S;t)\bar{j}, \tag{5}$$

where $\bar{i}$, $\bar{j}$ are the unit vectors.

Let us define the projection of the vector $\overline{A_{k-1}A_k}$ on the axis of the coordinate system by the following equation

$$\Delta S_k = S_k - S_{k-1}; \tag{6}$$

$$\Delta t_k = t_k - t_{k-1}. \tag{7}$$

Then for the vector $\overline{A_{k-1}A_k}$ the following relation is true

$$\begin{aligned}\overline{A_{k-1}A_k} &= (t_k - t_{k-1})\bar{i} + (S_k - S_{k-1})\bar{j} = \\ &= \Delta t_k \cdot \bar{i} + \Delta S_k \cdot \bar{j}.\end{aligned} \tag{8}$$

Taking into account the last equation, the equation (5) we write the (2) in the following form

$$A_k' = P(S;t)\cdot \Delta t_k + Q(S;t)\cdot \Delta S_k. \tag{9}$$

Then on the basis of the equations (3,4) taking into account the equation (9), we obtain the following

$$A_F \approx \sum_{k=1}^{n} P(t_k;S_k)\cdot \Delta t_k + Q(t_k;S_k)\cdot \Delta S_k. \tag{10}$$

Let ΔL_k be the arc lengt $\overset{\frown}{A_{k-1}A_k}$, and the value $d = \max\limits_{k=1,n} \Delta L_k$. Then the equation (10) can be written as follows, assuming that the exact value is the limit of the amount received, subject to the following arc $\overset{\frown}{A_{k-1}A_k}$ length to zero

$$A_F = \lim_{d\to 0}\sum_{k=1}^{n} P(t_k;S_k)\cdot \Delta t_k + Q(t_k;S_k)\cdot \Delta S_k. \tag{11}$$

The equation (11) means that with $n \to \infty$ each arc $\overset{\frown}{A_{k-1}A_k}$ shrinks to a point. Then, according to the definition of the curvilinear integral, the following equation operates

$$A_F = \lim_{n\to \infty}\sum_{k=1}^{n} P(t_k;S_k)\cdot \Delta t_k + Q(t_k;S_k)\cdot \Delta S_k. \tag{12}$$

On the ground that

$$\begin{aligned}&\sum_{k=1}^{n}\left[P(t_k;S_k)\cdot \Delta t_k + Q(t_k;S_k)\cdot \Delta S_k\right] = \\ &= \sum_{k=1}^{n}\left[P(t_k;S_k)\cdot \Delta t_k\right] + \sum_{k=1}^{n}\left[Q(t_k;S_k)\cdot \Delta S_k\right]\end{aligned} \tag{13}$$

and assuming that there are limits to the amounts that are on the right side of the last equality, we have the following

$$\begin{aligned}&\lim_{n\to\infty}\sum_{k=1}^{n}\left[P(t_k;S_k)\cdot \Delta t_k + Q(t_k;S_k)\cdot \Delta S_k\right] = \\ &= \lim_{n\to\infty}\sum_{k=1}^{n}\left[P(t_k;S_k)\cdot \Delta t_k\right] + \lim_{n\to\infty}\sum_{k=1}^{n}\left[Q(t_k;S_k)\cdot \Delta S_k\right].\end{aligned} \tag{14}$$

The boundaries that are right in the equation (14) can be considered as curvilinear integrals along the arcs $\overset{\frown}{AB}$, respectively, from the vector-functions $P(S;t)$ and $Q(S;t)$. Therefore, the equation (14) can be represented in the following form

$$\begin{aligned}&\int_{AB}\left[P(S;t)dt + Q(S;t)dS\right] = \\ &= \int_{AB} P(S;t)dt + \int_{AB} Q(S;t)dS.\end{aligned} \tag{15}$$

According to the definition of the curvilinear integral [6, 7], the force along the curved arc is equal to the integral along this arc. The last statement allows you to write the equation (15) as follows

$$A_F = \int_{AB} Pdt + \int_{AB} QdS, \tag{16}$$

where P and Q are the projections of the resultant force on the axis of the coordinate system of the curve of the motion.

On the other hand, as the target path is defined as the limit of the sum of elementary works $\overline{F_p}\cdot d\bar{r}$, it actually is expressed by the following integral

$$A_F = \int_{S_1}^{S_2} \overline{F_p}\cdot d\bar{r}. \tag{17}$$

Let us substitute in the last equation the vector of the resultant force equal to it, according to the basic law of mechanics, for the vector $\left(m\cdot\frac{d^2\bar{r}}{dt^2}+\overline{C_o}+\overline{C_\partial}\right)$, where C_o is the main resistance force, which is caused by the resistance of the air environment, internal friction in the electrical complex, and the interaction of the electrical complex and paths, whereas $C_д$ is the force of the additional resistance to movement, which is determined by the conditions of slopes and curved track sections, as a result of which we obtain the following equation

$$A_F=\int_{S_1}^{S_2}\left(m\cdot\frac{d^2\bar{r}}{dt^2}+\overline{C_o}+\overline{C_\partial}\right)d\bar{r}. \tag{18}$$

Let us turn to the integration time, using the substitution

$$d\bar{r}=\frac{d\bar{r}}{dt}\cdot dt. \tag{19}$$

Then we get the equation (18) as follows

$$A_T=\int_{S_1}^{S_2}\left(m\cdot\frac{d^2\bar{r}}{dt^2}+\overline{C_o}+\overline{C_\partial}\right)\cdot\frac{d\bar{r}}{dt}dt=$$
$$=\frac{m}{2}\cdot\left(\left(\frac{d\bar{r}}{dt}\right)^2\bigg|_{t_2}-\left(\frac{d\bar{r}}{dt}\right)^2\bigg|_{t_1}\right)+ \tag{20}$$
$$+\left(\overline{C_\partial}+\overline{C}\right)\cdot\left(r|_{t_2}-r|_{t_1}\right).$$

Analysing the last equation, we assume that the constant values of the coordinates of the start and end points of the curve of motion are (t_1; S_1) and (t_2; S_2), respectively, which is determined by the graphic chart of movement. Thus, we conclude that to minimize the work that is performed by the resultant force to move the body with mass m, it is necessary to decrease the value $\left(\frac{d\bar{r}}{dt}\right)$.

Since according to the content of the curve of motion the following system of constraints is applied for the coordinates of the start and end points of the curve of motion (t_1; S_1) and (t_2; S_2)

$$\begin{cases} t_1<t_2 \\ S_1<S_2 \end{cases}, \tag{21}$$

then under the condition of the constant values of the coordinates of the start and end points of the curve of motion the following options of passing the curve $S=S(t)$ shown in Fig. 3 are possible:

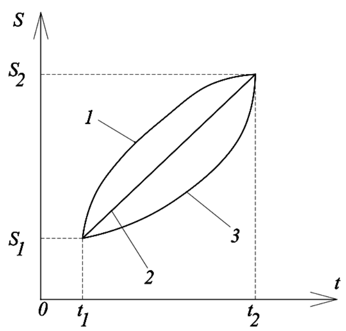

Fig. 3. Options of passing for the curve S = S(t)on a fixed part of the curve of motion

- a convex form of the curve $S=S(t)$ – curve 1;
- $S=S(t)$ is a straight line – curve 2;
- the concave shape of the curve $S=S(t)$ – curve 3.

As it is well known, at every point, the derivative of the function is equal to the slope of the tangent line to the curve of this function [6, 7]. Let us choose an arbitrary point on the curve of motion of Fig. 3 and draw a tangent to it (Fig. 4).

In this alternate version, the placement of the tangent point is at the same time level for the first characteristic curve $S=S(t)$.

As it can be seen in this case we have the following relation

$$\text{tg}\,\beta_3<\text{tg}\,\beta_2<\text{tg}\,\beta_1, \tag{22}$$

which suggests that this version of the distribution of the curve $S=S(t)$ on the line AN_i (the index i is the position number of the point N for each curve, respectively) to minimize the value of the derivative of this function on the section of such a peculiar shape.

Let us consider the second possibility of the peculiar shape of the curve of motion – the one with which a certain section approaches the straight line.

Let us choose an arbitrary point on the curve of motion in Fig. 3 and draw a tangent to it (Fig. 5).

In this alternate version, the placement of the tangent point is at the same time level for the second peculiar section of the curve $S=S(t)$. As you can see in this case we have the following formula

$$\text{tg}\,\beta_3<\text{tg}\,\beta_2\approx\text{tg}\,\beta_1 \tag{23}$$

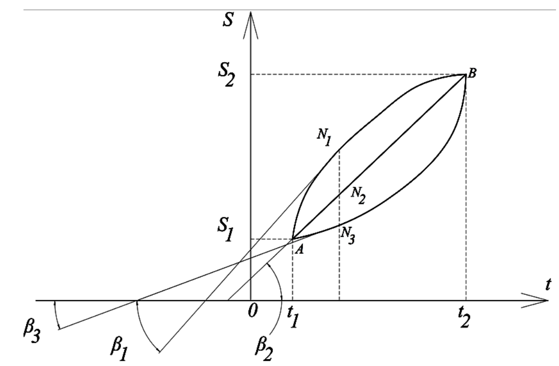

Fig. 4. First property of tangents to graphs S = S(t)

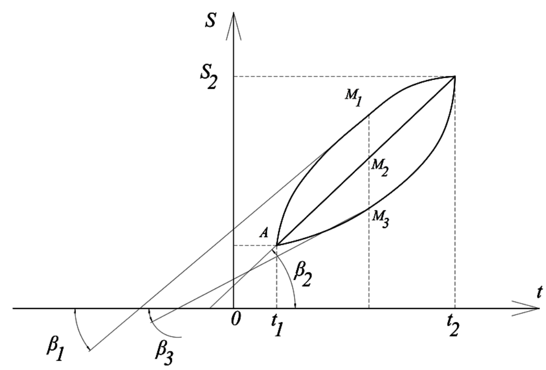

Fig. 5. The second property of tangents to graphs S = S(t)

which suggests that this version of the distribution of the curve $S = S(t)$ at the section near the point M_i (the index i is the position number of the point M for each curve, respectively), which is a straight line that minimizes the value of the derivative of this function at the section of the direct form.

When the sections of the functions $S = S(t)$ similar to those of parallel lines can be considered a neighbourhood of a point M_i, the following formula will be used

$$\operatorname{tg}\beta_3 \approx \operatorname{tg}\beta_2 \approx \operatorname{tg}\beta_1. \quad (24)$$

Let us consider the second possible case, the one in which a certain section approaches a straight line.

Let us choose an arbitrary point on the curve of the movement in Fig. 3 and draw a tangent to it (Fig. 6).

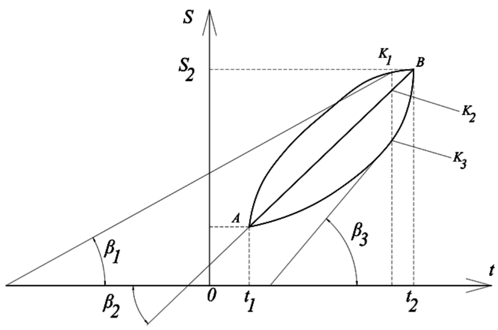

Fig. 6. The third property of tangents to graphs $S = S(t)$

In this alternate version, the placement of the tangent point is at the same time level for the third characteristic curve. As you can see, in this case we have the following formula

$$\operatorname{tg}\beta_1 < \operatorname{tg}\beta_2 < \operatorname{tg}\beta_3. \quad (25)$$

According to the (25) in this area, the value of the derivatives of functions in a neighbourhood of a point K_i (the index i is the position number of point K for each curve, respectively) does not allow minimizing the value of the derivative of this function in the section of this form (the third curve). This option requires a separate explanation.

Given the characteristics of the curve as a geometric interpretation of the curve of motion of the electrotechnical complex, it is impossible to plot the curve of motion with the ratios that can be represented in the following collective

$$\begin{cases} t_k < t_{k+1} \\ S_{k+1} < S_k \end{cases} \cup \begin{cases} t_k > t_{k+1} \\ S_{k+1} < S_k \end{cases}, \quad (26)$$

where k is an arbitrary number of the curve $S = S(t)$, and

$$\lim_{m\to\infty} \sum_{k=1}^{m} S_k = \overset{\frown}{AB}. \quad (27)$$

Then we introduce restrictions to the values of the corresponding coordinates, which define the following system

$$\begin{cases} t_k < t_{k+1} \\ S_k < S_{k+1} \end{cases}. \quad (28)$$

With that considering that the expression (27) is correct. The system of constraints (28) actually means "irreversibility" of time and eliminates a situation in which the electrotechnical complex drives past the destination point to which it was directed, and then comes back because it has driven by. Mathematically, we excluded the case from consideration in which $\operatorname{tg}\beta_i < 0$, as in this case $\beta_i \geq \frac{\pi}{2}$, what is shown in Fig. 7.

In Fig. 8, let us consider curve 3 corresponding to the case analysed in Fig. 6.

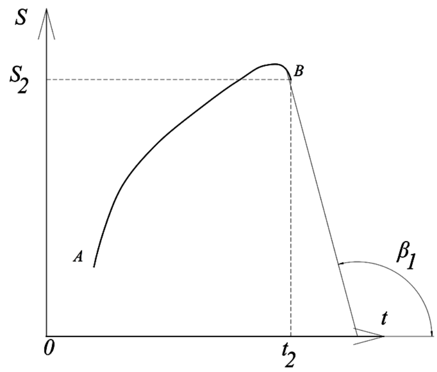

Fig. 7. Impossible distribution of the curve $S = S(t)$

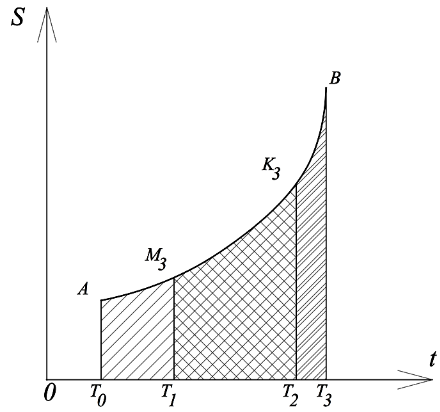

Fig. 8. Diagram for explanation of the area under the graph $S = S(t)$ in different cases of the slope of the given curve about the x-axis

It is evident that for the area *П,* i. e. shaded figures in Fig. 8 (under the relevant sections of the curve of motion), the following ratio is fulfilled

$$\Pi_{AM_3T_1T_0} + \Pi_{M_3K_3T_2T_1} > \Pi_{K_3BT_3T_2}. \quad (29)$$

Increasing the length of the corresponding arc K_0B, resulting in dramatic change of the slope of the curve $S = S(t)$ and the expression (29) losing its correctness with certain proportions, is impossible, because in fact it will mean a significant increase in speed, which is a limited value. In addition, the value of the maximum angle of inclination of the tangent to the graph of the curve is determined by the marginal values of jerk and acceleration and, therefore, has a certain value fixed for the given electrical complex in a particular area of the distance of movement.

For the phases of the electrical complex movement – acceleration → motion at established speed → braking, the following shape of the curve $S = S(t)$ shown in Fig. 9 is intrinsic

In Fig. 9, the acceleration section will correspond to the arc *AF*, which is actually the element of the contour of the ellipse, since there is a change of velocity in time, i. e. the change of the ratio between the sections of the road, covered over the same time period.

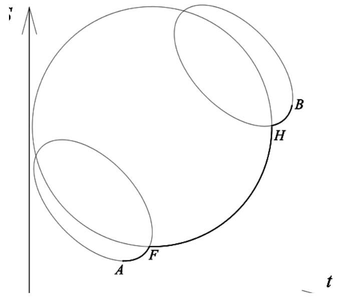

Fig. 9. The characteristic shape of the curve S = S(t) for different phases of movement of the electrotechnical complex

In Fig. 9, the movement section with constant velocity will correspond to the arc *FH*, which is actually the element of the circle, as there is a constant ratio between the sections of the road, covered over the same time period.

In Fig. 9, the braking section will correspond to the arc *HB*, which is a circumference of an ellipse, as deceleration occurs in time, i.e. reduction of the ratio between the sections of the road, covered over the same time period.

Let us consider the section of the motion with constant velocity (Fig. 10).

To complete the inequality (29) we introduce the following constraints

$$\Delta L_{F_3O_5} > \Delta L_{O_5H_3}, \quad (30)$$

considering also that the restrictions on the maximum values of velocity, acceleration and jerk in this case are implemented.

This should also run the ratio

$$\Delta L_{F_1O_4} > \Delta L_{O_4H_1}, \quad (31)$$

which will lead to the execution of equations (23, 29).

For different phases of movement of the electrotechnical complex, all the sections of the curve $S = S(t)$ discussed above in their shape follow the shape of the curve discussed between two arbitrary points of the movement earlier in this study. That is, for each phase of movement it is necessary to solve the optimization problem of the distribution curve $S = S(t)$ and implement it accordingly.

As it is known [8, 9], the length of the arc *AB* is such a border to which the length of the polygonal curve inscribed in this arc tends, when the length of the largest segment tends to zero. That is, the following formula is valid

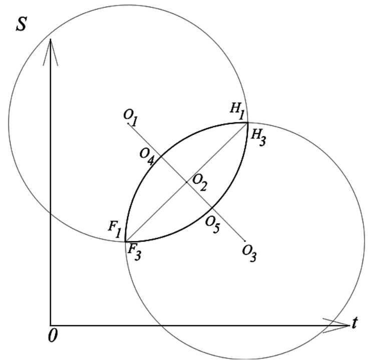

Fig. 10. The curve $S = S(t)$ for the motion with constant velocity

$$L_{\widehat{AB}} = \lim_{\max \Delta L_k \to 0} \sum_{k=1}^{m} \Delta L_k. \quad (32)$$

Based on the fact that the managing an electrical complex is a continuous function, i.e. the thrust force is continuous, then at a certain time interval, assuming constant values of the coordinates of the start and end point of the curve of motion – $(t_1; S_1)$ and $(t_2; S_2)$ accordingly, the function $S = S(t)$ and its derivative are also continuous functions. Then it can be argued that the bound (32) exists.

Let

$$\Delta S_k = S(t_k) - S(t_{k-1}). \quad (33)$$

Then, given that time cannot take on negative values, we have

$$\Delta S_k = \sqrt{(\Delta S_k)^2 + (\Delta t_k)^2} = \Delta t_k \cdot \sqrt{1 + \left(\frac{\Delta S_k}{\Delta t_k}\right)^2}. \quad (34)$$

Using Lagrange's theorem [1], we will get

$$\frac{\Delta S_k}{\Delta t_k} = \frac{S(t_k) - S(t_{k-1})}{t_k - t_{k-1}} = S'(\tau_k) \quad (35)$$

and

$$t_{k-1} < \tau_k < t_k. \quad (36)$$

Then on the basis of formulas (35, 36) we have

$$\Delta S_k = \Delta t_k \cdot \sqrt{1 + (S'(\tau_k))^2}. \quad (37)$$

Thus, we come to the conclusion that the length of the inscribed polygonal equals

$$L_m = \sum_{k=1}^{m} \left[\Delta t_k \cdot \sqrt{1 + (S'(\tau_k))^2} \right]. \quad (38)$$

Based on the continuity of the derivative of the function $S = S(t)$ we suggest that the function $\sqrt{1 + (S'(\tau_k))^2}$ is also continuous. Then the limit of integral sums exists and it equals to the integral defined

$$L_{\widehat{AB}} = \lim_{\max \Delta L_k \to 0} \sum_{k=1}^{m} \left[\Delta t_k \cdot \sqrt{1 + (S')^2} \right] \quad (39)$$

or

$$L_{\widehat{AB}} = \int_{t_1}^{t_2} \sqrt{1 + \left(\frac{dS}{dt}\right)^2} \, dt. \quad (40)$$

Conclusions and recommendations for further research in the area.

1. Thus, we received the arc length of the curve of motion of the electrical complex on the basis of coordinate values of the start and end points of the curve of motion, $(t_1; S_1)$ and $(t_2; S_2)$ respectively. Comparing this formula with the formula for finding the area of a figure formed by the arc in its rotation around the *x*-axis, we see that this formula is included in the formula for calculating the area as a multiplier.

2. Therefore, minimization of the area that an arc forms $S = S(t)$ rotating about the *x*-axis also leads to a decrease in the arc length $S = S(t)$. That is, the formulas (30, 31) in this case will also be executed automatically.

3. A mathematical model shows the prospects of further research in this direction for obtaining other criteria depending on practical needs.

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Мета. Описання в загальному вигляді змісту критерію мінімізації площі під кривою руху електротехнічного комплексу.

Методика. У роботі використані методи варіаційного числення, що поєднані з використанням підходів механіки тіла для системного дослідження властивостей рухомого об’єкта. Під оптимальним керуванням рухом електротехнічного комплексу розумівся такий вибір послідовності режимів керування тяговим електроприводом, який забезпечує мінімізацію критерію оптимальності при виконанні всіх обмежень, що накладаються на параметри та умови роботи електроприводу. Під критерієм оптимальності розумівся кількісний показник, що характеризує прийняте рішення – вибір керування рухом електротехнічного комплексу.

Результати. Створена аналітична модель використання критерію мінімізації площі під кривою руху електротехнічного комплексу. Доведено, що мінімізація площі, що її утворює дуга $S = S(t)$ при своєму обертанні навколо осі абсцис, призводить також до зменшення довжини дуги $S = S(t)$ та відповідного зменшення величини тягової роботи.

Наукова новизна. Уперше запропоноване та обґрунтоване використання критерію мінімізації площі під кривою руху для розробки алгоритмів роботи електроприводів рухомих електромеханічних об’єктів, що дозволяє забезпечувати мінімальне значення тягової роботи. На відміну від існуючих, запропонований критерій дозволяє реалізувати оптимальні динамічні властивості електротехнічного комплексу при русі.

Практична значимість. На основі розробленого критерію визначена процедура розрахунку параметрів оптимальної кривої руху, що дозволяє будувати системи автоматичного регулювання й управління системами тягових електроприводів, здійснювати випробування, удосконалення, ремонт та діагностику систем рухомого електротехнічного комплексу.

Ключові слова: *рухомий комплекс, робота, мінімізація витрат, крива руху, електрична передача*

Цель. Описание в общем виде содержания критерия минимизации площади под кривой движения электротехнического комплекса.

Методика В работе использованы методы вариационного исчисления, которые объединены с использованием подходов механики тела для системного исследования свойств подвижного объекта. Под оптимальным управлением движением электротехнического комплекса понимался такой выбор последовательности режимов управления тяговым электроприводом, который обеспечивает минимизацию критерия оптимальности при выполнении всех ограничений, которые накладываются на параметры и условия работы электропривода. Под критерием оптимальности понимался количественный показатель, который характеризует принятое решение – выбор управления движением электротехнического комплекса.

Результаты. Создана аналитическая модель использования критерия минимизации площади под кривой движения электротехнического комплекса. Доказано, что минимизация площади, которую образовывает дуга $S = S(t)$ при своем обращении вокруг оси абсцисс, приводит также к уменьшению длины дуги $S = S(t)$ и соответствующему уменьшению величины тяговой работы.

Научная новизна. Впервые предложено и обосновано использование критерия минимизации площади под кривой движения для разработки алгоритмов работы электроприводов подвижных электромеханических объектов, что позволяет обеспечивать минимальное значение тяговой работы. В отличие от существующих, предложенный критерий позволяет реализовать опти-

мальные динамические свойства электротехнического комплекса при движении.

Практическая значимость. На основе разработанного критерия определена процедура расчетов параметров оптимальной кривой движения, что позволяет проектировать системы автоматического регулирования и управления системами тяговых электроприводов, осуществлять испытание, усовершенствование, ремонт и диагностику систем подвижного электротехнического комплекса.

Ключевые слова: *подвижный комплекс, работа, минимизация затрат, кривая движения, электрическая передача*

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SYNTHESIS OF HIGH-VOLTAGE CONVERTER FOR ELECTROTECHNOLOGY

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СИНТЕЗ ВИСОКОВОЛЬТНОГО ПЕРЕТВОРЮВАЧА ДЛЯ ЕЛЕКТРОТЕХНОЛОГІЇ

Purpose. Development of the theory of design of the generators of high-voltage pulses, creation of a generator with less loss of energy.

Methodology. The high-voltage converter scheme synthesis was done by the synthesis algorithm with variable structure based on the graph of state changes. The synthesis procedure is the ideal arrangement of keys in a circuit with a constant structure so that the nature of electromagnetic processes in the received circuit with variable structure conforms to the originally specified graph of state changes.

Findings. The synthesis of high-voltage circuit converter was performed in accordance with the requirements of energy loss reduce: the assurance of the of IGBT operation modes, which reduce current and voltage loading of the device, and allow switching the transistor with zero current or voltage, or in the case of neutral switching.

Originality. A new circuit of a converter was synthesized. It can halve the working voltage of the primary capacitive storage, and reduce the current and voltage loading of the power switches. Thereby, it becomes possible to increase the allowable frequency of operation of the high-voltage converter in comparison with a single-cycle circuit of the converter.

Practical value. The proposed method of implementation and the high-voltage converter device can be used in electrotechnics of cleaning of sulphur-containing gases from sulphur dioxide, since the high-voltage converter has better mass and size (by reducing the amount of magnetic knots) than similar devices based on single-cycle thyristor schemes and loses less energy (efficiency of 70 % vs. 60 %).

Keywords: *electrotechnology, high-voltage converter, graph of state change*

Definition of the problem: The primary task of modern competitive industrial production is the mastery of high-performance technologies based on sustainable use of natural resources, saving energy and reducing harmful emissions [1, 2]. The development of such technologies is facilitated by the use of high-voltage pulse technique of the nanosecond and submicrosecond bands in particular for the purposes of purification of waste gases of metallurgical production from sulfur dioxide.

Unsolved aspects of the problems. The development of highly efficient method of neutralization of sulfur-containing gases is based on the method of processing gases of electric discharge and promising method of cleaning gases from sulfur dioxide with crucial assistance of activated absorption solution. The key factor is the power specific volume which can be created in the discharge gap. From this point of view, pulsed action of high voltage (tens to hundreds of kilovolts) with low pulse duration (no more than several hundreds of ns) is of great interest; under the action a streamer discharge is formed and the transition discharge to the form of the spark discharge is prevented, since the spark discharge leads to a sharp drop of the specific power by volume in the discharge chamber. To form a streamer discharge it is necessary to develop a high-voltage impulsive energy sources – a high-voltage impulsive converter.

In the middle of the last century, high-voltage pulse power sources began to be created for research programmes with a view to their use in particle accelerators and fusion processes. In this direction the main efforts are concentrated on getting record-high output power.

A large number of scientists have been engaged in the problem of the design and creation of the high-voltage pulse generator: I. S. Garber, L. A. Meerovich, G. A. Mesyats,

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