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## ESTIMATING ECONOMIC EQUIVALENT OF REACTIVE POWER IN THE SYSTEMS OF ENTERPRISE ELECTRIC POWER SUPPLY

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## ДО ПИТАННЯ ОЦІНКИ ЕКОНОМІЧНОГО ЕКВІВАЛЕНТА РЕАКТИВНОЇ ПОТУЖНОСТІ В СИСТЕМАХ ЕЛЕКТРОПОСТАЧАННЯ ПІДПРИЄМСТВ

**Purpose.** To analyze the development of the notion “Economic equivalent of reactive power” (EERP) and deals with definitions of this concept in historical perspective.

**Methodology.** The method of researches is based on the theory of the electrical engineering at the use of methods of regulations reactive power.

**Findings.** It is emphasized that the impact of the voltage change in the network on the alteration of resistance losses from the var flow should be taken into account while doing the corresponding mathematical derivation and introducing numerical illustration. Hence, absence of necessity to consider capacitive generation of overhead lines has been proved computationally. Peculiarities of the regulating effect of voltage related to the reactive power and EERP have been substantiated, which allows to treat them jointly within the framework of electromagnetic compatibility concept. It is stressed that in computing EERP it is reasonable to use assessed values of parameters, which is illustrated by several examples.

**Originality.** Feasibility of using the method of incremental rates with partial derivatives has been theoretically substantiated. Incompleteness and incorrectness of the input information impact on the value of EERP have been assessed. It was proved that regulatory documents or instructions are vital to standardize recommended methods of calculations.

**Practical value.** Approaches to project practices are also in the centre of attention. Analysis of the obtained results allowed to give a quantitative assessment to the impact of voltage on EERP, as well as to the values of minimal losses of reactive power. An approximate formula for assessing resistance of the external grid in computing EERP has been derived.

**Keywords:** reactive power, economic equivalent, methods of calculations

**Problem statement.** The notion of “Economic equivalent of reactive power” (EERP) coefficient  $C_e$  was introduced into practice of projecting compensation of the reactive power in industrial electrical networks in the 1950–1960s as a characteristic

of supplementary loss of the active power in time or per unit of additional reactive load, kW/kVar. In reference literature or educational books, one can find alternative names for this parameter – loss coefficient, adaptation coefficient, loss reduction coefficient etc. We shall further use  $C_e$  instead of EERP.

A number of EERP definitions reflect its physical essence. For example, “EERP is understood as increment of active power losses in the entire network together with increase in the reactive power of the network unit (substation, power plant)”. Others define EERP as specific active power loss increment related to reactive power in the network node. Some sources refer to RIPL – relative increment of power losses [1, 2]. RIPL has been widely applied to the solution of optimization problems (optimum distribution of sources and loads in power engineering, modes of power systems, most feasible composition of the operating machinery), and to the research into relative power loss, which became the basis for the development of EERP.

$C_e$  value is used in several mode calculations: for example, computation of voltage and reactive power regulation, designing rational substation layout and determining the number of operating transformers or other electric equipment, optimizing calculations in electrical circuits of enterprises and power systems etc.

**Analysis of the previous research.** It is hardly possible to get exact expressions for specific loss increments. V.A. Venikov claims that change of power in the node is analogous to the values of other nodes. That is why it is a generally accepted practice to use approximate formulas taking into account allowances that even for the maximum deviations of active and reactive power the corresponding parameters in nodes are considered unaltered. From practice we know that  $C_e$  and  $C_l$  vary significantly.  $C_l$  is related to losses of power and energy, being the coefficient characterising active power loss increment under the change of active load. Thus, according to measurements’ results at enterprise central substations, the values of  $C_e$  and  $C_l$  differ sufficiently (by 1.5–2 times and more) in winter and summer, in the daytime maximum and summer load minimum: the same is true for nodes of 110 kV mains [3 and others].

It is known from literature (textbooks by L. M. Zeltsburg, A. A. Taitz etc.) that values of  $C_e$  have been determined with the help of approximate formula for incidental loss of active power  $\Delta P_Q$  during the change of the node reactive power by  $\Delta Q$

$$C_e = \frac{\Delta P_Q}{\Delta Q}. \quad (1)$$

The simplest transformation (skipping elementary derivation) allows to obtain

$$C_e = \frac{2QR}{U^2}, \quad (2)$$

where  $R$  – network resistance from the source buses (power plant, substation) to load buses;  $Q$  – reactive power on the consumer substation buses.

$C_e$  values known from literature are rather approximate in respect to power supply schematic and the number of transformer steps. Thus, their usage in computing reactive power compensation, though helpful in a way, resulted in notable error.

Transition from calculations on the basis of finite increments to computing  $C_e$  by the method of incremental losses known already in the 1920–1930ss, with the use of partial derivatives and optimization methods, has become an impetus for the development of EERP.

Here is the formula for computing EERP

$$C_e = \frac{d(\Delta P)}{dQ}.$$

After the necessary transformations for  $C_e$ , we obtain the above formula (1), which serves as the basis for deriving other formulas used in computing  $C_e$ .

Let us turn to characters and symbols used in the “Method for calculating payment for the flow of reactive power between power supplying facility and consumers”. We will mark economic equivalent of the reactive power  $C_e$  by  $D$ . Formula (2) presents expression for  $D_1$ , reflecting reduction of active power losses in the network related to reactive power flows

$$D_1 = \frac{2QR}{U^2}.$$

**Research materials and results.** It is known that losses change related to the reactive (as well as active) power flow depends sufficiently on the voltage change, voltage losses comprising about a half of the total loss in 6–10 kV networks and about  $\frac{2}{3}$  – in networks of higher voltage [1, 3]. Considering the voltage level

$$D = D_1 + \left( \frac{\partial \Delta P}{\partial U} \right) \left( \frac{\partial U}{\partial Q} \right),$$

where  $D_2 = \left( \frac{\partial \Delta P}{\partial U} \right) \left( \frac{\partial U}{\partial Q} \right)$  and

$$D = D_1 + D_2.$$

The solution below presents expression for  $D$  with the following order of transformations

$$\frac{\partial \Delta P}{\partial u} = 2 \frac{P^2 + Q^2}{U^3} R = 2 \frac{\Delta P}{U},$$

where  $\Delta P = \frac{P^2 + Q^2}{U^2} R$  – active power losses;  $R$  – active network resistance.

$$\Delta U = \frac{PR + QX}{U}; \quad \frac{\partial U}{\partial Q} = \frac{X}{U},$$

where  $\Delta U$  – voltage loss;  $X$  – reactive network resistance.

$$D_2 = -2 \frac{P^2 + Q^2}{U^3} R \cdot \frac{X}{U} = -2 \frac{P^2 + Q^2}{U^4} RX = 2 \frac{\Delta P}{U^2} X. \quad (3)$$

Expression (3) can be presented as

$$D_2 = \frac{2\Delta P_x}{\text{tg}\varphi}, \quad (4)$$

where  $\Delta P_* = \frac{\Delta P}{P}$  – relative value of active power losses;

$\text{tg } \varphi$  – value of a parameter in the network node (on the buses at the central substation CS, main stepdown substation MSS, load-centre substation LCS etc.).

For deriving expression (4), we used the known equation

$$X = \frac{U^2}{Q} = \frac{U^2}{P \cdot \text{tg } \varphi}.$$

Eventually

$$D = D_1 + D_2 = 2 \frac{QR}{U^2} + 2 \frac{\Delta P}{U^2} X$$

or

$$D = 2 \frac{QR}{U^2} + 2 \frac{\Delta P_*}{\text{tg } \varphi}.$$

As an example, let us find the value for  $D$  according to the expression (4) for the network with 110 kV voltage if  $Q = 20$  MVar,  $P = 50$  MW,  $R = 12.45$  O,  $X = 20$  O

$$D = \frac{2 \cdot 20}{110^2} + \frac{2(50^2 + 20^2)}{110^4} \cdot 12.45 \cdot 20 \approx 0.05.$$

Thus,  $D \approx 0.05 = C_e$ .

The calculations above do not take into account generation of reactive power  $Q_c$ , conditioned by 110 kV overhead power transmission line (OPTL). Let us consider parameter  $Q_c$ , using pi circuit of OPTL substitution [1]

$$D = D_1 + D_2 + D_c.$$

According to [1], for 110 kV line 50 km long we will get

$$Y = 134.5 \cdot 10^{-6} \text{ S},$$

where  $Y$  – capacity susceptance of lines.

The value

$$Q_c = U^2 Y = 110^2 \cdot 134.5 \cdot 10^{-6} = 1.62 \text{ MVar}.$$

Coefficient  $D_c$ , considering capacity generation of 110 kV OPTL will be

$$D_c = -2Q \frac{RX}{UY} = \frac{2 \cdot 1.62 \cdot 12.45 \cdot 12.15}{UY} \approx 34 \cdot 10^{-5}.$$

It is obvious that, capacity generation being small, there is no necessity to take into account  $D_c$ .

Calculation of  $D$  values adjusted to buses of low voltage is one of the tasks for the automated control system [3, 4]. These data allow to formulate requirements for reactive power compensation.

**Analysis of the obtained solution.** In order to estimate quantitatively the voltage effect, that is the significance of parameter  $D_2$ , let us find the  $\delta$ -relationship  $D_2/D_1$

$$\delta_2 = 2 \frac{P^2 + Q^2}{U^4} RX \cdot \frac{U^2}{2QR} = \frac{\Delta P_*}{\text{tg } \varphi} K_X.$$

In derivation, we have considered the known relationships

$$X = \frac{U^2}{Q} \quad \text{and} \quad Q = P \text{tg } \varphi; \quad \Delta P_* = \frac{\Delta P}{P}, \quad K_X = \frac{X}{R},$$

where  $X$  and  $R$  – active and reactive resistances.

Relationship  $\delta_2 = \frac{\Delta P_*}{\text{tg } \varphi} K_X$  is called coefficient of node voltage level effect on the value of  $C_e$ .

Values of  $K_X$  in 10, 35, 110 kV lines are within 1–3.

Assessed values of  $\delta_2 = \frac{\Delta P_*}{\text{tg } \varphi} K_X$  in nodes of 110 kV electric network are  $\frac{0.05 \div 0.1}{0.25 \div 0.35} = 0.1 \div 0.4$ , for nodes

of 35 kV networks –  $0.3 \div 0.4$ , 10 kV networks –  $0.1 \div 0.2$ .

Values of  $D = D_1 + D_2$ , are usually within  $0.01 \div 0.15$ .

It is interesting to find the values of  $Q = Q_{\text{opt}}$  related to the minimum loss from the reactive power flow.

It is possible to conclude from the expression for  $D$  that voltage growth results in loss reduction both from active and reactive power flows. In this case, minimum loss will be reported at  $Q_{\text{opt}}$  determined by  $D = 0$  [1], not at  $Q = 0$  which would take place if we took into account only loss reduction in the network at the expense of reactive power drop. Solution of this equation becomes easier if we reduce both members by  $\frac{2R}{U^2}$ ; as a

result we obtain the square equation

$$Q_{\text{opt}} + \frac{P^2 + Q_{\text{opt}}^2}{U^2} X = 0 \quad \text{or} \quad Q_{\text{opt}}^2 X + U^2 Q_{\text{opt}} + P^2 X = 0,$$

with the solution

$$Q_{\text{opt}} = \frac{-U^2 + \pm \sqrt{U^4 - 4P^2 X^2}}{2X}.$$

It is evident that the solution is true if there is “+” before the root.

Let us analyze the following example: power  $P = 30$  MW is transmitted along 110 kV line 50 km long ( $X = 0.4 \cdot 50 = 20$  O). The sought-for quantity is

$$Q_{\text{opt}} = \frac{-110^2 + \pm \sqrt{110^4 - 4 \cdot 30^2 \cdot 20^2}}{2 \cdot 20} \approx -1.5 \text{ MVar} \neq 0.$$

This result testifies to the existence of reactive power source in the network.

Computations of  $C_e$  values done within the frames of the present research, also reported in [3, 4], allowed to conclude that the results obtained for average computational parameters values and nominal equipment data differ by 30–50 % for MSS or LCS of the enterprise under 110 kV voltage, by 15–25 % for network nodes under 35 kV and by 10–15 % for network nodes under 10 kV. It is essential to consider the significant effect of incomplete and incorrect input information in the tasks of power supply design. As a matter of fact,

30–50 % of the data necessary for computation of  $C_e$  are not known. In most cases, mode characteristics of power networks (levels and laws governing the voltage change on the boundary bus), presence of distributed generation sources in the network, existence of isolated generating plants, non-linear loads (medium and large enterprises use more than 90 % of electric equipment in transformed way), as well as equivalent error etc. are not commonly taken into account in  $C_e$  computation.

Principles and approaches to correct dealing with specifics of loads and structure of enterprises power supply systems should be substantiated if their feasibility is proven. Hence there is no necessity to consider maximum number of effective factors in  $C_e$  computation which makes application of complex programs that take those factors into account unreasonable in some calculations of industrial electric networks, given the probability of significant errors in computations.

In order to correctly use  $C_e$  values in computing reactive power compensation and calculating the cost of the reactive power flow, it is necessary in some cases to compile regulating instructions (or other documents like “Regulating instructions for computing short circuit currents”), with the view to standardising (or otherwise consolidating) computational methods, parameters and operational modes of electric networks as well as acceptable calculation error.

It is common practice to use basic (normal) power supply diagrams for MSS or LCS in  $C_e$  computation. These substations are usually fed by two overhead lines (of 110 kV and higher voltage) located either on different towers or in different places. Consumers are supplied with 10 (6) kV voltage from 110 kV buses via transformers. Consumers are responsible for feeding loads at 0.4 kV. Capacitor banks for reactive power compensation are installed in networks of 0.4 to 6 (10) kV [2–4].

*Small enterprises* (about 10 % of the total number of transformers) work under the load of 3–5.000 kW under transformer capacity up to 8 MV·A. Cabels are used for feeding transformers.

*Medium enterprises* can work under the load of 10 kV and about 12 MW capacity, or 6 kV and 10 MW, with median  $\cos \varphi \sim 0.9$ .

*Big enterprises* (6–10 %) of 100 MW (and higher) load are equipped with outdoor switch gear of 110 kV and transformers of 110/10 kV as well as their own co-generation unit.

*Extra big enterprises* of 1000 MW (and higher) load do not differ from energy systems in their parameters.

In  $C_e$  computation, reactive power values are taken either from power facilities data or from calculations that are a mandatory part of engineering documentation. If there are no data for the boundary parameters, electric system resistance (from boundary buses, i. e. CS, MSS or LCS etc.) can be determined approximately via power and/or short circuit current  $S_{sc}$  (known or calculated) through formula

$$x_C \approx 2 \frac{U^2}{S_{sc}}$$

To analyse and compute voltage modes and related tasks of reactive power compensation, it is common to use the value of regulating effect of reactive power load. The related equations as a rule use the value of economic equivalent of the reactive power. Expressions describing regulating effect and assessment of the economic equivalent of the reactive power are identical in structure. Peculiarities of voltage regulating effect assessment in terms of reactive power and economic equivalent of the reactive power explain the necessity to interpret them jointly within the frames of electromagnetic compatibility concept.

Knowledge of  $C_e$  specifics allows to get its assessed (rough) values. They make it possible to estimate the order of  $C_e$  values and evaluate correctness of complicated calculations. Sometimes, for the purposes of operation, it is sufficient to determine assessed values of  $C_e$ .

A great number of comparative calculations applied to power supply systems of enterprises and power systems (with 110 kV and lower voltage) allowed to obtain an approximate formula

$$C_e = 2\Delta P_* \operatorname{tg} \varphi,$$

where  $\operatorname{tg} \varphi$  – median value of  $\operatorname{tg} \varphi$  of the feeding network.

Analysis of consumer statistic characteristics show that voltage increase by 1 % results in consumer active load growth by 1 %, and reactive power by 2 %, i. e. we can assume that

$$\frac{\partial P}{\partial u} \approx \frac{P}{U} \cdot \frac{\partial Q}{\partial u} - 2 \frac{Q}{U}.$$

Regulatory effect of voltage along the reactive power being

$$K_{U,Q} \approx \frac{1}{S_{sc}},$$

where  $S_{sc}$  – short circuit power in the network node.

For transformers of 6–10 kV and 110 kV

$$C_e \approx C_p \operatorname{tg} \varphi,$$

while for any section without network taps

$$C_e = 2\Delta P_*.$$

### Conclusions.

1. Implementation of approaches based on the usage of the relative power losses in the network, in the form of partial derivatives of the studied processes’ parameters, confirms correctness of the obtained results, characterizing economic equivalent of the reactive power.

2. It is proven that the value of the second component of  $C_e$  ( $D_2$ , defining the effect of the voltage level) does not exceed  $D_1$  depending on the voltage level by 10–45 %. In this case, it is not necessary to consider capacity generation of overhead lines at 100 kV (and higher nominal voltages) in view of its negligible influence on  $C_e$  level.

3.  $C_e$  computations done on the basis of nominal data of electrical equipment, the effect of incomplete and incorrect input information, non-linearity of loads, errors of equivalent values – make it unneces-

sary to consider a big number of influential factors, while accounting for significant (to 50 %) error.

4. The research substantiated feasibility of doing  $C_e$  assessment against the background of electromagnetic compatibility concept.

5. The paper presented sufficiently simple expressions which allow to assess the order of  $C_e$  values enabling to judge about the correctness of more complicated computations.

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**Мета.** Уявити історію розвитку поняття „економічний еквівалент реактивної потужності“ (ЕЕРП), розглянути визначення цього поняття, прийняті в різні роки.

**Методика.** Методика досліджень заснована на теорії електротехніки при використанні способів передачі та регулювання реактивної потужності.

**Результати.** Наголошено на необхідності врахування впливу змін напруги в електричній мережі на зміну активних втрат від перетоків реактивної потужності. З урахуванням цієї обставини виведені відповідні математичні вирази, наведено числовий приклад. Також з урахуванням цієї обставини, за допомогою розрахунку, обґрунтована відсутність необхідності врахування ємнісної генерації повітряних ліній. Наведене обґрунтування особливостей регулюючого ефекту напруги по реактивній потужності і ЕЕРП, що дозволяє розглядати їх спільно в рамках концепції електромагнітної сумісності. Підкреслено доцільність використання в розрахунках ЕЕРП оціночних значень параметрів, наведено ряд прикладів.

**Наукова новизна.** Надано теоретичне обґрунтування доцільності використання методу відносних приростів з використанням часткових похідних. Був оцінений вплив неповноти й неточ-

ності вихідної інформації на значення ЕЕРП, показана необхідність розробки керівних вказівок (або подібних документів), в яких повинні бути стандарти, рекомендовані методи розрахунків.

**Практична значимість.** Відзначені також обґрунтування підходів, прийнятих у проектній практиці. Аналіз отриманого рішення дозволив кількісно оцінити вплив напруги на ЕЕРП, а також значення мінімальних втрат реактивної потужності. Представлена наближена формула для оцінки опору зовнішньої мережі при розрахунку ЕЕРП.

**Ключові слова:** реактивна потужність, економічний еквівалент, методи розрахунку

**Цель.** Представить историю развития понятия „экономический эквивалент реактивной мощности“ (ЭЭРМ), рассмотреть определения этого понятия, принятые в разные годы.

**Методика.** Методика исследований основана на теории электротехники при использовании способов передачи и регулирования реактивной мощности.

**Результаты.** Подчеркнута необходимость учета влияния изменений напряжения в электрической сети на изменение активных потерь от перетоков реактивной мощности. С учетом этого обстоятельства сделан вывод соответствующих математических выражений, приведен числовой пример. Также с учетом этого обстоятельства, с помощью расчета, обосновано отсутствие необходимости учета емкостной генерации воздушных линий. Приведены обоснования особенностей регулирующего эффекта напряжения по реактивной мощности и ЭЭРМ, что позволяет рассматривать их совместно в рамках концепции электромагнитной совместимости. Подчеркнута целесообразность использования в расчетах ЭЭРМ оценочных значений параметров, приведен ряд примеров.

**Научная новизна.** Дано теоретическое обоснование целесообразности использования метода относительных приростов с использованием частных производных. Было оценено влияние неполноты и некорректности исходной информации на значение ЭЭРМ, показана необходимость разработки руководящих указаний (или подобных документов), в которых должны быть стандарты, рекомендуемые методы расчетов.

**Практическая значимость.** Отмечены также обоснования подходов, принятых в проектной практике. Анализ полученного решения позволил количественно оценить влияние напряжения на ЭЭРМ, а также значение минимальных потерь реактивной мощности. Представлена приближенная формула для оценки сопротивления внешней сети при расчете ЭЭРМ.

**Ключевые слова:** реактивная мощность, экономический эквивалент, методы расчета

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