UDC 534.12; 517.4; 622.23

O. I. Khytriak[1], Cand. Sc. (Tech.),
L. F. Dziuba[2], Cand. Sc. (Tech.), Assoc. Prof.,
I. V. Kuzio[3], Dr. Sc. (Tech.), Prof.

1 – Hetman Petro Sahaidachnyi National Army Academy, Lviv, Ukraine, e-mail: khytriakolga@gmail.com
2 – Lviv State University of Life Safety, Lviv, Ukraine, e-mail: lidadz111@gmail.com
3 – Lviv Polytechnic National University, Lviv, Ukraine

THE INFLUENCE OF VISCOELASTIC PROPERTIES OF THE BELT ON ITS DYNAMIC CHARACTERISTICS

О. І. Хитряк[1], канд. техн. наук,
Л. Ф. Дзюба[2], канд. техн. наук, доц.,
І. В. Кузьо[3], д-р техн. наук, проф.

1 – Національна академія сухопутних військ імені гетьмана Петра Сагайдачного, м. Львів, Україна, e-mail: khytriakolga@gmail.com
2 – Львівський державний університет безпеки життєдіяльності, м. Львів, Україна, e-mail: lidadz111@gmail.com
3 – Національний університет „Львівська політехніка“, м. Львів, Україна

ВПЛИВ В'ЯЗКОПРУЖНИХ ВЛАСТИВОСТЕЙ ПАСА НА ЙОГО ДИНАМІЧНІ ХАРАКТЕРИСТИКИ

Purpose. The purpose of the article is to create a viscoelastic dynamic model of flexible axially-moving belt of different mechanisms considering the influence of the material mechanical properties on its dynamic characteristics in case of transverse vibrations.

Methodology. Methodology for flexible belt transverse oscillation investigation is based on asymptotical methods of nonlinear mechanics and wave theory of movement.

Findings. Analytical relations for amplitude and frequency of transverse vibration definition for flexible viscoelastic axially-moving belts are obtained in the research. The influence of viscous and elastic belt material properties on its frequency response is analyzed.

Originality. For the first time, based on a created dynamic model of flexible viscoelastic axially-moving belt, its dynamic performance during the transverse vibrations is identified analytically and the influence of material mechanical properties on these characteristics is studied

Practical value. The offered flexible belt transverse oscillation investigation method allows determining the influence of longitudinal movement speed and viscoelastic material properties on the main parameters of dynamic process. Obtained analytical relations can be the basis for engineering calculations of components and mechanisms, part of which are axially-moving flexible belts.

Keywords: *mathematical model, the wave theory of motion, viscoelasticity, transverse vibrations, belt drive, the perturbation methods*

Introduction. Topicality and literature review. The drive belt as a flexible traction belt drive element is the most important element that defines transmission efficiency and durability. Exploitation of the machine belt drives is usually accompanied by oscillatory processes with increase in the dynamic loads on the belt. Consequently, its length increases and durability decreases prematurely. Therefore, study of dynamic phenomena in belt drive flexible elements is prerequisite for reliable and efficient operation of drives.

The description of the belt dynamics is usually based on a mathematical model [1–4]. Separately in [1] the nonlinear governing equations of motion for viscoelastic moving belt are established by using the generalized Hamilton's principle. In the work [1], based on the linear viscoelastic differential constitutive law the generalized equation of motion is derived for a moving belt with geometric non-linarites. Also in [3] taking into account the translation effect by the material derivative and the nonlinearity by the Lagrangian strain the governing equation is given. As one-dimension axially-moving string with viscoelastic properties the belt is considered in [4]. Thus, the belt can be considered as one-dimension flexible axially-moving body, which oscillates in transverse direction.

Considering the speed of longitudinal movement of the specified belt usually leads to significant mathematical difficulties in solving problems. In works [5–7] these difficulties are overcome by adapting the wave theory of motion to this type of problems. Based on this relatively simple equations for engineering calculations are obtained. In particular, one of the fundamental works is [5], in which regarding the axially-moving one-dimension system the solution of non-perturbed problem is suggested to submit as a superposition of two waves with different lengths and the same frequency. The parameters of these waves are defined in [6]; analytical solution for two-dimensional flexible axially-moving bodies is given. In [7] the interaction with the environment is taken into account for two-dimension case.

However, in these works the belt is considered as completely elastic. This assumption usually leads to inaccurate results. In general, the mechanical properties of the belt depend on the belt design, material and its elements. Belt elements – the cord, a pillow and wrapper are made from highly polymeric materials [8]. The behavior of the majority of polymeric materials under the influence of mechanical loadings is considered viscoelastic [4]. There are plastic deformations that are accompanied by energy dissipation due to internal loss mechanism in the loaded belts [1, 3]. The dissipation, reducing noise and vibration affects the dynamic characteristics of moving belts.

Considering the specified, the purpose of this work is to create a viscoelastic dynamic model of the flexible axially-moving belt and to research the influence of the mechanical characteristics of the belt material on the parameters of its transverse vibrations. It should be noted that a similar objective was formulated in the works [1, 3, 4], but it is difficult to say that the used methods for mathematical calculations (finite differences or multiple scale methods) are proximate. For the analytical investigation of the influence of viscoelastic belt properties on amplitude and frequency of its transverse vibration it is advisable to use the basic idea of works [5–7].

Problem definition. To achieve the goal let us describe the dynamic processes in the belt as in a flexible element. The belt is modeled in the form of a one-dimensional elastic body. Its transverse vibrations are described by differential equation with partial derivatives that are obtained from Newton's second law [3]

$$\rho\frac{\partial^2 u}{\partial t^2}=\left(\frac{T}{A}+\sigma\right)\frac{\partial^2 u}{\partial x^2}+\left(\frac{\partial u}{\partial x}\right)^2\frac{\partial \sigma}{\partial x}, \tag{1}$$

where $u(x, t)$ is transverse displacement of a cross section of moving flexible belt with x coordinate in arbitrary moment of time t; A is cross-sectional area; σ is normal stress; ρ is density of material; T is force of an initial tension.

The subscript in the formulas means differentiation on the corresponding variable, in particular $\frac{\partial u}{\partial t}$ is the first partial derivative on time t from movement, $\frac{\partial^2 u}{\partial x^2}$ is the first partial derivative on coordinate x from movement, etc.

The equation (1) is written with Lagrange variables. Considering the occurrence of the longitudinal belt movement, it is more appropriate to use the Euler coordinates (spatial coordinates) [1, 6, 9]. In the case, when the flexible belt moves with constant speed v in the direction of the Ox axis, transition formulas from the Lagrange variables to Euler variables take place [9]

$$\frac{d}{dt}=\frac{\partial}{\partial t}+\frac{\partial}{\partial x}\frac{dx}{dt};\quad \frac{dx}{dt}=v\Rightarrow\frac{d}{dt}=\frac{\partial}{\partial t}+v\frac{\partial}{\partial x};$$
$$\frac{d^2}{dt^2}=\frac{\partial^2}{\partial t^2}+2v\frac{\partial^2}{\partial x\partial t}+v^2\frac{\partial^2}{\partial x^2}, \tag{2}$$

where the first term on the right side of the formula (2) is a local acceleration component, the second is a Coriolis acceleration component, and the last is a component of centripetal acceleration. Based on this we present the differential equation (1) as

$$\rho\left(\frac{\partial^2 u}{\partial t^2}+2v\frac{\partial^2 u}{\partial x\partial t}+v^2\frac{\partial^2 u}{\partial x^2}\right)=\left(\frac{T}{A}+\sigma\right)\frac{\partial^2 u}{\partial x^2}+\left(\frac{\partial u}{\partial x}\right)^2\frac{\partial \sigma}{\partial x}. \tag{3}$$

Without the loss of generality, we consider part of a traction branch of a belt in length l, that equals the distance between the start point a belt contact to the pulley. Considering that contact of the belt with the pulleys is constantly irrevocable, we assume that there are not transverse displacements in the contacts points of a belt to pulleys. It enables us to attach boundary conditions (4) to differential equation (3)

$$u(x,t)\big|_{x=0}=u(x,t)\big|_{x=l}=0. \tag{4}$$

To count the viscoelastic material characteristics of the belt we use the linear differential relation of viscoelastic theory, which connects stress $\sigma(t)$ and strain $\varepsilon(t)$ [3]

$$\sigma(t)=E^*\varepsilon(t). \tag{5}$$

In the formula (5) E^* is the equivalent to Young's modulus that is a certain differential operator with respect to time and takes into account the instantaneous elasticity, elastic aftereffect and viscosity of material. In many cases, it is enough to submit viscoelastic behavior in a limited time frame, considering only one or two terms of the operator E^*.

Only geometric nonlinearity, which takes place due finite elongation, is considered in this paper. For axially-moving belt in case of sufficient value of the oscillation amplitude the deformation in the longitudinal direction Ox is

$$\varepsilon(x,t)=\frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2. \tag{6}$$

Then expression (5) for normal tension takes the form

$$\sigma(x,t)=E^*\left(\frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2\right). \tag{7}$$

Considering (7) in (3) we obtain

$$\rho\frac{\partial^2 u}{\partial t^2}+2\rho v\frac{\partial^2 u}{\partial x\partial t}+\left(\rho v^2-\frac{T}{A}\right)\frac{\partial^2 u}{\partial x^2}=$$
$$=E^*\left(\frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2\right)\cdot\frac{\partial^2 u}{\partial x^2}+\frac{\partial u}{\partial x}\cdot\frac{\partial}{\partial x}\left(E^*\left(\frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2\right)\right). \tag{8}$$

Differential equation (8) differs from the equation of motion of axially-moving purely elastic flexible elements [5] in the fact that ordinary elastic modulus E_0 is replaced by equivalent modulus E^*. Therefore, equation (8) is the basis for studying transverse vibrations of axially-moving flexible one-dimensional bodies, elastic characteristics which are described by arbitrary viscoelastic law, for example, Kelvin-Voigt, Maxwell, Maxwell-Kelvin and others. Since Drive belts are made from high-polymeric materials, that expedient representation of their viscoelastic behavior is Kelvin-Voigt model [3]. According to this model the properties of material are given in the form of two in parallel connected viscous and elastic elements. As two elements of model are parallel, deformations in them are identical $\varepsilon(t) = \varepsilon_1(t) = \varepsilon_2(t)$. Then the normal stress is equal to the sum of stresses in each element $\sigma(t) = \sigma_1(t) + \sigma_2(t)$.

Taking into account that $\sigma_1(t) = E_0\varepsilon(t)$ and $\sigma_2(t) = \eta\varepsilon_t(t)$, where η is the coefficient of dynamic viscosity, we obtain the differential ratio, that connects stress and strain in case of viscoelastic Kelvin-Voigt model and equivalent Young's modulus

$$\sigma(t) = E_0\varepsilon(t) + \eta\frac{\partial\varepsilon(t)}{\partial t}; \quad E^* = E_0\left(1 + \frac{\eta}{E_0}\frac{\partial}{\partial t}\right). \tag{9}$$

Given the (9) differential equation of transverse vibrations of flexible moving belt becomes

$$\frac{\partial^2 u}{\partial t^2} + 2v\frac{\partial^2 u}{\partial x\partial t} - (\alpha^2 - v^2)\frac{\partial^2 u}{\partial x^2} = \lambda\left(\tilde{E}\left(\frac{\partial u}{\partial x}\right)^2\frac{\partial^2 u}{\partial x^2} + 2\tilde{\eta}\frac{\partial^2 u}{\partial x^2}\frac{\partial^2 u}{\partial x\partial t}\frac{\partial u}{\partial x} + \tilde{\eta}\frac{\partial^3 u}{\partial x^2\partial t}\left(\frac{\partial u}{\partial x}\right)^2\right), \tag{10}$$

where $\tilde{E} = \frac{3E_0}{2\rho\lambda}$; $\tilde{\eta} = \frac{\eta}{\rho\lambda}$; $\alpha^2 = \frac{T}{A\rho}$; λ is a small parameter, which in the right hand of equation means small value of nonlinear force component compared to recovery force.

Method of solving. The problem is to find the solution of equation (10) with boundary conditions (4). The specified problem belongs to the class of weakly nonlinear, which allows using general principles of perturbation methods for construction of its solution [10]. According to one of these methods, in particular Krylov-Bogolyubov-Mitropolsky's method, we present the solution of equation (10) in the form of asymptotic series [5, 10]

$$u(x, t) = U_0(a, x, \psi) + \lambda U_1(a, x, \psi) + \lambda^2 U_2(a, x, \psi) + \lambda^3 \ldots, \tag{11}$$

where $\psi = \omega t + \varphi$; φ is the initial phase of oscillation; a is amplitude; ω is frequency; $U_0(a, x, \psi)$ is the solution of lineal (when $\varepsilon = 0$) analogue of problem (10), (4); $U_1(a, x, \psi)$, $U_2(a, x, \psi)$ are unknown 2π-periodic on ψ functions, which satisfy the boundary condition, that derive from (4), that is

$$U_1(a, x, \psi)\big|_{x=0} = 0; \quad U_1(a, x, \psi)\big|_{x=l} = 0. \tag{12}$$

Having used the results of the linear axially-moving models of one-dimensional bodies study, that are obtained in [5] we define the function $U_0(a, x, \psi)$.

The single-frequency solution of the nonperturbed equation ($\varepsilon = 0$), which corresponds to (10), is interpreted in this work as a superposition of two waves (reflected and direct) with wave numbers κ and χ respectively

$$U_0(t, x, y) = a(\cos(\kappa x + \omega t + \varphi) - \cos(\chi x - \omega t - \varphi)), \tag{13}$$

where

$$\kappa = \frac{k\pi(\alpha + v)}{\alpha l}; \quad \chi = \frac{k\pi(\alpha - v)}{\alpha l};$$
$$\chi = \frac{k\pi(\alpha - v)}{\alpha l}; \quad k = 1, 2, \ldots.$$

These waves have different lengths, identical initial phases, amplitudes and frequencies.

Due to their finite length driving belts can be referred to the so-called systems with limited size. It is considered that in such systems, nonlinear forces influence only laws of change amplitude and frequency of the dynamic process in time [5, 7, 10]. We specify these laws, in the first approximation, by using ordinary differential equations which are the following [5–7]

$$\frac{da}{dt} = \lambda\Lambda(a); \quad \frac{d\psi}{dt} = \omega + \lambda\Xi(a), \tag{14}$$

where the right sides, in other words functions $\Lambda(a)$, $\Xi(a)$ can be found so that formula (11) satisfies the original equation (10) with the required degree of accuracy, if to substitute functions of time for the place of parameters a and ψ, which are determined by differential equations (14).

Considering the above-mentioned, the single-solution approach to boundary problem (10), (4) in the first approximation can be represented as

$$u(x, t) = a(\cos(\kappa x + \psi) - \cos(\chi x - \psi)) + \lambda U_1(a, x, \psi). \tag{15}$$

On function $U_1(a, x, \psi)$ and its partial derivatives on ψ and x including to the second order the additional conditions are imposed, namely: these function must not contain the items proportional to the principal harmonicas in expansions, that

$$\int_0^{2\pi} U_1(a, x, \psi)\begin{Bmatrix}\cos\psi \\ \sin\psi\end{Bmatrix} d\psi = 0. \tag{16}$$

Twice differentiating by linear and time-variable and taking into account (14), we get

$$\frac{\partial^2 u(x,t)}{\partial t^2} = -\Big(a\left\{2\omega^2 + 2\lambda\omega\Xi(a) + \lambda^2 + \ldots\right\} + 2\omega a\left\{\omega + \lambda\Xi(a)\right\} + \left\{\lambda^2\frac{d\Lambda(a)}{da}\Lambda(a) + \lambda^3\ldots\right\}\Big) \times$$
$$\times\big(\cos(\kappa x + \psi) - \cos(\chi x + \psi)\big) - 2\omega\lambda\Lambda(a)\big(\sin(\kappa x + \psi) + \sin(\chi x + \psi)\big) +$$
$$+\lambda\left(\frac{\partial^2 U_1(a,x,\psi)}{\partial a^2}\left\{\lambda^2\Lambda^2(a) + \lambda^3 + \ldots\right\} + \frac{\partial^2 U_1(a,x,\psi)}{\partial\psi^2}\left\{\omega^2 + 2\lambda\omega\Xi(a) + \lambda^2 + \ldots\right\} + \right.$$

$$+\frac{\partial^2 U_1(a,x,\psi)}{\partial a \partial \psi}\left\{\lambda\omega\Lambda(a)+\lambda^2+\ldots\right\}+$$
$$+\frac{\partial U_1(a,x,\psi)}{\partial a}\left\{\lambda^2\frac{d\Lambda(a)}{da}\Lambda(a)+\lambda^3+\ldots\right\}+$$
$$+\frac{\partial U_1(a,x,\psi)}{\partial \psi}\left\{\lambda^2\frac{d\Xi(a)}{da}\Lambda(a)+\lambda^3+\ldots\right\}\Bigg)+\lambda^2\ldots;$$

$$\frac{\partial^2 u(x,t)}{\partial x^2}=a\big(-\kappa^2\cos(\kappa x+\psi)+\chi^2\cos(\chi x-\psi)\big)+$$
$$+\lambda\frac{\partial^2 U_1(a,x,\psi)}{\partial x^2}+\lambda^2\ldots;$$

$$\frac{\partial^2 u(x,t)}{\partial x\partial t}=\lambda\Lambda(a)\big(\kappa\sin(\kappa x+\psi)-\chi\sin(\chi x-\psi)\big)+$$
$$+a\big(\kappa\cos(\kappa x+\psi)+\chi\cos(\chi x-\psi)\big)\left\{-2a\omega-a\lambda\Xi(a)\right\}-$$
$$-\lambda\left\{\frac{\partial^2 U_1(a,x,\psi)}{\partial x\partial a}\lambda\Lambda(a)+\frac{\partial^2 U_1(a,x,\psi)}{\partial x\partial\psi}\big(\omega+\lambda\Xi(a)\big)\right\}+$$
$$+\lambda^2\ldots.$$

Substituting obtained relationships in equation (10), considering (16), after equating of coefficients in the same powers of λ, we obtain the first approach differential equation, which connects unknown functions $U_1(a, x, \psi)$, $\Lambda(a)$, $\Xi(a)$ with known values

$$L(U_1)=\omega^2\frac{\partial^2 U_1(a,x,\psi)}{\partial\psi^2}+2\nu\omega\frac{\partial^2 U_1(a,x,\psi)}{\partial x\partial\psi}-$$
$$-(\alpha^2-\nu^2)\frac{\partial^2 U_1(a,x,\psi)}{\partial x^2}= \quad (17)$$
$$=f_1(a,x,\psi)+2\Psi(x)\Lambda(a)+2a\Theta(x)\Xi(a),$$

where

$$\Psi(x)=[(\omega+\kappa\nu)\sin(\kappa x)+(\omega-\chi\nu)\sin(\chi x)];$$
$$\Theta(x)=[(\omega+\kappa\nu)\cos(\kappa x)-(\omega-\chi\nu)\cos(\chi x)]);$$
$$f_1(a,x,\psi)=\tilde{E}a^3\big[\kappa\sin(\kappa x+\psi)-\chi\sin(\chi x-\psi)\big]^2\times$$
$$\times\big[\chi^2\cos(\chi x-\psi)-\kappa^2\cos(\kappa x+\psi)\big]+2\tilde{\eta}\omega a^3\times$$
$$\times\big[\chi^2\cos(\chi x-\psi)-\kappa^2\cos(\kappa x+\psi)\big]\times$$
$$\times\big[\kappa\cos(\kappa x+\psi)+\chi\cos(\chi x-\psi)\big]\times$$
$$\times\big[\kappa\sin(\kappa x+\psi)-\chi\sin(\chi x-\psi)\big]+$$
$$+\tilde{\eta}\omega a^3\big[\kappa^2\sin(\kappa x+\psi)+\chi^2\sin(\chi x-\psi)\big]\times$$
$$\times\big[\kappa\sin(\kappa x+\psi)-\chi\sin(\chi x-\psi)\big]^2.$$

The property (16) enables to obtain the system of algebraic equations which connects unknown functions from differential equation (17)

$$\Psi(x)\Lambda(a)+\Theta(x)\Xi(a)=\frac{-\lambda}{2\pi}\int_0^{2\pi}f_1(a,x,\psi)\cos\psi d\psi;$$
$$\Theta(x)\Lambda(a)-a\Psi(x)\Xi(a)=\frac{\lambda}{2\pi}\int_0^{2\pi}f_1(a,x,\psi)\sin\psi d\psi. \quad (18)$$

After averaging over linear variable from the system of equations (18) we obtain

$$\Lambda(a)=\frac{-\lambda l}{4k^2\pi^3(V^2-\alpha^2)}\times$$
$$\times\int_0^l\int_0^{2\pi}f_1(a,x,\psi)\left\{\Psi(x)\cos\psi+\Theta(x)\sin\psi\right\}d\psi dx;$$
$$\Xi(a)=\frac{\lambda l}{4k^2\pi^3 a(V^2-\alpha^2)}\times$$
$$\times\int_0^l\int_0^{2\pi}f_1(a,x,\psi)\left\{\Psi(x)\sin\psi-\Theta(x)\cos\psi\right\}d\psi dx.$$

Hence, in the first approximation, the dynamic process in flexible belt with boundary conditions (4) is described by relation (15) in which parameters a and ψ are defined by ordinary differential equations

$$\frac{da}{dt}=\frac{-\lambda\tilde{\eta}k^4\pi^4\left(7\nu^4+6\nu^2\alpha^2+3\alpha^4\right)}{8l^4\alpha^4\left(\alpha^4-\nu^4\right)}a^3; \quad (19)$$

$$\frac{d\psi}{dt}=\omega+\frac{\lambda\tilde{E}k^3\pi^3\left(7\nu^4+6\nu^2\alpha^2+3\alpha^4\right)}{8l^3\alpha^3\left(\alpha^4+\nu^4\right)}a^2. \quad (20)$$

As is in (20), the first approximation viscoelastic of the material does not affect the frequency of natural oscillations. Then this dependence coincides with the case when taking into account only the elastic properties of the material as a non-linear technical law [7].

Research results. Obtained analytical dependencies enable us to analyze the impact of elasticity module and the coefficient of dynamic viscosity on the main oscillation parameters.

For research the V-belt transmission is selected with four B-type belts of and anode cord, which transfer power from the engine of 10 kW with rotary speed of a shaft of the engine equal 1000 s^{-1}. Belt material density ρ = = 1304 kg/m^3; cross-sectional area of one belt A = 9.2 × × 104 sq.m; force of a preliminary tension of T = 1095 N; length of the site of a belt l = 0.84 m. The coefficient of dynamic viscosity η is connected with Jung's module of dependence where δ is chosen from 0.00001 to 0.01 [4].

In Fig. 1 the influence of the parameter η on the amplitude of oscillations in time range from 0 s to 3600 s for

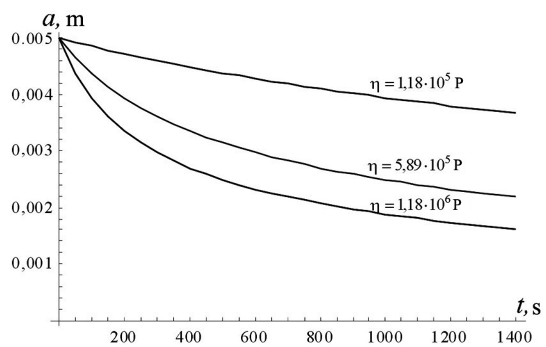

Fig. 1. Dependence of the transverse belt oscillation amplitude a on time t for various values of dynamic viscosity η

$v = 7$ m/s and $E_0 = 1.18 \cdot 10^8$ Pas is illustrated. At the time $t = 1440$ s the amplitude of oscillations is lower by 26 % for $\eta = 1.18 \cdot 10^5$ P, by 56 % for $\eta = 5.89 \cdot 10^5$ P and by 68 % for $\eta = 1.18 \cdot 10^6$ P compared with a given initial amplitude $a_0 = 0.005$ m.

From the graphics in Fig. 1 it is seen that amplitude greatly depends on the coefficient of dynamic viscosity η. For larger values of this parameter amplitude is lower and speed of its attenuation with time is higher.

In Fig. 2 the dependences of the frequency of natural oscillations on the longitudinal movement speed for the initial amplitude of movement $a_0 = 0.005$ m and a different type of cord are shown ($E_0 = 1.8 \cdot 10^8$ Pas for anode cord; $E_0 = 3.6 \cdot 10^8$ Pas for polyester cord and $E_0 = 4.2 \cdot 10^8$ Pas for viscose cord fabric).

The belt material is considered to be linear elastic ($\eta = 0$), because the dependence (20) shows that the coefficient of dynamic viscosity does not affect the frequency of oscillation. For larger values of elasticity modulus E_0 system nonlinearity is stronger and the natural frequency is greater. With increasing speed of longitudinal belt movement the frequency of oscillation decreases (Fig. 2).

Conclusions. Obtained analytical and graphical dependencies enable us to make conclusions about the influence of viscoelastic properties on the main parameters of dynamic process in flexible belts:

1. The speed of the belt longitudinal movement significantly impacts the amplitude-frequency oscillations characteristics. Separately its oscillations frequency increases with the speed of longitudinal movement.

2. The amplitude of fluctuations significantly depends on the viscous belt material characteristics. In case when the viscosity is ignored ($\eta = 0$), amplitude is a constant value in time.

3. The dynamic viscosity, from the first approximation, does not affect the frequency of natural oscillations.

4. The speed of amplitude decrease over time is greater for larger values of η. Having picked up the belt material with considerable viscosity, we can effectively reduce the vibration of amplitude without changing the frequency.

5. The Young's modulus E_0 does not affect the amplitude of oscillation in the first approximation; with E_0 increasing the natural frequency is increasing, too. So the choice of cord material can adjust the oscillation frequency of the belt.

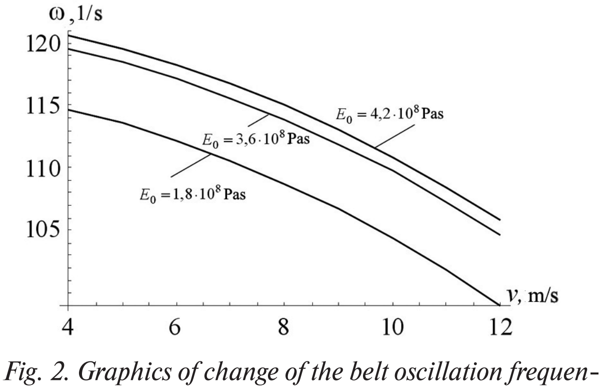

Fig. 2. Graphics of change of the belt oscillation frequency ω depending on the speed of longitudinal movement ν for different values of elasticity modulus E_0

References.

1. Chen, L. H., Zhang, W. and Liu Y. Q., 2007. Modeling of nonlinear oscillations for viscoelastic moving belt using generalized Hamilton's principle. *Journal of Vibration and Acoustics*, 129(1), pp. 128–132.

2. Shpachuk, V. P., 2016. Effect of mutually amplifying action two coordinate shock loading in problems of dynamics of knots of machines. *Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu*, 6(156), pp. 89–94.

3. Chen Li-Qun, Chen Hao and Lim, C. W., 2008. Asymptotic analysis of axially accelerating viscoelastic string. *International Journal of Engineering Science*, 46, pp. 976–985.

4. Gregor Cepona, Lionel Maninb and Miha Boltezara, 2009. Introduction of damping into the flexible multibody belt-drive model: A numerical and experimental investigation, *Journal of Sound and Vibration*, 324, pp. 283–296.

5. Kuzio, I. V., Sokil, B. I., Andruhiv, A. I. and Sokil, M. B., 2009. Asymptotic approximation of wave processes in lengthy bodies which are characterized by longitudinal movement. *The vibrations in engineering and technology*, 3(55), pp. 24–28.

6. Sokil, M. B. and Khytriak, O. I., 2011. The wave theory of motion in the study of vibrations of flexible drive and transport elements taking into account their longitudinal movement. *Military technical collected articles*, 1, pp. 102–105.

7. Khytriak, O. I., 2011. Generalization of the Van der Pol's method on boundary value problems for the two-dimensional Klein-Gordon's equation. *Scientific Bulletin: Collection of scientific works UDLTU*, 21(4), pp. 358–362.

8. Alspaugh, Mark, 2008. *Myers Bulk Material Handling by Conveyor Belt 7*, Society for Mining, Metallurgy and Exploration.

9. Agapov, M. S., Kuznetsov, E. B. and Shalashilin, V. I., 2008. Numerical solution of the strong nonlinear deformation problems in the Euler coordinates. In: *Selected problems of the strength of modern engineering. Collection of scientific articles*, pp. 24–33.

10. Andrianov, I. V. and Avreitsevitch, Y., 2013. *The methods of asymptotic analysis and synthesis of nonlinear dynamics and mechanics of deformable solid body, Institute of computer sciences*, Moscow, Izhevsk: Institute of Computer Studies.

Мета. Створення в'язкопружної динамічної моделі гнучкого поздовжньо-рухомого паса різних механізмів з урахуванням впливу механічних властивостей матеріалу на його динамічні характеристики у разі поперечних коливань.

Методика. Методика дослідження поперечних коливань гнучкого паса базується на асимптотичних методах нелінійної механіки та хвильовій теорії руху.

Результати. У роботі для гнучких в'язкопружних поздовжньо-рухомих пасів отримані аналітичні залежності для визначення амплітуди й частоти поперечних коливань. Проаналізовано вплив в'язких і пружних властивостей матеріалу паса на його амплітудно-частотні характеристики.

Наукова новизна. Полягає в тому, що вперше аналітично на підставі створеної динамічної моделі гнучкого в'язкопружного поздовжнього рухомого паса визначені його динамічні характеристики під час поперечних коливань і досліджено вплив механічних властивостей матеріалу на ці характеристики.

Практична значимість. Запропонована методика дослідження поперечних коливань поздовжньо-рухомих гнучких пасів дозволяє визначити вплив на основні параметри динамічного процесу швидкості поздовжнього руху та в'язкопружних властивостей матеріалу. Отримані аналітичні залежності можуть бути базою для інженерних розрахунків вузлів і механізмів, складовими частинами яких є поздовжньо-рухомі гнучкі паси.

Ключові слова: *математична модель, хвильова теорія руху, в'язкопружність, поперечні коливання, пасова передача, методи збурень*

Цель. Создание вязкоупругой динамической модели гибкого продольно-подвижного ремня различных механизмов с учетом влияния механических свойств материала на его динамические характеристики в случае поперечных колебаний.

Методика. Методика исследования поперечных колебаний гибкого ремня базируется на асимптотических методах нелинейной механики и волновой теории движения.

Результаты. В работе для гибких вязкоупругих продольно-подвижных ремней получены аналитические зависимости для определения амплитуды и частоты поперечных колебаний. Проанализировано влияние вязких и упругих свойств материала ремня на его амплитудно-частотные характеристики.

Научная новизна. Заключается в том, что впервые аналитически на основании созданной динамической модели гибкого вязкоупругого продольно-подвижного ремня определены его динамические характеристики во время поперечных колебаний и исследовано влияние механических свойств материала на эти характеристики.

Практическая значимость. Предложенная методика исследования поперечных колебаний продольно-подвижных гибких ремней позволяет определить влияние на основные параметры динамического процесса скорости продольного движения и вязкоупругих свойств материала. Полученные аналитические зависимости могут быть базой для инженерных расчетов узлов и механизмов, составными частями которых являются продольно-подвижные гибкие ремни.

Ключевые слова: *математическая модель, волновая теория движения, вязкоупругость, поперечные колебания, ременная передача, методы возмущений*

Рекомендовано до публікації докт. техн. наук М. М. Семераком. Дата надходження рукопису 26.04.16.

UDC 621.914.5

M. M. Pidhaietskyi, Cand. Sc. (Tech.), Assoc. Prof., A. R. Aparakin

Kirovohrad National Technical University, Kropyvnytskyi, Ukraine, e-mail: anton.aparakin@gmail.com

DYNAMIC AND KINEMATIC SYNTHESIS OF THE CONTOUR GEAR HOBBING'S SYSTEM

М. М. Підгаєцький, канд. техн. наук, доц., А. Р. Апаракін

Кіровоградський національний технічний університет, м. Кропивницький, Україна, e-mail: anton.aparakin@gmail.com

ДИНАМІЧНИЙ ТА КІНЕМАТИЧНИЙ СИНТЕЗ СИСТЕМИ ЧЕРВ'ЯЧНО-КОНТУРНОГО ЗУБОФРЕЗЕРУВАННЯ

Purpose. Development of the finishing gear hobbing method based on the interaction of the change of the tool cutting edge and the machined surface.

Methodology. The study includes the method of abstraction based on gear hobbing as the interaction of the tool cutting edge with the machined surface. In turn, the properties of the interaction of the tool cutting edge and the machined surface are studied with the help of models under computer simulated conditions.

Findings. The specifications for the finishing gear hobbing method have been assigned. Possible variations of system of the tool cutting edge interaction with the machined surface were shown; the level of stability of the cutting process with specified variations of interaction was determined in terms of computer modeling. A dynamic model of the tool cutting edge interaction with the machined surface has been created. The model considers the nature (geometry) of the interaction, tension, damping and disturbance of technological system (self-adjustment – tool – component part). The directions to increase astatism of system were determined.

Originality. For the first time the gear hobbing modeling method was created which shows the research opportunities of the tool cutting edge interaction with the cutting surface including the machined surface under conditions of both their coincidence, and non-coincidence. For the first time the finishing gear hobbing process of involute surfaces was created in which the cutting surface does not match the determined involute surface and is set at an angle to it.