ГЕОТЕХНІЧНА І ГІРНИЧНА МЕХАНІКА, МАШИНОБУДУВАННЯ

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B. V. Vinogradov, Dr. Sci. (Tech.), Professor,
D. O. Fedin, Cand. Sci. (Tech.), Associate Professor

State Higher Educational Institution "Ukrainian State University of Chemical Technology", Dnepropetrovsk, Ukraine, e-mail: bvvin@mail.ru; dafedin@mail.ru

THE STIFFNESS OF THE OPEN GEARING OF TUMBLING MILLS

Б. В. Виноградов, д-р техн. наук, проф.,
Д. О. Федін, канд. техн. наук, доц.

Державний вищий навчальний заклад „Український державний хіміко-технологічний університет", м. Дніпропетровськ, Україна, e-mail: bvvin@mail.ru; dafedin@mail.ru

ЖОРСТКІСТЬ ВІДКРИТИХ ЗУБЧАСТИХ ПЕРЕДАЧ БАРАБАННИХ МЛИНІВ

Purpose. Development of the calculation technique of gear wheels tooth pair stiffness in one-pair gearing by the finite elements method and its application for an estimation of tumbling mills open tooth gearings stiffness.

Technique. The calculation of stress state was carried out by the finite elements method. At the first stage the analytical model of a gearing was formed. Based on the key geometrical parameters and drawings the three-dimensional geometrical model of the tooth gearing was designed. The physical properties of the constructional materials were set. The geometrical model split into finite elements. The boundary conditions that ensure the kinematic stability of model, and the condition of contact of working surfaces of teeth were set. An external loading in the form of the torque applied to a seat contact surface of a gear wheel was set. The equations of balance were solved. The displacement unit of the contact line of the teeth was defined. The stiffness of the tooth pair was calculated.

Findings. The current situation in the field of the tooth gearings stiffness calculation was investigated. The special attention was paid to shortcomings of the existing standards with reference to the large-sized tooth gearings. We sowed the advantages of the finite elements method use for calculation of the tooth gearing elements strain state. The algorithm and a calculation technique of the tooth gearings stiffness by means of the finite elements method was presented. The geometrical model and the analytical model were validated. The algorithm of calculation includes creation of a geometrical model of gearing, definition of mechanical properties of constructional materials, splitting of geometrical model into finite elements, calculation of displacements and stiffness of gearing. The calculation results of the tumbling mill МШРГУ 4500x6000 gearing stiffness received by the developed technique, standard technique and results of an experimental determination were compared. There appeared a wide disagreement of the results received by the standard technique and by the experiments. It was shown that the developed technique allows for more exact estimation of the stiffness as it takes into account the design features of the ring gear.

Originality. The influence of the gear rim design on gearing stiffness was established. The quantitative estimation of the stiffness of the open tooth gearing of tumbling mill МШРГУ 4500x6000 was presented.

Practical value. The algorithm of calculation technique for the tumbling mills open tooth gearings stiffness by the finite elements method was developed.

Keywords: *tumbling mill, tooth gearing, mesh stiffness, finite elements method*

Introduction. Stiffness of toothed wheel gearing is one of the key parameters defining nonuniformity of load distribution along the length of contact lines, and, therefore, calculation of tooth gearings on durability [1, 2]. In particular, it concerns heavily loaded open tooth gearings of the tumbling mills whose capacity of drives reaches 7900 kW and more, and the transferred moment makes hundreds of thousands of newton by meter [2]. The calculation of tooth gearings is standardized [3]. Despite this fact, significant difficulties occur when stiffness is calculated. In particular, the calculations based on GOST 21354-87 do not consider a wheel pliability that leads to a considerable divergence of experimental results and calculations. Therefore, the problem of improvement of a calculation technique for tumbling mills tooth gearings stiffness is urgent.

Analysis of the research. Complexity of the determination of tooth gearings stiffness is that it depends on the loading conditions changing over time, and for large-sized gearings in many respects, it is defined by a ring gear design.

Systematic studying of gearing stiffness began in the 50-ies years of the last century. In 1950, W.A. Tuplin offered the first dynamic model of gearing with the concentrated weight and a spring of constant equivalent stiffness [4]. In 1953 Strauch H. offered a step model of change of gearing stiffness upon transition from one-pair gearing to two-pair one [4]. A. Seireg, D.R. Houser presented a model of tooth stiffness as a cantilever beam in 1970 [4]. All the models specified considered flexibility of a tooth. Other elements of gearing were accepted as rigid. Among domestic scientists, it is worth considering K.I. Zablonsky's works. By generalization of experimental data and the theory of elasticity, K.I. Zablonsky developed a calculation technique of gearing durability which provided the basis for the native standard GOST 21354-87.

With the development of computer technologies, there appeared models considering a nonlinear nature of gearing stiffness changes. In 1986 H.H. Lin, R. Huston [4] developed a computer program of gearing stiffness changing in time; it considered the bending deformations of a tooth as a cantilever beam, as well as contact deformations and deformations of the tooth basis.

Admitting linear deformation, under the defined loading the body stiffness k, N/mm, is determined as the relation of force F, N, to the corresponding displacement d, mm, by a formula

$$k = \frac{F}{d}.$$

According to contemporary conceptions [5–9], the load distributed along the contact line, causes accumulation of potential energy of elastic deformation U, J, which can be written down as follows

$$U = \frac{F^2}{2k}. \tag{1}$$

Potential energy of elastic deformation can be expressed as the sum of the contact component U_h, bending component U_B, shear compression component U_s, axial compression component U_A and component of tooth fillet foundation compression U_f

$$U = U_h + U_B + U_s + U_A + U_f. \tag{2}$$

All the components can be calculated with the equations of the theory of elasticity taking into account geometrical parameters of gearing. However, in most cases an exact expression for displacement d cannot be received. Depending on whether the contacting teeth are over a cut in a disk or over a spoke, stiffness will change from the maximum value to the minimum. The value of the maximum stiffness defines nonuniformity of load distribution and, therefore, durability of a gear wheel.

Considering (1) and (2), the general stiffness of tooth pair in gearing is defined as

$$k = \frac{1}{\frac{1}{k_h} + \frac{1}{k_{b1}} + \frac{1}{k_{s1}} + \frac{1}{k_{a1}} + \frac{1}{k_{f1}} + \frac{1}{k_{b2}} + \frac{1}{k_{s2}} + \frac{1}{k_{a2}} + \frac{1}{k_{f2}}}, \tag{3}$$

where k_h is contact stiffness, N/mm; k_{b1} k_{b2} is bending stiffness, N/mm; k_{s1} k_{s2} is shear stiffness, N/mm; k_{a1} k_{a2} is stiffness at axial compression, N/mm; k_{f1} k_{f2} is stiffness of tooth fillet foundation, N/mm; subscripts 1 and 2 denote pinion and gear correspondingly.

Contact stiffness in the (3) can be received according to the theory of Hertz [4]

$$k_h = \frac{\pi E b}{4\left(1 - \nu^2\right)},$$

where E is Young's modulus, Pa; b is tooth width, m; v is Poisson's ratio. Other components of stiffness have no exact expression and are often determined by using the semi-empirical formulas, which are the result of generalization of experimental data and results of calculation, such as in GOST 21354-87 or ISO6336-1. It is known that the stiffness value counted with a traditional technique when tooth is presented by the cantilever beam of a complicated form considerably differs from measured values [4]. For example, in GOST 21354-87 stiffness is a function of equivalent number of teeth z_{v1}, z_{v2} and their shifts x_1, x_2, mm, and has the following look

$$c' = f(z_{v1}, z_{v2}, x_1, x_2).$$

Unlike GOST 21354-87, current ISO6336-1 standard considers such factors as flexibility of a rim and a wheel disk with corresponding empirical factor of C_R considering flexibility

$$C_R = 1 + \frac{\ln\left(\frac{b_S}{b}\right)}{5e^{\frac{s_R}{5m}}},$$

where m is the tooth module, m; b_S, s_R are the linear sizes defined in [3].

The formula for calculation of factor $\boldsymbol{C_R}$ has a limited scope; it is very approximate and does not consider other factors, for example, inequality of stiffness along the tooth width.

According to the same standard [3], the finite elements method (FEM) gives more accurate values of stiffness due to the consideration of deformations of an involute profile tooth, deformations of a rim and a disk of a wheel of any design, as well as contact deformations [4].

Recently FEM has been more and more widely used for the analysis of tooth gearings. For example, using FEM Marunić and Gordana showed that the nonuniformity of load distribution [8] is observed in rather wide ring gears owing to action of edge effect even if the actual deviation of contact lines does not exist. Pedersen, Jorgensen [5] investigated one-pair and multipair contact in gears and determined a considerable influence of a wheel rim thickness on the stiffness of gearing. A bit earlier Vinogradov B.V. and Sladkovsky A.V. [10] investigated a ring gear of a tumbling mill and established essential influence of a design of a wheel body on mesh stiffness. However, FEM is a very knowledge-intensive method. Ensuring convergence of the contact problem solutions demands special skills and knowledge of a designer and limits the use of the method in work practice.

The objective of present study is to develop a reliable calculation technique for the maximum stiffness of a tooth pair at one-pair gearing of ring gears with the use of FEM and its application to estimate open gearing stiffness of tumbling mills.

Statement of the problems. Stiffening of tooth gearing results in load concentration along the length of contact lines due to errors of gearing and, therefore, to decrease in durability. With relation to durability calculation, it is very important to have a calculation technique for the maximum stiffness of a tooth pair in gearing. Therefore, it is necessary:

- to distinguish the parameters of the tooth gearing which are essentially influencing its stiffness and to create the block of entering data;
- to define the rational analytical model of a tooth gearing;
- to define the input data for calculation of the maximum stiffness of a tooth gearing;
- to confirm adequacy of the model.

Feasibility of the analytical model. In present study the analytical model with complete contact of teeth in a pitch point is investigated (Fig. 1).

The model includes a 3-dimensional geometrical model of a pinion and ring gear (Fig. 2) which are carried out according to the drawings and data in Table 1. Moreover, such features of the design as chamfers, fixing holes and the surrounding teeth are excluded as being insignificant. The ring gear 2 is a segment in 119° with fixed support, the pinion 1 is the continuous cylinder with one degree of freedom. The diameter of a landing surface 3 is no more than 2/3 of the diameter of the pinion. The torque according to data in Table 2 is attached to the pinion. The longitudinal plane of symmetry of ring gear tooth 4 coincides with the longitudinal plane of symmetry of a ring gear gusset 5.

Young's modulus $E = 2.1 \cdot 10^5$ MPa, and Poisson's ratio $v = 0.28$ are accepted as physic-mechanical properties of constructional materials. Boundary conditions are a motionless basis of a rim gear 2 and the cylindrical hinge on a landing surface of a pinion 1. The control of accuracy was carried out on the basis of the convergence analysis on displacements.

In Fig. 3 the block diagram of the problem solution is shown. The algorithm of calculation includes the following stages:

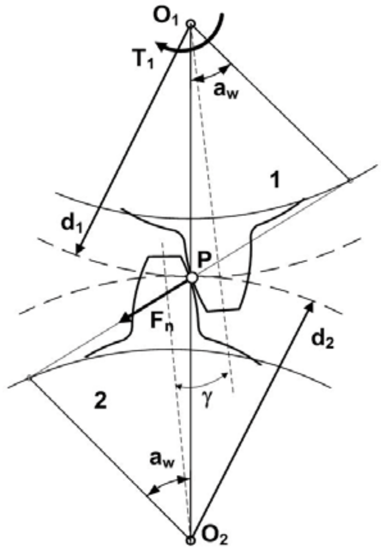

Fig. 1. Analytical model: 1 – pinion; 2 – ring gear; a_w – gearing angle; d_1 – pinion diameter; d_2 − ring gear diameter; F_n − normal force; O_1O_2 – centre distance; P – pitch point; T_1 − torque; γ – inclination angle

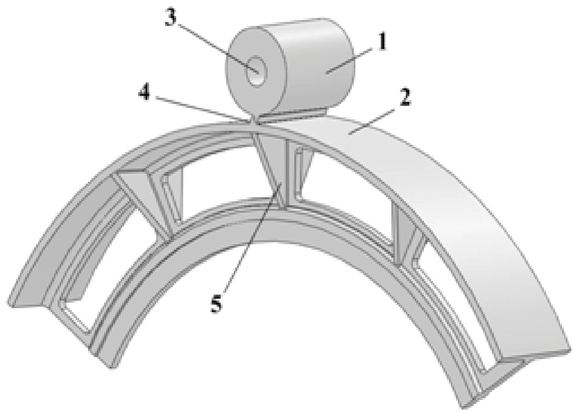

Fig. 2. Geometrical model of gearing: 1 − pinion; 2 – ring gear; 3 − landing surface of pinion; 4 −gear tooth; 5 − gusset

Table 1

Basic geometrical parameters of gearing

Parameters	Symbol, unit	Test case		МШРГУ 4500x6000	
		Pinion	Wheel	Pinion	Wheel
Number of teeth	z	32	64	28	252
Tooth width	b, mm	60	60	800	800
Base circle diameter	d, mm	166.667	333.334	700	6300
Gearing angle	a, °	20		20	
Module	m, mm	5		25	
Centre distance	a_w, mm	250		3511	

Table 2

Basic data on loading of tooth gearings

Parameters	Symbol, unit	Value	
		Test case	МШРГУ 4500x6000
Pinion rotation frequency	n_1, rev/min	1500	150
Torque	T_1, kN · m	1.97	159.155
Transmission ratio	u	2	9

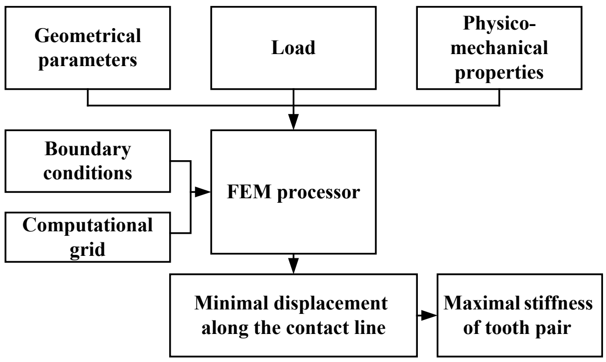

Fig. 3. Block diagram of a calculation technique for mesh stiffness

– creation of a three-dimensional model of gearing;

– determination of mechanical characteristics of constructional materials;

– determination of boundary conditions;

– creation of a computational grid;

– the solution of a contact problem with the finite elements method with convergence control;

– calculation of the minimal displacement along the contact line on the surface of a rim gear tooth;

– calculation of the maximal stiffness of a tooth pair in gearing with a formula

$$c' = \frac{F_n}{\delta \cdot b},$$

where F_n is full normal force, N; δ is the maximum displacement along the contact line in the direction of the line of gearing, μm. Normal force is calculated as

$$F_n = \frac{F_t}{\cos(\alpha_w)},$$

where F_t is tangential force, N; α_w is gearing corner, degrees.

The calculated values of stiffness for all problems with use of standards and the developed technique are given in Table 3.

The recommended generator of a computational grid is GMSH, the processor and the post-processor – Calculix, which are a free software.

Results of calculations and their discussion. The problem of determining the maximum stiffness of teeth in gearing used in GOST 21354-87 as an example of durability calculation is solved as a test case. Further the problem for an open tooth gearing of tumbling mill МШРГУ 4500x6000 is solved.

Table 3

Calculated values of mesh stiffness

Case		Maximal Stiffness of a Tooth Pair C′, kN/μm·mm		
		GOST 21354-87	ISO6336–1	FEM
Test case	Solid gear	17.3	14.5	13.4
	Shaped gear	–	13.7	8.2
МШРГУ 4500 × 6000	Solid gear	17.4	17.6	12.4
	Shaped gear	–	12.5	6.3

Basic data are given in Table 1, 2. Besides, to demonstrate the influence of a rim gear cross-section form on mesh stiffness, the problem of solid and T-shaped section of rim gear is solved (Fig. 4). The value of b_S is accepted to be 30 and 70 mm for a test case and real gear of МШРГУ respectively; the value of s_R is accepted to be 30 and 66 mm for a test case and real gear of МШРГУ respectively.

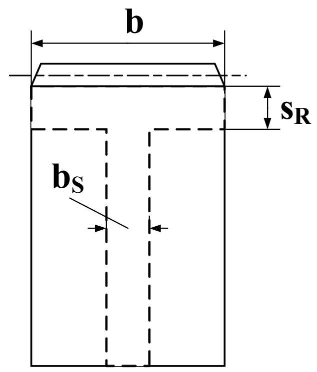

Fig. 4. Scheme of a rim gear profile: b – gear width; b_S – thickness of central disc; s_R – thickness of rim

The data shown in Table 3 testify to satisfactory coincidence of calculation stiffness results by the ISO and FEM methods for taste case, the difference of values has made no more than 8 %. The results of the calculation in accordance with the GOST 21354-87 method in all range of parameters are over-estimated. The GOST 21354-87 method does not carry out any technique of gear flexibility accommodation due to gear cross-section form at all.

The diameter and rim gear width being increased, the difference in values of stiffness between the ISO and FEM methods becomes considerable. The values of stiffness calculated with the ISO technique and FEM technique for a rim gear of tumbling mill МШРГУ 4500x6000 are 4.6-fold different. Earlier B. V. Vinogradov [2] experimentally investigated nonuniformity of load distribution along the length of contact lines in a tooth gearing of the mill МШРГУ 4500 × 6000. The received value of the factor of load distribution nonuniformity corresponds to stiffness of no more than 8 N/μm · mm that is 2.2 times as little as the stiffness received in accordance with the GOST 21354-87 method.

It is possible to explain the divergence of the results in the following way. The design of rim gears in gearings of heavy machinery has considerable features, and loads on teeth are high. The ISO6336-1 standard is developed for calculation of tooth gearings of the general machinery and does not consider influence of gussets and cuts in the central disk of a rim gear. Besides, the ISO6336-1 standard does not consider nonuniformity of load distribution and stiffness that occur in wide gears even at complete contact of teeth [6]. It is possible to consider the listed features only by conducting detailed research of the stress and strain state of gearing. Therefore, the description of the factor C_R is rather tentative and cannot be transferred to calculation of the heavy-loaded and large-sized tooth gearings.

Stiffness of tooth is not equal throughout the length. Upon transition of a point of gearing from tooth top to its basis, which is directly, related to the angle change γ (Fig. 1), stiffness changes. In Fig. 5 the dependence of the maximum stiffness of tooth pair in gearing on an angle γ corresponding to the tooth pair position over a spoke (line 1) and between spokes (line 2) is presented. Fig. 5 shows that stiffness is maximal, when the point of gearing coincides with a gearing pole. The central disk of a rim gear contains the cuts intended to facilitate wheel weight. At the same time, cuts reduce stiffness of a wheel. Therefore, upon gearing transition from a spoke to a spoke stiffness of gearing changes periodically. Fig. 5 demonstrates that in consequence of transition of tooth pair position concerning a gear spoke, stiffness changes by 40 % on average. Stiffness reaches the greatest value in the position

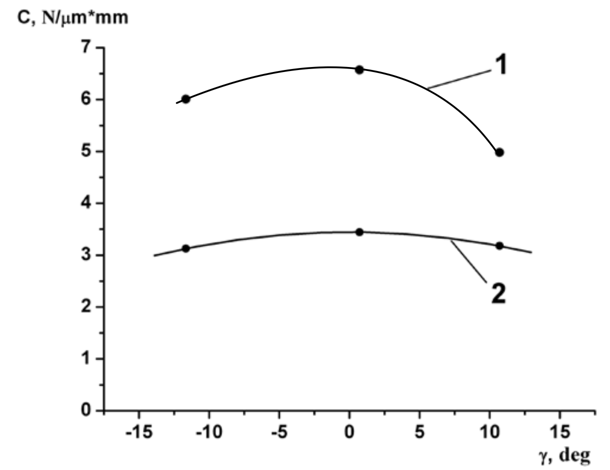

Fig. 5. Dependency of the tooth pair stiffness on angle γ: 1 – tooth pair position over a spoke; 2 – tooth pair position between spokes

of tooth pair over a spoke when the plane of symmetry of gear tooth coincides with the longitudinal plane of symmetry of a gusset. Therefore, this position of a tooth pair is the most dangerous from the point of view of gearing durability.

Conclusions. The technique developed in the present work based on FEM has the following advantage. Being a method of theory of elasticity, this technique allows investigating the stress and strain state of random-shaped gear wheels. The use of semi-empirical formulas to define the relation between loading and stress disappears is not required. Correlation of calculated values and values obtained by experiments makes the developed technique a priority while calculating tumbling mill gearing durability.

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Мета. Розробка методики розрахунку максимальної жорсткості пари зубів при однопарному зачепленні зубчастих коліс методом кінцевих елементів та її використання для оцінки жорсткості зачеплення відкритих зубчастих передач барабанних млинів.

Методика. Розрахунок напружено-деформованого стану здійснюється методом кінцевих елементів. На першому етапі формується розрахункова схема задачі. На базі геометричних параметрів та креслень здійснюється побудування тривимірної геометричної моделі зубчастого зачеплення. Задаються фізичні характеристики конструкційних матеріалів. Геометрична модель розбивається на кінцеві елементи. Задаються граничні умови, що забезпечують кінематичну незмінність моделі, та умови контакту робочих поверхонь зубів. Задається зовнішнє навантаження у вигляді крутного моменту, що прикладений до шестерні. Здійснюється чисельне розв'язання рівнянь рівноваги. Визначається мінімальне переміщення вздовж контактної лінії зубів. Обчислюється жорсткість пари зубів.

Результати. Досліджено сучасний стан питання щодо розрахунку жорсткості зубчастих передач. Особливу увагу приділено недолікам існуючих стандартів стосовно великогабаритних зубчастих передач. Показані переваги методу кінцевих елементів при розрахунку деформованого стану елементів зубчастої передачі. Представлені алгоритм і методика розрахунку жорсткості зубчастих передач за допомогою методу кінцевих елементів, здійснене обґрунтування геометричної моделі та розрахункової схеми. Алгоритм розрахунку включає побудову геометричної моделі передачі, завдання механічних властивостей конструкційних матеріалів, розбиття геометричної моделі на кінцеві елементи, розрахунок переміщень і жорсткості зачеплення. Зроблене порівняння результатів розрахунку жорсткості зубчастої передачі барабанної млини МШРГУ 4500 × 6000 за розробленою методикою, стандартною методикою та результатів експериментального дослідження. Встановлене значне розходження результатів стандартної методики та результатів експериментів. Показано, що розроблена методика дає більш точну оцінку жорсткості завдяки повнішому врахуванню конструктивних особливостей вінця.

Наукова новизна. Встановлено вплив конструкції зубчастого вінця на жорсткість зачеплення та представлена кількісна оцінка жорсткості відкритої зубчастої передачі барабанної млини МШРГУ 4500 × 6000.

Практична значимість. Розроблено алгоритм методики розрахунку жорсткості відкритих зубчастих передач барабанних млинів методом кінцевих елементів.

Ключові слова*: барабанний млин, зубчаста передача, жорсткість, метод кінцевих елементів*

Цель. Разработка методики расчета жесткости пары зубьев при однопарном зацеплении зубчатых колес методом конечных элементов и применение ее для оценки жесткости зацепления открытых зубчатых передач барабанных мельниц.

Методика. Расчет напряженно-деформированного состояния осуществляется методом конечных элементов. На первом этапе формируется расчетная схема задачи. На основании основных геометрических параметров и чертежей производится построение трехмерной геометрической модели зубчатой передачи. Задаются физические свойства конструкционных материалов. Геометрическая модель разбивается на конечные элементы. Задаются граничные условия, обеспечивающие кинематическую неизменяемость модели, и условия контакта рабочих поверхностей зубьев. Задается внешняя нагрузка в виде крутящего момента, приложенного к посадочной поверхности шестерни. Производится численное решение уравнений равновесия. Определяется минимальное перемещение вдоль контактной линии зубьев. Рассчитывается жесткость пары зубьев.

Результаты. Исследовано современное состояние вопроса о расчете жесткости зубчатых передач. Особое внимание уделено недостаткам существующих стандартов применительно к крупногабаритным зубчатым передачам. Показаны преимущества метода конечных элементов при расчете деформированного состояния элементов зубчатой передачи. Представлен алгоритм и методика расчета жесткости зубчатых передач с помощью метода конечных элементов, осуществлено обоснование геометрической модели и расчетной схемы. Алгоритм расчета включает построение геометрической модели передачи, задание механических свойств конструкционных материалов, разбиение геометрической модели на конечные элементы, расчет перемещений и жесткости зацепления. Произведено сравнение результатов расчета жесткости зубчатой передачи барабанной мельницы МШРГУ 4500 × 6000 по разработанной методике, стандартной методике и результатов экспериментального исследования. Установлено значительное расхождение результатов стандартной методики и результатов экспериментов. Показано, что разработанная методика дает более точную оценку жесткости благодаря более полному учету конструктивных особенностей венца.

Научная новизна. Установлено влияние конструкции зубчатого венца на жесткость зацепления и представлена количественная оценка жесткости открытой зубчатой передачи барабанной мельницы МШРГУ 4500 × 6000.

Практическая значимость. Разработан алгоритм методики расчета жесткости открытых зубчатых передач барабанных мельниц методом конечных элементов.

Ключевые слова: *барабанная мельница, зубчатая передача, жесткость, метод конечных элементов*

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