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## MODELING TOOLS OF INTERBUDGETARY REGULATION FOR MINING AREAS

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## МОДЕЛЬНИЙ ІНСТРУМЕНТАРІЙ МІЖБЮДЖЕТНОГО РЕГУЛЮВАННЯ ДЛЯ ШАХТАРСЬКИХ ТЕРИТОРІЙ

**Purpose.** To create models which allow carrying out the allocation of funds from the equity tax in the structure of <region> ↔ <Mining municipality> in due form.

**Methodology.** An approach to create models with the property change adaptation operating conditions is proposed. The approach is based on the analytical apparatus of the theory of stochastic automata operating in random environments and allows formally describing the behaviour of the agent in a measurable uncertainty created by revenues from taxes to the budgets of the lower levels of the budget system of the Russian Federation.

**Findings.** The design of stochastic automation exhibiting property of appropriateness of behaviour and asymptotic optimality is proposed. The machine adapts well to fluctuations in the external environment caused by variations in tax revenues in the budgets of mining municipal entities. We construct the matrix of the automation transition from one state to another under the influence of the input signals, identifying favourable and unfavourable reactions of the environment to the outputs of the machine. Expressions for the final probabilities of the machine stay in their states are obtained. The constructed mathematical models in the form of a probabilistic automaton are used in the decision support system for the management of inter-budgetary control for depressed areas, among which mining municipalities are listed.

**Originality.** The urgent task of creating a model tool contributing to the domination of the stimulating function of budgetary control over the levelling function in the <region> ↔ <Mining municipality> structure. An automation model for the formalization of a certain proportion of taxes in the order of the budget management allocation is developed. The advantage of the model is its adaptability to the conditions of uncertainty of effects of the environment, as well as the capacity for self-training due to the properties of appropriateness of behaviour and asymptotic optimality.

**Practical value.** The practical value of the research is in the application of the constructed models as part of a decision support system in the <region> ↔ <Mining municipality> structure.

**Keywords:** *interbudgetary regulation, mining municipalities, economic and mathematical models, stochastic automata, decision support systems*

**Introduction.** Despite the end of the transitional period of the reform of local self-government of the Russian Federation, the problem of financial sustainability of local authorities has remained unsolved. This is especially true of the mining municipalities. The specificity of the development of mining towns is characterized by a narrow industry-specific focus, where for a long period the industrial potential in the field of coal mining and its processing by metallurgical plants and enterprises of fuel and energy complex located in these areas has developed [1–6]. But currently, there is considerable deviation from the normal functioning of mining municipalities related to the restructuring of the coal industry. Financing of the technical aspects of the coal industry restructuring and implementation of

a complex of organizational measures of the state support programme for local development and employment for mining towns and villages are unable to create favourable conditions for sustainable development. As a result, socio-economic status of these territories is characterized by low productivity, loss of previously high socio-economic and human potential and, finally, poor life standards, a significant unemployment rate, poor demographics, etc. If we consider inter-budgetary relations in the <region> ↔ <mining municipal education> structure, they are dominated by the dependence on subsidies, leading to a strong fiscal dependence on higher authorities. The current imbalance between the lining and stimulating functions of intergovernmental relations contributes to the inability of local governments to address large-scale socio-economic problems. It is well known that within intergov-

ernmental relations the central position is held by relations of interbudgetary control coordinating financial relationships and ensuring the existence of the budgetary system under conditions of integrity and unity of the Federal state. One of the main functions of these relations is the effective alignment of level of budgetary security of municipalities, while maintaining stimulation of local authorities' interest in developing their own tax base. The purpose of the intergovernmental fiscal regulation at the regional level is the provision of municipal budgets with their own funds to carry out the local authorities' powers. Characterizing interbudgetary relations between constituent entities of the Russian Federation and the mining areas, it should be noted that they are developing in a path of the predominance of the alignment function repressing the incentive function. In this context there is a need to improve the mechanism of inter-budgetary relations between the region and the mining areas through the use of tools of the budgetary control enabling to provide the local budgets of mining towns with their own income. The budgetary regulation is a particularly important task of multilevel governance of the budgetary system, consisting in a partial redistribution of financial resources between budgets of different levels. Its successful solution depends largely on providing a secure economic base in the mining municipalities, because the basic part of incomes to local budgets is produced at the expense of the interbudgetary regulation. The stabilization of the financial situation of municipalities at the present time is of particular importance due to the fact that creating a sound economic base is a major prerequisite for the economic growth of the country.

**Analysis of the recent research.** In ongoing research relating to budgetary control, attention has been mainly paid to the formalization of their passive component that performs the function of alignment of budgetary security of territories – redistribution of the budget of the higher level of budgetary system of the Russian Federation to the budgets of a lower level in the form of grants, subsidies, and subventions. In these studies, mathematical models were developed to determine the amount of financial assistance provided by the regions to municipalities. Such models, of course, have played a positive role since they succeeded in reducing the influence of the subjective component on the value calculations while determining the value of grants, subventions, and subsidies. However, they were unable to halt the growth of the dependency taking place in municipalities, contribute to the reduction of subsidization and, as a consequence, the development of initiative and independence at the local level. The function of stimulating the interest of mining communities in their economic growth is imposed on the additional payments from taxes to be credited to the budget of a constituent entity of the Russian Federation, according to the standards, the value of which is determined on a formalized basis. The results of research given in this article are devoted to the development of economic and mathematical

models and tools used for decision-making when establishing the value of these deductions. The currently applied approach, which is based on the use of various types of heuristic algorithms that implement a certain proportion of tax revenue-sharing between the budgets, is not effective and causes the concentration of funds in the budgets of higher levels of the Russian budget system, a large deficit of local budgets, contrary to the principle of organization independence of local self-government.

Among the problems of budget management in the structure of <region> ↔ <mining municipality>, the following issue is of special interest: this is the task of establishing such values of standards of deductions from the tax revenues of the budget of the regional Russian budget system levels to the budgets of the municipal level, which lead to balance budgets in the vertical interests. Processes of making decisions on the value of allocations to the budgets of municipalities from federal and regional taxes to be paid to the regional budget should be based on the use of economic and mathematical models to predict the consequences of these decisions. The article proposes economic and mathematical models, formally describing the decision-making in the strategic objective of budgetary control, which consists in determining the rates of contributions to local budgets of mining towns from federal and regional taxes and charges to be paid to the regional budget.

**Unsolved aspect of the problem.** The control object, which is the budget of the mining of the municipality in terms of its substantive content, is presented as an abstract mathematical object “dynamic system”  $P$  [1, 2]. Let us consider the generalized coordinates of the system. Input control variables are the norms  $ST = (S_1, S_2, \dots, S_k)$  of deductions to the local budgets from federal and regional taxes and charges to be paid to the regional budget. Component  $S_i$  of the vector  $ST$  is a value of standard deductions to the local budget from the tax of  $i$  type. Revenues  $X(t)$  and expenditures  $Y(t)$  are considered as disturbances, while the value of residual budget  $Q(t)$  at the time  $t$  is regarded as the phase state of a dynamic system  $P$ . Strategic budget management control problem is formulated as follows: to establish such values  $S_i, i = \overline{1, k}$ , standards of deductions from the budget transferred with the regulation of taxes and charges, which result in a compromise of in-interests of the budgets of regions and municipalities.

**Objectives of the article.** To choose compromise solutions  $S_i, i = \overline{1, k}$ , vector objective function is proposed  $\tilde{\varphi} = (p, M_p, q, M_q)$ , where  $p$  is the assessment of the probability of the budget deficit;  $M_p$  is the evaluation of the expectation gap;  $q$  is the assessment of the likelihood of budget surplus;  $M_q$  is the assessment of mathematical expectations of surplus. The analytical expression of function-components are [3–5]

$$M_p = \frac{1}{N} \sum_{t=1}^N Q(t) \cdot I(t);$$

$$I(t) = \begin{cases} \frac{1}{2} \left( 1 - \frac{|Q(t)|}{Q(t)} \right), & \text{if } Q(t) \neq 0; \\ 0, & \text{if } Q(t) = 0 \end{cases}; \quad p = \frac{1}{N} \sum_{t=1}^N I(t);$$

$$Q(t) = Q(t-1) + X(t) + \text{Re } g(ST, t) - Y(t);$$

$$\text{Re } g(ST, t) = \sum_{i=1}^K S_i \cdot Rg_i(t);$$

$$M_q = \frac{1}{N} \sum_{t=1}^N Q(t) \cdot J(t);$$

$$J(t) = \begin{cases} \frac{1}{2} \left( 1 + \frac{|Q(t)|}{Q(t)} \right), & \text{if } Q(t) \neq 0; \\ 0, & \text{if } Q(t) = 0 \end{cases};$$

$$q = \frac{1}{N} \sum_{t=1}^N J(t),$$

where  $Rg_i(t)$  is the value of the tax payments of the  $i$  type according to which the value of the standard  $S_i$  of allocations to the local budget at the time  $t$  is determined ( $i$  type tax payments are made by individuals and legal entities of municipal entities);  $X(t)$  is the value of the municipal budget revenue from personal income at the time excluding funds of budget regulation;  $Y(t)$  is the magnitude of the budget expenditures of the municipality at the time  $t$ .

The components  $p$ ,  $M_p$  and  $q$ ,  $M_q$  of the vector function  $\tilde{\varphi}$  are controversial for several reasons.

Increasing values  $S_i$ ,  $i = \overline{1, k}$  result in increase in cash receipts in the budget of the municipality due to the tax revenue, and, thus, lead to decreasing  $p$ ,  $M_p$  functions and increasing  $q$ ,  $M_q$  functions. However, this reduces budget income of the subject of the Russian Federation. In order to increase these revenues it is necessary to reduce  $S_i$ ,  $i = \overline{1, k}$  value, but this leads to increasing functions  $p$ ,  $M_p$ , i.e. an increase in the deficit in the local budget. In this context, decision making on the values of the relative components of the vector  $ST = (S_1, S_2, \dots, S_k)$  occurs in conflict situations caused by contradictory character of changes in  $p$ ,  $M_p$  and  $q$ ,  $M_q$  functions.

A compromise between the interests of the budgets of the regional and municipal levels of the budget system of the Russian Federation in defining  $S_i$ ,  $i = \overline{1, k}$  values is determined as a result of multi-objective optimization problem solution with respect to the vector target functions  $\tilde{\varphi} = (p, M_p, q, M_q)$ .

The essence of the problem is to find such a solution of  $ST \in \tilde{ST}$  ( $\tilde{ST}$  is the set of feasible solutions), which minimizes the value of all components of the vector  $\tilde{\varphi}$  in one way or another. Since the functions  $p$ ,  $M_p$  and  $q$ ,  $M_q$  do not reach the minimum at the same point, this formulation is not entirely correct [1].

By solution of multi-objective optimization problem is meant a subset of  $\tilde{ST}$ , in which function values  $p$ ,  $M_p$ , and  $q$ ,  $M_q$  would correspond to some of the intuitive notions of “best” values for these functions in

pursuit of them at the same time to a minimum on the set  $\tilde{ST}$ .

Representations of the best decisions are formalized on the basis of Pareto optimality [1]. As a solution to the multi-objective optimization problem, the authors regard such  $S_i$ , that cannot be improved regarding the criteria  $p$ ,  $M_p$ , except the deterioration according to the criteria  $q$ ,  $M_q$  (i.e., the optimal solution according to Pareto) [1].

In other words, as solutions such  $ST^* \subset \tilde{ST}$  are considered, that for any  $ST \subset \tilde{ST}$  of that  $q(ST^*) \leq q(ST)$  and  $M_q(ST^*) \leq M_q(ST)$  it follows that  $p(ST^*) > p(ST)$  and  $M_p(ST^*) > M_p(ST)$ :

$$\forall ST \in \tilde{ST}, \exists ST^* \subset \tilde{ST} / q(ST^*) \leq q(ST) \wedge \\ \wedge M_q(ST^*) \leq M_q(ST) \Rightarrow p(ST^*) > p(ST) \wedge \\ \wedge M_p(ST^*) > M_p(ST).$$

Consequently, the point  $ST^*$  will be considered Pareto efficient if there is such a point  $ST \in \tilde{ST}$  that  $q(ST^*) < q(ST) \wedge p(ST^*) < p(ST)$ .

The disadvantage of this approach is the multiplicity of Pareto optimal solutions. This drawback is overcome by using the apparatus of game theory.

**Presentation of the main research.** For decision-making in the allocation of tax revenues transferred according to the budget regulation, the authors constructed a model of the stochastic automaton  $A$  operating in a random environment [1]. The reaction of the environment is described by the vector

$$C = \langle (p, q_1), (p_2, q_2), \dots, (p_n, q_n) \rangle,$$

components of which take the values of the probabilities of the winning  $p_i$  of machine  $A$  in the state  $\varphi_i$  and estimating the probability  $q_i$  of loss and in the same condition. Elements of set  $\{\varphi_i\}_{i=1}^n$  of states of machine  $A$  are all possible sets  $\varphi_i = \langle S_{i1}, S_{i2}, \dots, S_{ik} \rangle$  whose components  $S_{il}$ ,  $j = \overline{1, 2}$ ,  $l = \overline{1, k}$  reflect the values of assets of the rate of deductions from personal taxes of the higher-level budget to the lower level of the budgetary system of the Russian Federation [3, 4]. Quantities  $S_{il}$  take values from a continuum of sets ( $S_{il} \in [0, 1]$ ). For transition from a continuum to a finite set, sampling of structural states  $S_{il}$  is done, which consists in partitioning the interval  $[0, 1]$  into a finite number of line segments with the coordinates of the ends  $[\alpha_i, \alpha_{i+1}]$ ,  $[\alpha_i, \alpha_{i+1}] \cap [\alpha_j, \alpha_{j+1}] = 0$ ,  $i = \overline{1, k}$ ,  $j = \overline{1, k}$ , with  $i \neq j$  and  $[\alpha_i, \alpha_{i+1}] \cap [\alpha_j, \alpha_{j+1}] = 0$ ,  $i = \overline{1, k}$ ,  $j = \overline{1, k}$ , with  $i \neq j$ . As the  $S_{il}$ , coordinates of the ends  $[\alpha_i, \alpha_{i+1}]$  are taken. Then, as a structural condition  $\varphi_i$ , a set  $\langle \alpha_{i1}, \alpha_{i2}, \dots, \alpha_{ik} \rangle$  can be considered; the number of all possible sets is equal to the number of combinations of coordinate lines  $[\alpha_i, \alpha_{i+1}]$  and is  $N = (n + 1)^k$ , where  $n$  is the number of segments and  $k$  is the number of types of governing taxes.

Thus, we consider that the stochastic automation  $A$  has  $N = (n + 1)^k$  of states  $(\varphi_1, \varphi_2, \dots, \varphi_N)$ ,  $\varphi_i = \langle \alpha_{i1}, \alpha_{i2}, \dots, \alpha_{ik} \rangle$ .

The authors propose the design of a stochastic automation  $A$  used for the decision-making model regarding the magnitude of the standards of deductions to the budgets of mining municipalities from paying taxes going to the regional budget. Stochastic machine  $A$  has  $N$  states of  $(\varphi_1, \varphi_2, \dots, \varphi_N)$  reflecting various combinations of sets of values transmitted from the standards of deductions in the order of budgetary and regulatory taxes in the budget level  $i$ .

As output  $Q_j$ ,  $j = \overline{1, N}$  of the machine in the condition  $\varphi_j$  the level of current stock of funds accumulated in the budget for this level is examined.

The machine has two values of the input variable, referred to as “penalty” with  $\delta = 1$  and “non-default” with  $\delta = 0$ , and is immersed in a stationary random environment  $C = C(p_1, p_2, \dots, p_N)$ . If at the time  $t$  the machine has performed an action  $Q_j(t)$ , then at the moment  $(t + 1)$  its input receives a signal  $\delta(t + 1) = 0$  with an estimation of the probability  $p_j$ , and a signal  $\delta = 1$  with an estimation of the probability  $q_j = (1 - p_j)$ .

The machine is penalized if the value of current stock of funds in the budget will be  $Q_i < 0$ , i. e. if in the budget there occurs some deficiency.

The authors have proposed a design of the machine with a matrix of transitions from one state to another under the influence of the input signals  $\delta = 0$  (the matrix  $a_{ij}(0)$ ) and  $\delta = 1$  (matrix  $a_{ij}(1)$ ) having the form [6, 7]

$$\|a_{kl}(1)\| = \begin{vmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \end{vmatrix};$$

$$\|a_{kl}(0)\| = \begin{vmatrix} 0 & 1 & 0 & \dots & 0 & 0 & 0 \\ \gamma & 0 & (1-\gamma) & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & \gamma & 0 & (1-\gamma) \\ 0 & 0 & 0 & \dots & 0 & 1 & 0 \end{vmatrix},$$

where  $\gamma$  is selective tactics of the machine when penalized. The probability  $p_{ij}$  of transition of the automation from the state  $\varphi_i$  to the state  $\varphi_j$  will be calculated as follows: The transition probability matrix is of the form

$$p_{ij} = q_i a_{ij}(0) + p_i a_{ij}(1).$$

The transition probability  $\|p_{ij}\|$  matrix is of the form

$$\|p_{kl}\| = \begin{vmatrix} q_1 & p_1 & 0 & \dots & 0 & 0 & 0 \\ \gamma \cdot p_2 & q_2 & (1-\gamma)p_2 & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \gamma \cdot p_{N-1} & q_{N-1} & (1-\gamma)p_{N-1} \\ 0 & 0 & 0 & 0 & 0 & p_n & q_n \end{vmatrix}.$$

The matrix  $\|p_{ij}\|$  is stochastic because  $\sum_i p_{ij} = 1$ .

The system of equations for determining the final probability of the machine being in a particular state can be written as [8, 9]

$$\begin{cases} r_1 = q_1 r_1 + \gamma p_2 r_2 \\ r_2 = p_1 r_1 + q_2 r_2 + \gamma p_3 r_3 \\ r_3 = (1-\gamma) p_2 r_2 + q_3 r_3 + \gamma p_4 r_4 \\ \dots \\ r_{N-1} = (1-\gamma) p_{N-2} r_{N-2} + q_{N-1} r_{N-1} + p_N r_N \\ r_N = (1-\gamma) p_{N-1} r_{N-1} + q_N r_N \end{cases}$$

Expressions for the final probabilities  $r_i$ ,  $i = \overline{1, N}$  obtained by solving the systems of equations are of the form

$$r_1 = \frac{1}{1 + p_1 \sum_{i=2}^{N-1} \left( \frac{1}{p_i} \frac{(1-\gamma)^{i-2}}{\gamma^{i-1}} + \frac{p_1}{p_N} \frac{(1-\gamma)^{N-2}}{\gamma^{N-2}} \right)};$$

$$r_2 = \frac{p_1}{\gamma p_2 \left( 1 + p_1 \sum_{i=2}^{N-1} \frac{1}{p_i} \frac{(1-\gamma)^{i-2}}{\gamma^{i-1}} + \frac{p_1}{p_N} \frac{(1-\gamma)^{N-2}}{\gamma^{N-2}} \right)};$$

$$r_N = \frac{p_1 (1-\gamma)^{N-2}}{\gamma^{N-2} p_N \left( 1 + p_1 \sum_{i=2}^{N-1} \frac{1}{p_i} \frac{(1-\gamma)^{i-2}}{\gamma^{i-2}} + \frac{p_1}{p_N} \frac{(1-\gamma)^{N-2}}{\gamma^{N-2}} \right)}.$$

The authors previously evaluated the adequacy of the proposed automation model for decision support on interbudgetary regulation [7].

The tools for checking the adequacy of the model are based on mathematical apparatus which allows evaluating its ability to make decisions under conditions of uncertainty no worse than a person acting under these conditions. As a criterion of adequacy of the model of the decision-making process, appropriateness measurement of behaviour and asymptotic optimality of the machine is used. In [7] it is proved that the machines of the proposed structure possess the property of appropriateness of behaviour and asymptotic optimality. As a measure of appropriateness and asymptotic optimality, expected payoff is used

$$M = \sum_{i=1}^N r_i p_i.$$

Terms of appropriateness of behaviour and asymptotic optimality are

$$M < \frac{\sum_{i=1}^N p_i}{N} \quad \text{and} \quad \lim M(\gamma, p) = p_{\min},$$

correspondingly.

Thus, the machine proposed for modelling decision-making is arranged appropriately, i. e. while operating, it will win more often and lose more rare-



ly. If you compare its average pay-off with a pay-off that it will receive, when choosing their actions randomly according to the final probability, the latter is much greater. With unlimited increase in memory capacity, this machine behaves no worse than the person who knows winning probabilities in each state in advance: it performs an action whose probability is maximal.

For decision-making on the budgetary regulation leading to the harmonization of the interests of the budgets of various levels of the budgetary system of the Russian Federation, the model of collective behaviour of stochastic machines  $A_1$ , and  $A_2$  is proposed which operate at municipal and regional levels. The operation of the machine  $A_1$  is aimed at minimizing the budget deficit of the mining municipalities, and that of the machine  $A_2$  is to minimize the surplus.

Due to the fact that the interaction between the machines  $A_1$  and  $A_2$  is to provide a selection of some of trade-off alternatives of values of the standards of deductions in the budget of the municipality from the Federal and regional taxes to be credited to the local budget, as a tool to describe this interaction the analytical apparatus of game theory is used.

A game  $\varphi$  of the machines  $A_1$  and  $A_2$  consists of a sequence of laps  $\varphi_1, \varphi_2, \dots, \varphi_n$ . At the same time, a lap  $\varphi_i$  of a game  $\varphi$  is played in time  $t$  is a set of strategies  $\varphi_i = (\varphi_k^{(1)}, \varphi_\ell^{(2)})$ ,  $k = \overline{1, n}$ ,  $\ell = \overline{1, n}$ , selected by the machines involved in the game  $A_1$  and  $A_2$ .

If  $\varphi = \{\varphi_i\}$  is a set of laps,  $\varphi \subseteq \{\varphi_i^{(1)}\}_{i=1}^n \times \{\varphi_i^{(2)}\}_{i=1}^n$ . The outcome  $\delta_i(t+1)$  of the game  $\varphi_i = (\varphi_k^{(1)}, \varphi_\ell^{(2)})$  is a set  $(\delta_k^{(1)}(t+1), \delta_\ell^{(2)}(t+1))$  of the value inputs, where  $\delta_k^{(1)}(t+1) = 0$  if the machine  $A_j$  has won, and  $\delta_\ell^{(2)}(t+1) = 0$  if the machine  $A_j$  has failed. The task of the machine  $A_1$  and  $A_2$  structures and assessment of probabilities  $p_i(\delta_i^{(1)}(t+1), \delta_i^{(2)}(t+1))$ ,  $i = \overline{1, n}$  of various outcomes of these laps determines the game of the machines.

The game  $\varphi$  is a zero-sum game. The probability that the machine  $A_1$  will benefit due to its  $i$ -strategy is determined as:  $v_{ij} = r_i \cdot (1 - p_i)$ . The values  $v_i$  form a square matrix  $\|v_{ij}\|$ , and are accepted as payments. The solution of the game in the form of mixed strategies  $R_1 = (p_1, p_2, \dots, p_n)$ ,  $R_2 = (\sigma_1, \sigma_2, \dots, \sigma_n)$  allows determining the compromise value of the standards of deductions to the local budgets as transmission of budget management of taxes and duties, leading to balance of the interests of the budgets of the regional and municipal levels of the budget system of the Russian Federation. The authors propose a formal description of the solution of the game machines in the form of mixed strategies which is to define estimates of probabilities  $\pi_1, \pi_2, \dots, \pi_n$  and  $\sigma_1, \sigma_2, \dots, \sigma_n$  of application of the strategies of the first and second players, respectively.

Thus, the system of inequalities are compiled

$$\begin{cases} \pi_1 \cdot v_{11} + 0 \cdot \pi_2 + \dots + 0 \cdot \pi_m \geq \xi \\ 0 \cdot \pi_1 + \pi_2 \cdot v_{22} + \dots + 0 \cdot \pi_m \geq \xi; \\ \dots \\ 0 \cdot \pi_1 + 0 \cdot \pi_2 + \dots + \pi_n \cdot v_{nn} \geq \xi \end{cases}$$

$$\begin{cases} \lambda_1 \cdot v_{11} + 0 \cdot v_{12} + \dots + 0 \cdot v_{1n} \geq 1 \\ 0 \cdot v_{11} + \lambda_2 \cdot v_{22} + \dots + 0 \cdot v_{1n} \geq 1 \\ \dots \\ 0 \cdot v_{11} + 0 \cdot v_{12} + \dots + \lambda_n \cdot v_{nn} \geq 1 \end{cases}$$

where  $\xi$  is the merit value of the first player,

$$\lambda_1 = \frac{\pi_1}{\xi}; \quad \lambda_2 = \frac{\pi_2}{\xi}; \quad \dots \quad \lambda_m = \frac{\pi_m}{\xi}.$$

Due to the fact that  $\sum_{i=1}^n \pi_i = 1$ ,  $\sum_{i=1}^n \lambda_i = \frac{1}{\xi}$  and  $\xi$  is the

merit value of the first player, then the objective function for the solution of a linear programming problem, which is the solution of the game in the form of mixed strategies, is  $\lambda_1 + \lambda_2 + \dots + \lambda_m \Rightarrow \min$ .

The solution  $\lambda_1 = \frac{1}{v_{11}}$ ,  $\lambda_2 = \frac{1}{v_{22}}$ , ...,  $\lambda_n = \frac{1}{v_{nn}}$  of the

linear programming problem has allowed defining analytical expressions for the multimodal strategy of the

first player  $\pi_1 = \lambda_1 \cdot \xi = \frac{\lambda_1}{\sum_{i=1}^n \lambda_i}$ ;  $\pi_2 = \lambda_2 \cdot \xi = \frac{\lambda_2}{\sum_{i=1}^n \lambda_i}$ ; ...;

$$\pi_m = \lambda_m \cdot \xi = \frac{\lambda_m}{\sum_{i=1}^n \lambda_i}.$$

These expressions allow you to define a set of coefficients for a compromise solution.

At the same time, compromise solutions for the values  $S_i^k$  of the standards of deductions to the local budgets from the Federal and regional taxes to be credited to the budget of the region, are defined as

$$S_1^* = \sum_{i=1}^n S_{i1} \cdot \pi_i; \quad S_2^* = \sum_{i=1}^n S_{i2} \cdot \pi_i; \quad \dots; \quad S_l^* = \sum_{i=1}^n S_{il} \cdot \pi_i.$$

The developed economic-mathematical models, on a formalized basis, allow making such decisions in the allocation of tax revenues between the region and mining municipality, which meet the condition for achieving a compromise between the interests of the budgets of various levels. This provides the basis for implementing effective policies of results-oriented budget management.

**Conclusions and recommendations for further research.** The model proposed in the article to determine the distribution proportion of income from taxes in the <region> ↔ <mining municipality> structure, on a formalized basis, allows determining the rates of deductions from Federal and regional taxes to the state regulation, which gives an opportunity to quantify the effect of made decisions and contributes to strengthening of the stimulating function of inter-budget regulation. The developed automate model is adaptive to changes in the operation conditions, caused by variations in income from taxes and also has the property of appropriateness of behaviour and asymptotic optimal-

ity, which allows its use as a formal equivalent of the decision-makers. The model of game machines allows choosing the standards of deductions from Federal and regional taxes to the mining municipal entities on the basis of the terms of a compromise of the interests of the budgets of different levels. The created model is included in the system of support of decision-making and in future its introduction into the process of budgetary control for the mining communities is planned.

### References/Список літератури

1. Matveeva, L. G., 2014. The innovative potential of industry in the south of Russia: the tool management in order to reduce regional-asymmetry. *Regional Economy. South of Russia*, No. 1, pp. 42–54.

Матвеева Л. Г. Инновационный потенциал промышленности юга России: инструментарий управления в целях снижения региональной асимметрии / Л. Г. Матвеева // Региональная экономика. Юг России. – 2014. – № 1. – С. 42–54.

2. Matveeva, L. G. and Chernova, O. A., 2013. Modelling of managerial processes by resource flows for development of peripheral areas. *Terra Economicus*, Vol. 11, No. 3–2, pp. 84–88.

Матвеева Л. Г. Моделирование процессов управления ресурсными потоками в целях развития периферийных территорий / Л. Г. Матвеева, О. А. Чернова // Terra Economicus. – 2013. – Т. 11. – № 3–2. – С. 84–88.

3. Matveeva, L. G. and Chernova, O. A., 2014. Indicative approach to forecasting and strategic management of the development of coal mines. *Sustainable development of mountain territory*, No. 3 (21), pp. 96–100.

Матвеева Л. Г. Индикативный подход к прогнозированию и стратегическому управлению развитием угледобывающего предприятия / Л. Г. Матвеева, О. А. Чернова // Устойчивое развитие горных территорий. – 2014. – № 3(21). – С. 96–100.

4. Matveeva, L. G., Nikitaeva, A. Y. and Chernova, O. A., 2015. Prospects and development potential of the southern regions of Russia in the anti-Russian economic sanctions. *Regional economy: Theory and practice*, No. 17 (392), pp. 2–12.

Матвеева Л. Г. Перспективы и потенциал развития регионов юга России в условиях антироссийских экономических санкций / Л. Г. Матвеева, А. Ю. Никитаева, О. А. Чернова // Региональная экономика: теория и практика. – 2015. – № 17(392). – С. 2–12.

5. Chernova, O. A. and Chernova M. G., 2014. Growth forming and limiting factors of innovative development of the former mining towns of the Rostov area. *Economy and society*, No. 1–2 (10), pp. 1074–1080.

Чернова О. А. Ростоформирующие и лимитирующие факторы инновационного развития бывших шахтерских городов Ростовской области / О. А. Чернова, М. Г. Чернова // Экономика и социум. – 2014. – № 1–2(10). – С. 1074–1080.

6. Chernova, O. A., 2010. Problems of formation of innovative development pathway of depressive miner

territories of Rostov Region. *Regional Economy. Theory and practice*, No. 13, pp. 58–63.

Чернова О. А. Проблемы формирования траектории инновационного развития депрессивных шахтерских территорий Ростовской области / О. А. Чернова // Региональная экономика. Теория и практика. – 2010. – № 13. – С. 58–63.

7. Streltsova, E. D., 2001. Application of stochastic automata to model complex systems with time-varying nature of the behaviour. *News of Higher Educational Institutions. Electrical engineering*, No. 3, pp. 76–78.

Стрельцова Е. Д. Применение стохастических автоматов для моделирования сложных систем с изменяющимся во времени характером поведения / Е. Д. Стрельцова // Изв. вузов. Электромеханика. – 2002. – № 3. – С. 76–78.

**Мета.** Створення моделей, що дозволяють на формальній основі здійснювати частковий розподіл коштів від сплати податків у структурі <region> ↔ <шахтарське муніципальне утворення>.

**Методика.** Запропоновано підхід для створення моделей, що мають властивість адаптації до змін умов функціонування. Підхід базується на аналітичному апараті теорії стохастичних автоматів, що функціонують у випадкових середовищах, та дозволяє формально описувати поведінку агента в умовах вимірюваної невизначеності, створюваної надходженнями від сплати податків до бюджетів нижчого рівня бюджетної системи РФ.

**Результати.** Запропонована конструкція стохастичного автомата, що має властивість доцільності поведінки та асимптотичної оптимальності. Автомат добре пристосовується до флуктуацій зовнішнього середовища, викликаних варіаціями податкових доходів у бюджетах шахтарських муніципальних утворень. Побудовані матриці переходів автомата з одного стану в інший під дією входних сигналів, що ідентифікують сприятливу та несприятливу реакцію зовнішнього середовища на виходи автомата. Отримані вирази для фінальних імовірностей перебування автомата у своїх станах. Побудована математична модель у вигляді ймовірнісного автомата використовується в системі підтримки прийняття рішень з управління міжбюджетним регулюванням для депресивних територій, до числа яких занесені шахтарські муніципальні утворення.

**Наукова новизна.** Поставлене актуальне завдання створення модельного інструментарію, сприяючого домінуванню стимулюючої функції міжбюджетного регулювання над вирівнюючою функцією у структурі <region> ↔ <шахтарське муніципальне утворення>. Побудована автоматна модель для формалізації визначення пропорцій розподілу податків у порядку бюджетного регулювання. Перевага моделі полягає в її адаптивності до умов невизначеності впливів зовнішнього середовища, а також у здатності до самонавчання за рахунок властивості доціль-

ності поведінки та асимптотичної оптимальності.

**Практична значимість.** Практична значимість проведених досліджень полягає у виконанні побудованих моделей у складі системи підтримки прийняття рішень у структурі <регіон> ↔ <шахтарське муніципальне утворення>.

**Ключові слова:** міжбюджетне регулювання, шахтарські муніципальні утворення, економіко-математичні моделі, стохастичні автомати, системи підтримки прийняття рішень

**Цель.** Создание моделей, позволяющих на формальной основе осуществлять долевое распределение средств от уплаты налогов в структуре <регион> ↔ <шахтёрское муниципальное образование>.

**Методика.** Предложен подход для создания моделей, обладающих свойством адаптации к изменениям условий функционирования. Подход базируется на аналитическом аппарате теории стохастических автоматов, функционирующих в случайных средах, и позволяет формально описывать поведение агента в условиях измеримой неопределённости, создаваемой поступлениями от уплаты налогов в бюджеты нижестоящего уровня бюджетной системы РФ.

**Результаты.** Предложена конструкция стохастического автомата, обладающего свойством целесообразности поведения и асимптотической оптимальности. Автомат хорошо приспособляется к флуктуациям внешней среды, вызванным вариациями налоговых доходов в бюджетах шахтёрских муниципальных образований. Построены матрицы переходов автомата из одного состояния в другое под действием входных сигналов, идентифицирующих благоприятную и неблагоприятную реакции внешней среды на выхо-

ды автомата. Получены выражения для финальных вероятностей пребывания автомата в своих состояниях. Построенная математическая модель в виде вероятностного автомата используется в системе поддержки принятия решений по управлению межбюджетным регулированием для депрессивных территорий, в число которых занесены шахтёрские муниципальные образования.

**Научная новизна.** Поставлена актуальная задача создания модельного инструментария, способствующего доминированию стимулирующей функции межбюджетного регулирования над выравнивающей функцией в структуре <регион> ↔ <шахтёрское муниципальное образование>. Построена автоматная модель для формализации определения пропорций распределения налогов в порядке бюджетного регулирования. Преимущество модели состоит в её адаптивности к условиям неопределённости воздействий внешней среды, а также в способности к самообучению за счёт свойства целесообразности поведения и асимптотической оптимальности.

**Практическая значимость.** Практическая значимость проведённых исследований состоит в использовании построенных моделей в составе системы поддержки принятия решений в структуре <регион> ↔ <шахтёрское муниципальное образование>.

**Ключевые слова:** межбюджетное регулирование, шахтёрские муниципальные образования, экономико-математические модели, стохастические автоматы, системы поддержки принятия решений

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