UDC 536.24

M. H. Berdnyk, Cand. Sc. (Phys.-Math.), Assoc. Prof.

State Higher Educational Institution "National Mining University", Dnipro, Ukraine, e-mail: mgb2006@ukr.net

MATHEMATICAL MODEL OF AND METHOD FOR SOLVING THE NEUMANN GENERALIZED HEAT-EXCHANGE PROBLEM FOR A CYLINDER WITH HOMOGENEOUS LAYERS

М. Г. Бердник, канд. фіз.-мат. наук, доц.

Державний вищий навчальний заклад „Національний гірничий університет", м. Дніпро, Україна, e-mail: mgb2006@ukr.net

МАТЕМАТИЧНА МОДЕЛЬ І МЕТОД РІШЕННЯ УЗАГАЛЬНЕНОЇ ЗАДАЧІ НЕЙМАНА ТЕПЛООБМІНУ КУСКОВО-ОДНОРІДНОГО ЦИЛІНДРА

Purpose. To develop a new generalized mathematical model of temperature distribution in the homogeneous-layer cylinder in the form of the physicomathematical boundary-value problem for the heat conduction equation, and to solve the obtained boundary-value problem.

Methodology. To use the known Laplace and Fourier finite integral transformations, and to apply the obtained new integral transformation to the homogeneous-layer space.

Findings. A non-stationary temperature field in the solid circular cylinder with outside radius R in the polar coordinate system (r, φ), layers of which were homogeneous to the direction of the polar radius r, and which rotated with a constant angular velocity around the axis OZ, was defined with taking into account finite velocity of the heat conduction. The heat-transfer properties of each of the layers do not depend on the temperature in case of ideal contact between the layers and if no internal sources of the heat are available. At the initial moment of time, the cylinder temperature is constant G_0, and heat flow $G(\varphi)$on the outside surface of the cylinder is known.

Originality. The mathematical model of the distribution of the temperature field in a piecewise homogeneous cylinder in the form of the Neumann boundary-value problem for the hyperbolic heat conduction equation was developed for the first time. A new integral transformation was created for the space with homogeneous layers, with the help of which it became possible to present a temperature field in the solid homogeneous-layer circular cylinder in the form of convergence orthogonal series by Bessel and Fourier functions.

Practical value. The obtained solution of the generalized boundary-value problem of heat exchange in the rotating cylinder with taking into account finite velocity of the heat conduction can be used for modeling temperature fields occurred in different technical systems (satellites, forming rolls, turbines, etc.).

Keywords: *Neumann boundary-value problem, integral transformation, relaxation time*

The problem statement, analysis of the recent research and publications. The phenomenological theory of heat conduction assumes that velocity of the heat conduction is an infinite value [1]. However, impact of finiteness of velocity value on the heat exchange is essential for the highly-intensive processes occurring, for example, at explosions, high rotational velocities, in supersonic flows, etc. [2, 3].

The [4] shows that in case of generalized Fourier's law of the heat conduction, the energy-transport equation is true for the one-dimensional, homogeneous and stationary space.

In the [4], a generalized heat conduction equation is presented for the moving element of the solid medium with taking into account finiteness of the velocity and heat-conduction values.

The objective of this work is to develop a new generalized mathematical model of temperature distribution in the homogeneous-layer cylinder in the form of physicomathematical boundary-value problem for the heat conduction equation, and to solve the obtained boundary-value problem, solution of which can be used for controlling the temperature fields.

As review of scientific papers shows, heat exchange in the rotating cylinders has not been studied fully yet [4]. It is shown that numerical methods are not always effective for studying non-stationary non-axis-symmetrical problems of heat-exchange of cylinders, which rotate at high rates [4].

Thus, it is stated in [4] that conditions for reliability of calculations by finite element method and finite difference method, which are used for calculating non-stationary non-axis-symmetrical temperature fields of the rotating cylinders are described by the same characteristics and can be expressed in the following way

$$1-\frac{\Delta F_0}{\Delta\varphi^2}\geq 0;\quad \frac{1}{\Delta\varphi}-\frac{Pd}{2}\geq 0,$$

where F_0 is Fourier's criterion, Pd is Predvoditelev's criterion.

If $Pd = 10^5$ and, consequently, corresponds to the angular velocity $\omega = 1.671$ sec^{-1} of rotation of metal cylinder with the radius of 100 mm, then variables $\Delta\varphi$ and ΔF_o should comply with the following conditions

$$\Delta\varphi \leq 2\cdot 10^{-5};\quad \Delta F_o \leq 2\cdot 10^{-10}.$$

In case of uniformly cooled cylinder when $Bi = 5$ (Bi is criterion Bio) time period needed for temperature to reach 90 % of stationary state is equal to Fo ≈ 0.025 [4]. It means that, within this period of time, at least $1.3\cdot 10^8$ operations should be fulfilled in order to reach the stationary temperature distribution.

Moreover, it should be mentioned that it would be necessary to make $3.14\cdot 10^5$ calculations within one cycle of computation as the inside state of the ring should be characterized by $3.14\cdot 10^5$ points. It is obvious that this number of calculations needed for getting a numerical result is unrealistic.

Therefore, we will employ integral transformations for solving boundary-value problems which occur during mathematical modeling of the non-stationary heat-exchange processes in rotating cylinders.

The problem statement. Let us consider calculation of a non-stationary temperature field in a solid circular cylinder with outside radius R in cylindrical coordinate system (r, φ, z) and with homogeneous layers to the direction of the polar radius r, with taking into account finite velocity of the heat conduction. The cylinder rotates with constant angular velocity ω around the axis OZ. The heat-transfer properties of each of the layers do not depend on the temperature in case of ideal contact between the layers, and provided no internal sources of the heat are available. At the initial moment of time, the cylinder temperature is constant G_0, and the heat flow $G(\varphi)$ on the outside surface of the cylinder is known.

The relative temperature $\theta(\rho, \varphi, t)$ of the cylinder can be expressed in the following way

$$\theta(\rho,\varphi,t)=\begin{cases}\theta_1(\rho,\varphi,t) & \text{if } \rho\in(\rho_0,\rho_1)\\ \theta_2(\rho,\varphi,t) & \text{if } \rho\in(\rho_1,\rho_2)\end{cases}. \quad (1)$$

The relative temperatures $\theta_s(\rho, \varphi, t)$ of the s^{th} layer of the cylinder are calculated by the formulas

$$\theta_s(\rho,\varphi,t)=\frac{T_s(\rho,\varphi,t)-G_0}{T_{\max}-G_0},$$

where $T_s(\rho, \varphi, t)$ is the temperature of the s^{th} layer of the cylinder; $T_{\max}$ is the maximal temperature of the cylinder; $\rho=\dfrac{r}{R}$; $s = 1.2$.

Solution of the problem. In the [4], a generalized heat-conduction equation is presented for the moving element of solid medium with taking into account finiteness of the velocity and heat-conduction values. According to the [4], a generalized equation for the energy balance of a solid body, which rotates with constant angular velocity ω around axis OZ, and whose transfer properties do not depend on the temperature, and no internal sources of the heat are available, can be written in the following way

$$\gamma\, c\left\{\frac{\partial T}{\partial t}+\omega\frac{\partial T}{\partial\varphi}+\tau_r\left[\frac{\partial^2 T}{\partial t^2}+\omega\frac{\partial^2 T}{\partial\varphi\partial t}\right]\right\}= =\lambda\left[\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial\varphi^2}+\frac{\partial^2 T}{\partial z^2}\right], \quad (2)$$

where γ is density of the medium; c is specific heat capacity; λ is heat conductivity coefficient; $T(\rho, \varphi, z, t)$ is the temperature of the medium; t is time; τ_r is relaxation time.

Mathematically, the problem of defining relative temperature $\theta(\rho, \varphi, t)$ (1) of cylinder consists of integration of hyperbolic differential equations (2) of heat conduction into domains

$$D_s = \{(\rho, \varphi, t)\mid \rho\in(\rho_{s-1}, \rho_s),\ \varphi\in(0, 2\pi),\ t\in(0, \infty)\},$$

which, with taking into consideration the accepted assumptions, can be written as

$$\frac{\partial\theta_s}{\partial t}+\omega\frac{\partial\theta_s}{\partial\varphi}+\tau_r\frac{\partial^2\theta_s}{\partial t^2}+\tau_r\omega\frac{\partial^2\theta_s}{\partial\varphi\partial t}= =\alpha_s^2\left[\frac{\partial^2\theta_s}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial\theta_s}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2\theta_s}{\partial\varphi^2}\right] \quad (3)$$

for initial conditions

$$\theta_s(\rho,\varphi,0)=0;\quad \frac{\partial\theta_s(\rho,\varphi,0)}{\partial t}=0; \quad (4)$$

for boundary condition

$$\int_0^t \left.\frac{\partial\theta_2}{\partial\rho}\right|_{\rho=\rho_2} e^{\frac{\zeta-t}{\tau_r}}\, d\zeta = V(\varphi); \quad (5)$$

for condition of ideal heat contact

$$\theta_1(\rho_1, \varphi, t) = \theta_2(\rho_1, \varphi, t); \quad (6)$$

$$\lambda_1\frac{\partial\theta_1(\rho_1,\varphi,t)}{\partial\rho}=\lambda_2\frac{\partial\theta_2(\rho_1,\varphi,t)}{\partial\rho}, \quad (7)$$

and for boundedness condition on the cylinder axis

$$\theta_1(\rho, \varphi, t) < \infty, \tag{8}$$

where $\rho_1 = \frac{R_1}{R}$; $\rho_0 = 0$; $\rho_2 = 1$; R_1 is radius of the layer boundary; λ_s is heat conductivity coefficient; γ_s is density; c_s is specific heat capacity; $a_s = \frac{\lambda_s}{c_s \gamma_s}$ is thermal diffusivity of the s^{th} layer of the cylinder; $\alpha_s^2 = \frac{a_s}{R^2}$; $s = 1.2$; $V(\varphi) = \frac{G(\varphi)\,\tau_r}{\lambda(T_{\max} - G_0)}$; $G(\varphi) \in C(0, 2\pi)$.

In this case, the solution of the boundary-value problem (3–8) $\theta_s(\rho, \varphi, t)$ is twice continuously differentiated by ρ and φ, once – by t in the domain D and continuous on the $\bar{D}$ [5], i.e. $\theta_s(\rho,\varphi,t) \in C^{2,1}(D) \cap C(\bar{D})$, and functions $V(\varphi)$, $\theta_s(\rho, \varphi, t)$ can be decomposed into the Fourier complex series [5]

$$\begin{Bmatrix} \theta_s(\rho,\varphi,t) \\ V(\varphi) \end{Bmatrix} = \sum_{n=-\infty}^{+\infty} \begin{Bmatrix} \theta_{s,n}(\rho,t) \\ V_n \end{Bmatrix} \cdot \exp(in\varphi), \tag{9}$$

where

$$\begin{Bmatrix} \theta_{s,n}(\rho,t) \\ V_n \end{Bmatrix} = \frac{1}{2\pi} \int_0^{2\pi} \begin{Bmatrix} \theta_s(\rho,\varphi,t) \\ V(\varphi) \end{Bmatrix} \cdot \exp(-in\varphi) d\phi, \tag{10}$$

and

$$\theta_{s,n}(\rho,t) = \theta_{s,n}^{(1)}(\rho,t) + i\theta_{s,n}^{(2)}(\rho,t); \quad V_n = V_n^{(1)} + iV_n^{(2)},$$

where i is an imaginary unit.

In view of the fact that $\theta_s(\rho, \varphi, t)$ is a real-valued function, let us confine ourselves by considering only $\theta_{s,n}(\rho, t)$ for $n = 0,1,2, \ldots$, because $\theta_{s,n}(\rho, t)$ and $\theta_{s,-n}(\rho, t)$ are complex conjugates. By putting the values of functions from (9) into (3–8) we can obtain the following system of differential equations

$$\frac{\partial\theta_{s,n}^{(i)}}{\partial t} + \vartheta_n^{(i)}\theta_{s,n}^{(m_i)} + \tau_r \frac{\partial^2\theta_{s,n}^{(i)}}{\partial t^2} + \tau_r \vartheta_n^{(i)} \frac{\partial\theta_{s,n}^{(m_i)}}{\partial t} =$$
$$= \alpha_s^2 \left[\frac{\partial^2\theta_{s,n}^{(i)}}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial\theta_{s,n}^{(i)}}{\partial\rho} - \frac{n^2}{\rho^2}\theta_{s,n}^{(i)} \right],$$

with initial conditions

$$\theta_{s,n}^{(i)}(\rho,0) = 0; \quad \frac{\partial\theta_{s,n}^{(i)}(\rho,0)}{\partial t} = 0,$$

with boundary condition

$$\int_0^t \left.\frac{\partial\theta_{2,n}^{(i)}}{\partial\rho}\right|_{\rho=\rho_2} e^{\frac{\zeta - t}{\tau_r}} d\zeta = V_n^{(i)},$$

with condition of ideal heat contact

$$\theta_{1,n}^{(i)}(\rho_1,t) = \theta_{2,n}^{(i)}(\rho_1,t);$$

$$\lambda_1 \frac{\partial\theta_{1,n}^{(i)}(\rho_1,t)}{\partial\rho} = \lambda_2 \frac{\partial\theta_{2,n}^{(i)}(\rho_1,t)}{\partial\rho},$$

and with boundedness condition on the cylinder axis

$$\theta_{1,n}^{(i)}(\rho,t) < \infty,$$

where $\vartheta_n^{(1)} = -\omega n$; $\vartheta_n^2 = \omega n$; $m_1 = 2$; $m_2 = 1$; $i = 1.2$.

Let us employ the Laplace integral transformation [5] for the system of differential equations (10)

$$\tilde{f}(s) = \int_0^\infty f(\tau)e^{-s\tau}d\tau.$$

As a result, we obtain the following system of ordinary differential equations relating to $\tilde{\theta}_n^{(i)}$

$$s\tilde{\theta}_{s,n}^{(i)} + \vartheta_n^{(i)}\left(\tilde{\theta}_{s,n}^{(m_i)} + \tau_r s\tilde{\theta}_n^{(m_i)}\right) + \tau_r s^2\tilde{\theta}_{s,n}^{(i)} =$$
$$= \alpha_s^2 \left[\frac{d^2\tilde{\theta}_{s,n}^{(i)}}{d\rho^2} + \frac{1}{\rho}\frac{d\tilde{\theta}_{s,n}^{(i)}}{d\rho} - \frac{n^2}{\rho^2}\tilde{\theta}_{s,n}^{(i)} \right], \tag{11}$$

with boundary condition

$$\left.\frac{\partial\tilde{\theta}_n^{(i)}}{\partial\rho}\right|_{\rho=1} = \tilde{V}_n^{(i)}, \tag{12}$$

with condition of ideal heat contact

$$\tilde{\theta}_{1,n}^{(i)}(\rho_1,t) = \tilde{\theta}_{2,n}^{(i)}(\rho_1,t); \tag{13}$$

$$\lambda_1 \frac{\partial\tilde{\theta}_{1,n}^{(i)}(\rho_1,t)}{\partial\rho} = \lambda_2 \frac{\partial\tilde{\theta}_{2,n}^{(i)}(\rho_1,t)}{\partial\rho}, \tag{14}$$

and with boundedness condition on the cylinder axis

$$\tilde{\theta}_{1,n}^{(i)}(\rho,t) < \infty, \tag{15}$$

where $\tilde{V}_n^{(i)} = V_n^{(i)}\left(1 + \frac{1}{s\tau_r}\right)$, $(i = 1.2)$.

In order to solve the boundary-value equation (11–15), let us make an integral transformation

$$\bar{f}(\mu_{n,k}) = \int_{\rho_0}^{\rho_2} \frac{Q_0(\mu_{n,k}\rho)}{\alpha(\rho)}\rho f(\rho)d\rho =$$
$$= \sum_{s=1}^{2} \int_{\rho_{s-1}}^{\rho_s} \frac{Q_s(\mu_{n,k}\rho)}{\alpha_s^2}\rho f(\rho)d\rho, \tag{16}$$

where

$$Q_0(\mu_{n,k}\rho), \alpha(\rho) = \begin{cases} Q_1\left(\frac{\mu_{n,k}}{\alpha_1}\rho\right), \alpha_1^2 & \text{if } \rho \in (\rho_0, \rho_1) \\ Q_2\left(\frac{\mu_{n,k}}{\alpha_2}\rho\right), \alpha_2^2 & \text{if } \rho \in (\rho_1, \rho_2) \end{cases}.$$

Own functions $Q(\mu_{n,k}\rho)$ and own values can be defined by solving the Sturm–Liouville problem

$$\frac{d^2Q_s}{d\rho^2} + \frac{1}{\rho}\frac{dQ_s}{d\rho} - \frac{n^2}{\rho^2} + \frac{\mu_{n,k}^2}{\alpha_s^2}Q_s = 0; \tag{17}$$

$$Q_1(0)<\infty,\quad \frac{\partial Q_2\left(\frac{\mu_{n,k}}{\alpha_2}\rho_2\right)}{\partial\rho}=0; \tag{18}$$

$$Q_1\left(\frac{\mu_{n,k}}{\alpha_1}\rho_1\right)=Q_2\left(\frac{\mu_{n,k}}{\alpha_2}\rho_1\right); \tag{19}$$

$$\lambda_1\frac{\partial Q_1\left(\frac{\mu_{n,k}}{\alpha_1}\rho_1\right)}{\partial\rho}=\lambda_2\frac{\partial Q_2\left(\frac{\mu_{n,k}}{\alpha_2}\rho_1\right)}{\partial\rho}, \tag{20}$$

where $s = 1.2$.

By solving the Sturm—Liouville problem (17—20) we obtain

$$Q_1\left(\frac{\mu_{n,k}}{\alpha_1}\rho\right)=\frac{J_n\left(\frac{\mu_{n,k}}{\alpha_1}\rho\right)}{J_n\left(\frac{\mu_{n,k}}{\alpha_1}\rho_1\right)}; \tag{21}$$

$$Q_2\left(\frac{\mu_{n,k}}{\alpha_2}\rho\right)=\frac{\Psi\left(\frac{\mu_{n,k}}{\alpha_2}\rho\right)}{\Psi\left(\frac{\mu_{n,k}}{\alpha_2}\rho_1\right)},$$

where

$$\Psi\left(\frac{\mu_{n,k}}{\alpha_2}\rho\right)=\frac{\mu_{n,k}}{\alpha_2}\left[Y_n'\left(\frac{\mu_{n,k}}{\alpha_2}\rho_2\right)J_n\left(\frac{\mu_{n,k}}{\alpha_2}\rho\right)-J_n'\left(\frac{\mu_{n,k}}{\alpha_2}\rho_2\right)Y_n\left(\frac{\mu_{n,k}}{\alpha_2}\rho\right)\right],$$

where J_n, Y_n are the Bessel functions of the 1st and 2nd types of the nth order, correspondingly [4].

The own values $\mu_{n,k}$ can be defined by solving the transcendental equation

$$\frac{\mu_{n,k}J_n'\left(\frac{\mu_{n,k}}{\alpha_1}\rho_1\right)}{\alpha_1 J_n\left(\frac{\mu_{n,k}}{\alpha_1}\rho_1\right)}=\sigma\frac{H\left(\frac{\mu_{n,k}}{\alpha_2}\rho_1\right)}{\Psi\left(\frac{\mu_{n,k}}{\alpha_2}\rho_1\right)}, \tag{22}$$

where

$$H\left(\frac{\mu_{n,k}}{\alpha_2}\rho\right)=\frac{\mu_{n,k}}{\alpha_2}\left[Y_n'\left(\frac{\mu_{n,k}}{\alpha_2}\rho_2\right)J_n'\left(\frac{\mu_{n,k}}{\alpha_2}\rho\right)-J_n'\left(\frac{\mu_{n,k}}{\alpha_2}\rho_2\right)Y_n'\left(\frac{\mu_{n,k}}{\alpha_2}\rho\right)\right];\quad \sigma=\frac{\lambda_2\alpha_1}{\lambda_1\alpha_2}.$$

The formula of the inverse transformation can be written as

$$f(\rho)=\sum_{n=1}^{\infty}\frac{Q_0(\mu_{n,k}\rho)}{\left\|Q_0(\mu_{n,k}\rho)\right\|^2}\bar f(\mu_{n,k}), \tag{23}$$

where

$$\left\|Q_0(\mu_{n,k}\rho)\right\|^2=\frac{\rho_1^2}{2\alpha_1^2}\left\{\left(1-\frac{n^2\alpha_1^2}{\mu_{n,k}^2\rho_1^2}\right)+\left[\frac{\mu_{n,k}J_n'\left(\frac{\mu_{n,k}}{\alpha_1}\rho_1\right)}{\alpha_1 J_n\left(\frac{\mu_{n,k}}{\alpha_1}\rho_1\right)}\right]^2\right\}+$$
$$+\frac{\rho_2^2}{2\alpha_2^2}\left(1-\frac{n^2\alpha_2^2}{\mu_{n,k}^2\rho_2^2}\right)\left[\frac{\Psi\left(\frac{\mu_{n,k}}{\alpha_2}\rho_2\right)}{\Psi\left(\frac{\mu_{n,k}}{\alpha_2}\rho_1\right)}\right]-$$
$$-\frac{\rho_1^2}{2\alpha_2^2}\left\{\left(1-\frac{n^2\alpha_2^2}{\mu_{n,k}^2\rho_1^2}\right)+\left[\frac{H\left(\frac{\mu_{n,k}}{\alpha_2}\rho_1\right)}{\Psi\left(\frac{\mu_{n,k}}{\alpha_2}\rho_1\right)}\right]^2\right\}.$$

By applying the integral transformation (16), where its own functions $Q_s(\mu_{n,k}\rho)$ are defined by formulas (20, 21), and its own values can be defined by solving the transcendental equation (22), to the system of integral equations (11) and by with taking into accounts (1) we can obtain the following system of ordinary algebraic equations relating to $\bar{\bar{\theta}}_n^{(i)}$

$$s\bar{\bar{\theta}}_n^{(i)}+\vartheta_n^{(i)}\left(\bar{\bar{\theta}}_n^{(m_i)}+\tau_r s\bar{\bar{\theta}}_n^{(m_i)}\right)+\tau_r s^2\bar{\bar{\theta}}_n^{(i)}=$$
$$=q_{n,k}\left(\frac{Q_2\left(\frac{\mu_{n,k}}{\alpha_2}\right)\tilde V_n^{(i)}}{\mu^2_{n,k}}-\bar{\bar{\theta}}_n^{(i)}\right), \tag{24}$$

where $i = 1.2$; $q_{n,k}=\mu_{n,k}^2$.

By solving the system of equations (24) we obtain

$$\tilde{\bar{\theta}}_n^{(i)}=\alpha_{n,k}\frac{\tilde V_n^{(i)}v_{n,k}+(-1)^{i+1}\omega n\tilde V_n^{(m_i)}(1+s\tau_r)}{(v_{n,k})^2+\omega^2n^2(1+s\tau_r)^2}, \tag{25}$$

where $i = 1.2$; $\alpha_{n,k}=Q_2\left(\frac{\mu_{n,k}}{\alpha_2}\right)$; $v_{n,k}=\tau_r s^2+s+q_{n,k}$.

By applying the Laplace formulas of inverse transformation to the expression of the function (25) we obtain the original functions

$$\bar\theta_n^{(1)}(\mu_{n,k},t)=\sum_{j=1}^{2}\zeta_{n,k}(s_j)\left\{\tilde V_n^{(1)}(s_j)\cdot\left[(2\tau_r s_j+1)+\tau_r\omega ni\right]+\right.$$
$$\left.+\tilde V_n^{(2)}(s_j)\cdot\left[\tau_r\omega n-(2\tau_r s_j+1)i\right]\right\}(e^{s_jt}-1)+$$
$$+\sum_{j=3}^{4}\zeta_{n,k}(s_j)\left\{\tilde V_n^{(1)}(s_j)\cdot\left[(2\tau_r s_j+1)-\tau_r\omega ni\right]+\right.$$
$$\left.+\tilde V_n^{(2)}(s_j)\cdot\left[\tau_r\omega n+(2\tau_r s_j+1)i\right]\right\}(e^{s_jt}-1); \tag{26}$$

$$\begin{aligned}\overline{\theta}_n^{(2)}(\mu_{n,k},t) &= \sum_{j=1}^{2}\zeta_{n,k}(s_j)\Big\{\tilde{V}_n^{(2)}(s_j)\cdot\Big[\big(2\tau_r s_j+1\big)+\tau_r\omega ni\Big]- \\ &-\tilde{V}_n^{(1)}(s_j)\cdot\Big[\tau_r\omega n-(2\tau_r s_j+1)i\Big]\Big\}(e^{s_jt}-1)+ \\ &+\sum_{j=3}^{4}\zeta_{n,k}(s_j)\Big\{\tilde{V}_n^{(2)}(s_j)\cdot\Big[(2\tau_r s_j+1)-\tau_r\omega ni\Big]- \\ &-\tilde{V}_n^{(1)}(s_j)\cdot\Big[\tau_r\omega n+(2\tau_r s_j+1)i\Big]\Big\}(e^{s_jt}-1),\end{aligned} \qquad (27)$$

where $\zeta_{n,k}(s_j)=\dfrac{0.5s_j^{-1}\alpha_{n,k}}{(2\tau_r s_j+1)^2+(\tau_r\omega n)^2}$, and the values s_j for $j = 1, 2, 3, 4$ are defined by formulas

$$s_{1,2}=\frac{(\tau_r\omega ni-1)\pm\sqrt{(1+\tau_r\omega ni)^2-4\tau_r q_{n,k}}}{2\tau_r};$$

$$s_{3,4}=\frac{(\tau_r\omega ni+1)\pm\sqrt{(1-\tau_r\omega ni)^2-4\tau_r q_{n,k}}}{2\tau_r}.$$

By this way and by taking into account formulas of inverse transformation (9) and (23), we obtain a temperature field of the homogeneous-layer circular cylinder to the direction of the polar radius, which rotates with constant angular velocity ω around the axis *OZ*, with taking into account finite velocity of the heat conductivity

$$\theta(\rho,\varphi,t)=\sum_{n=-\infty}^{+\infty}\left\{\sum_{k=1}^{\infty}\Big[\overline{\theta}_n^{(1)}(\mu_{n,k},t)+i\cdot\overline{\theta}_n^{(2)}(\mu_{n,k},t)\Big]\times\frac{Q_0(\mu_{n,k}\rho)}{\|Q_0(\mu_{n,k}\rho)\|^2}\right\}\cdot\exp(in\varphi),$$

where values of $\overline{\theta}_n^{(1)}(\mu_{n,k},t)i\overline{\theta}_n^{(2)}(\mu_{n,k},t)$ are defined by the formulas (26, 27).

Conclusions. With the help of the new developed integral transformation, it is possible to define a temperature field of the homogeneous-layer circular cylinder, to the direction of the polar radius, which rotates with constant angular velocity ω around the axis *OZ*, with taking into account the finite velocity of the heat conductivity and in the form of convergence orthogonal series by Bessel and Fourier functions. The obtained analytical solution of the generalized boundary-value problem of heat exchange in the rotating cylinder with taking into account finite velocity of the heat conduction can be used for modeling temperature fields which occur in different technical systems (satellites, forming rolls, turbines, etc.).

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Мета. Розробка нової узагальненої математичної моделі температурних розподілів у кусково-однорідному циліндрі у вигляді крайової задачі математичної фізики для рівняння теплопровідності й розв'язання отриманої крайової задачі.

Методика. Застосовування відомих інтегральних перетворень Лапласа, Фур'є, а також розробленого нового інтегрального перетворення для кусково-однорідного простору.

Результати. Знайдено нестаціонарне температурне поле суцільного кругового циліндра зовнішнього радіуса R у полярній системі координат (r, φ), кусково-однорідного в напрямку полярного радіуса r, що обертається з постійною кутовою швидкістю навколо вісі *OZ*, з урахуванням кінцевої швидкості поширення тепла. Теплофізичні властивості в кожному шарі не залежать від температури за умови ідеального теплового контакту між шарами, а внутрішні джерела тепла відсутні. У початковий момент часу температура циліндра постійна G_0, а на зовнішній поверхні циліндра відомий тепловий потік $G(\varphi)$.

Наукова новизна. Уперше розроблена математична модель розподілу температурного поля в кусково-однорідному циліндрі у вигляді крайової задачі Неймана для гіперболічного рівняння теплопровідності . Уперше розроблено нове інтегральне перетворення для кусково-однорідного простору, за допомогою якого знайдено температурне поле суцільного кусково-однорідного кругового циліндра у вигляді збіжних ортогональних рядів за функціями Бесселя й Фур'є.

Практична значимість. Знайдений аналітичний розв'язок узагальненої крайової задачі теплообміну циліндра, що обертається, з урахуванням скінченності величини швидкості поширення тепла може знайти застосування при модулюванні температурних полів, які виникають у багатьох технічних системах (у супутниках, прокатних валках, турбінах і т. і.).

Ключові слова: *крайова задача Неймана, інтегральне перетворення, час релаксації*

Цель. Разработка новой обобщенной математической модели температурных распределений в кусочно-однородной цилиндре в виде краевой задачи математической физики для уравнения теплопроводности и решение полученной краевой задачи.

Методика. Применение известных интегральных преобразований Лапласа, Фурье, а также разработанного нового интегрального преобразования для кусочно-однородного пространства.

Результаты. Найдено нестационарное температурное поле сплошного кругового цилиндра внешнего радиуса R в полярной системе координат, кусочно-однородного в направлении полярного радиуса *r*, который вращается с постоянной угловой скоростью вокруг оси *OZ*, с учетом конечной скорости распространения тепла. Теплофизические свойства в каждом слое не зависят от температуры при идеальном тепловом контакте между слоями, а внутренние источники тепла отсутствуют. В начальный момент времени температура цилиндра постоянная, а на внешней поверхности цилиндра известный тепловой поток.

Научная новизна. Впервые разработана математическая модель распределения температурного поля в кусочно-однородной цилиндре в виде краевой задачи Неймана для гиперболического уравнения теплопроводности. Впервые разработано новое интегральное преобразование для кусочно-однородного пространства, с помощью которого найдено температурное поле сплошного кусочно-однородного кругового цилиндра в виде сходящихся ортогональных рядов по функциям Бесселя и Фурье.

Практическая значимость. Найденное аналитическое решение обобщенной краевой задачи теплообмена цилиндра, что вращается, с учетом конечности величины скорости распространения тепла, может найти применение при моделировании температурных полей, которые возникают во многих технических системах (в спутниках, прокатных валках, турбинах и т. д.).

Ключевые слова: *краевая задача Неймана, интегральное преобразование, время релаксации*

Рекомендовано до публікації докт. техн. наук М. О. Алексєєвим. Дата надходження рукопису 04.06.16.

UDC 681.5:519.2:622.778

I. K. Mladetsky[1], Dr. Sc. (Tech.), Prof.,
Ya. H. Kuvaiev[1], Cand. Sc. (Tech.), Assoc. Prof.,
N. S. Pryadko[2], Dr. Sc. (Tech.), Senior Research Fellow

1 – State Higher Education Institution "National Mining University", Dnipro, Ukraine, e-mail: kuvaievig@gmail. com
2 – Institute of Technical Mechanics of National Academy of Sciences of Ukraine, State Space Agency of Ukraine, Dnipro, Ukraine

CONTROL REGULARITIES OF THE USEFUL MINERAL EXTRACTION FROM ORE FEED STREAM WITH BALL GRINDING. CORRELATION ANALYSIS

І. К. Младецький[1], д-р техн. наук, проф.,
Я. Г. Куваєв[1], канд. техн. наук, доц.,
Н. С. Прядко[2], д-р техн. наук, старш. наук. співроб.

1 – Державний вищий навчальний заклад „Національний гірничий університет", м. Дніпро, Україна, e-mail: kuvaievig@gmail. com
2 – Інститут технічної механіки НАН України, Державне Космічне Агентство України, м. Дніпро, Україна

ЗАКОНОМІРНОСТІ КЕРУВАННЯ ВИЛУЧЕННЯМ КОРИСНОГО МІНЕРАЛУ З РУДОПОТОКІВ ІЗ КУЛЬОВИМ ПОДРІБНЕННЯМ. КОРЕЛЯЦІЙНИЙ АНАЛІЗ

Purpose. To define the indicative events characterizing a process state on "useful mineral content in ore –useful mineral content in a concentrate" control channel at an ore concentrating plant with ball grinding using correlation analysis method with the purpose of an operator exclusion out of a control circuit.

Methodology. Correlation analysis of the control object parameters.

Findings. According to the mutually correlation function a range of indicative events is determined which characterize a process state on "useful mineral content in ore –useful mineral content in a concentrate" control channel.

Originality. Comparison of numerical values of recession time and the equivalent delay of autocorrelation and correlation functions of the general iron content and a time constant of a dressing process line obtained in the normal operation of ore concentrating factory with ball grinding has been made out for the first time.

Practical value. The indicative events obtained by the type and parameters of the correlation functions between the content of a useful mineral in the feedstock and concentrate can be used for automatic system creation for situation-dependent process control of ferrous and non-ferrous ore dressing as both at separate ball grinding sections, and at separate plants which include these sections.