UDC 621.923.1

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MATHEMATICAL MODELING OF ABRASIVE GRINDING WORKING PROCESS

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МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ РОБОЧОГО ПРОЦЕСУ АБРАЗИВНОГО ШЛІФУВАННЯ

Purpose. 3D-modeling of working process of abrasive grinding of structural steel by unit grain and the individual grains under different conditions of overlap and altitude.

Methodology. Mathematical modeling of working processes of abrasive grinding of structural steel on steel 45 using the finite element method was performed. An electro-corundum wheel with ceramic binder was used as an abrasive tool. To simulate the surface of the tool the matrix method of coordinate transformation is applied.

Findings. Contact problem of tool interaction of grain and the workpiece has been solved. The problem of modeling the cutting process in a chip formation and, consequently, the formation of new surfaces has been solved. Distribution of plastic strain, temperature, shear strain, strain rate, internal pressure, charts of cutting forces and kinetic energy were obtained.

Originality. A general model of the tool surface whose calculation in MathCAD environment made it possible to obtain a graphical representation of various instrumental surfaces was obtained. As a result of mathematical modeling of working process abrasive grinding by individual grains with different variants of overlapping grains, characteristic curves of cutting forces and other parameters of the working process were obtained.

Practical value. The results can form the basis of new structural steel processing technologies, developing management methodology for workflow of abrasive grinding.

Keywords: *finite element method, abrasive grinding, structural steel, abrasives, corundum, ceramic bond*

Introduction. Final processing has a great impact on the quality of surface layer of parts and, at this regard, on their performance properties. The most common method is the final finishing grinding, which provides high accuracy in the manufacture of parts. But with the use of grinding, fell, cracks, internal tension may appear on the details. That is what causes the need for a comprehensive study of the causes that give rise to such defects, to find the ways of eliminating them.

Today predicting methods for 3D-modeling of abrasive grinding structural steel, which is based on the finite element method, are not paid sufficient attention. Currently, scientific schools that develop scientific bases of modeling and research workflows of cutting and diamond grinding of hard and of fragile materials [1] are working actively.

There is no analysis by finite element method for stress-strain state of contact patch of abrasive wheel and workpiece of plastic material, which makes it impossible to create a complete thermomechanical model of grinding.

In the process of grinding, in the contact area complex physical phenomena occur which relate to heat transfer and mechanical interaction of bodies; so to improve abrasive tools and methods for their treatment is a difficult and responsible task.

That is why to solve this problem we used software packages that implement the finite element method to study the “tie-grain-workpiece” system.

Analysis of the recent research. Grinding of details is a complex scientific and applied problem that has significant economic importance for many industries.

Modern software based on advanced mathematical modeling technology can solve complex analytical problems [2]. Grinding is actively implemented into production, so modeling of processes has found its use in abrasive processing through the works of research centers and teams of scientists, including: A. I. Hrabchenko, V. L. Dobroskok, V. A. Fedorovych; H. A. Petasiuk; O. F. Yenikyeiev [3], H. V. Sokolovska, T. L. Shcherbak [3], Ya. O. Shakhbazov, V. A. Storoschuk, I. M. Hriner and others. However, despite the above mentioned works, we can witness the lack of attention to the mathematical analysis of structural steels by the finite element method.

Unsolved aspects of the problem. Considering the need for a complete thermomechanical model of grinding of structural steel, a general mathematical model that describes the tool surface of various cutting tools has been further developed.

Objectives of the article. 3D-modeling of working process of abrasive grinding of structural steel by unit grain and the individual grains under different conditions of overlap and altitude.

Presentation of the main research. Much attention has been paid to optimization of products and production processes in the industry in recent years. Therefore, technologies of virtual modeling that allow performing optimization in the short term and at the lowest cost using a full-scale experiment as a verified one have been advanced considerably. Due to this, a detail with the optimal size, properties and cost can be obtained even at the technology design stage. And only small adjustments are required at the implementation phase of the process.

The present paper describes the methodology of implementing virtual grinding treatment using numerical methods, including the finite element method. The need for numerical methods is due to the impossibility of the analytical solution of differential equations of plasticity theory in partial derivatives in general form for any geometrically complex areas.

According to sources [1, 2, 4] currently several methods for numerical modeling of deformation processes and thermal conductivity are known: finite difference method (FDM), finite element method (FEM), boundary element method (BEM). They include digitization of space by imposing the mesh. Due to problems with mesh distortion through large deformations in these methods, mesh-free methods (MFM) have been developed and are beginning to be applied: smoothed particle hydrodynamics method (SPH), element free Galerkin method (EFG) and others.

These numerical methods have a single principle in common: discretization of an area (division into separate parts of forms known in advance) where the problem is solved, analytical solution of differential equations in each of these areas, combining the solutions considering boundary conditions in the system of equations, that contain unknown displacement and temperature at characteristic points (nodes) in the region, solving the system of equations.

The basis of the finite element method (FEM) involves discretization of the study area by elementary geometric figures – finite elements – as triangles and quadrangles in the plane; tetrahedra and parallelepipeds in space. Despite the fact that the form of finite elements can be quite arbitrary, and their edges are not necessarily straight segments, all finite element of the study area are interconnected at the nodes. This is due to the unknown value of these nodes that analytical expression is developed for displacements, strains, stresses and temperatures at any point inside the field of a finite element, and, hence, of the entire study area.

Differentiating these expressions for each finite element and substituting them in the original system of differential equations we obtain a system of equations for the unknown values at the key points.

However, the realization of FEM has certain known difficulties, the most significant of which is the complexity of building and maintaining quality in the calculation of finite element meshes that is the mesh formed by finite elements of correct form. The more distorted the form of finite elements is, the bigger the calculation error for pressure and temperature fields is in it.

To achieve the purpose it is necessary to explore the "tie-grain-workpiece" system (with different locations of grains) as mathematical modeling, using the following software:

- SolidWorks 2013 – for simulation of 3D-model of "tie-grain-workpiece";
- ANSYS WorkBench 14.0 – to create a finite-element mesh;
- LS-PREPOST – to enter the data and analyze the results;
- LS-DYNA – for direct calculation of the three-dimensional non-linear thermo-mechanical problems.

Simulation of abrasive processing is performed in the following order [1]:

1. Creating 3D-models of individual elements of the "tie-grain-workpiece" system.
2. Compiling created elements in a single system.
3. Creating a finite-element mesh.
4. Importing 3D-model into pre-postprocessor LS-PREPOST.
5. Data input – parameters of contact interaction between the elements, properties of materials, cutting conditions, etc.
6. Modeling abrasive processing in the established system. Analysis of the results.

To understand the process of modeling it is necessary to conduct modular 3D-modeling of theoretical tool surfaces. It is necessary use the coordinate transformation matrix [2, 5].

The radius vector of coordinate origin

$$e_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$

Matrices of coordinate transformations when moving forward along the axes x, y, z, respectively

$$A1(q) = \begin{bmatrix} 1 & 0 & 0 & q \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; A2(q) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & q \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};$$

$$A3(q) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Matrices of coordinate transformations during rotation around the axes x, y, z, respectively

$$A4(q) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(q) & -\sin(q) & 0 \\ 0 & \sin(q) & \cos(q) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};$$

$$A5(q)=\begin{bmatrix}\cos(q) & 0 & \sin(q) & 0\\ 0 & 1 & 0 & 0\\ -\sin(q) & 0 & \cos(q) & 0\\ 0 & 0 & 0 & 1\end{bmatrix};$$

$$A6(q)=\begin{bmatrix}\cos(q) & -\sin(q) & 0 & 0\\ \sin(q) & \cos(q) & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}.$$

Thus, using the general equation we can describe any instrumental surface [6]

$$Rp(\theta,\psi) = A5(\theta)\cdot A3(S)\cdot A2(R1)\cdot A6(\psi)\cdot A2(\rho)\cdot e^4, \quad (1)$$

where θ, ψ, S, $R1$, ρ are parameters of moving along and around the axis.

Now in MathCAD 14 modifying certain parts of equation (1) we can directly create three-dimensional models of tool surface (Fig. 1).

Creation of 3D-models for grinding is performed in SolidWorks 2013. First, the individual elements (workpiece, tie, grain) are created, afterwards they are combined in the overall system (Fig. 2), and then imported into ANSYS WorkBench 14.0 – to create a finite element mesh.

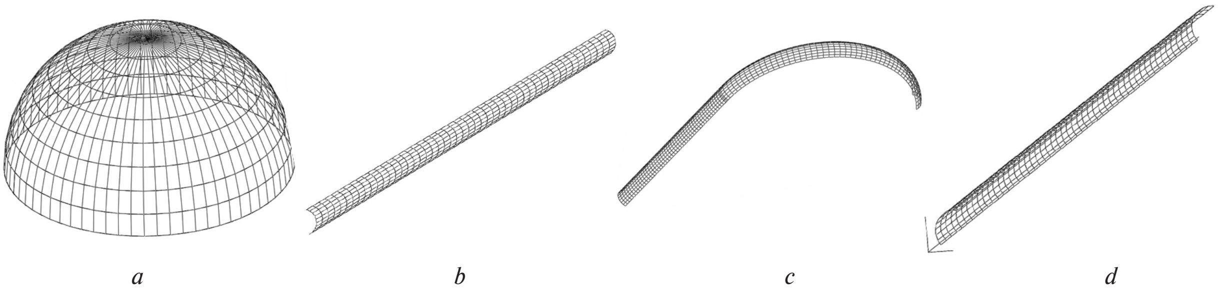
a *b* *c* *d*

Fig. 1. Examples of theoretical tool surfaces of various cutting tools:
a – grain; b – the passing cutter; c – a round cutter; d – bevel mill

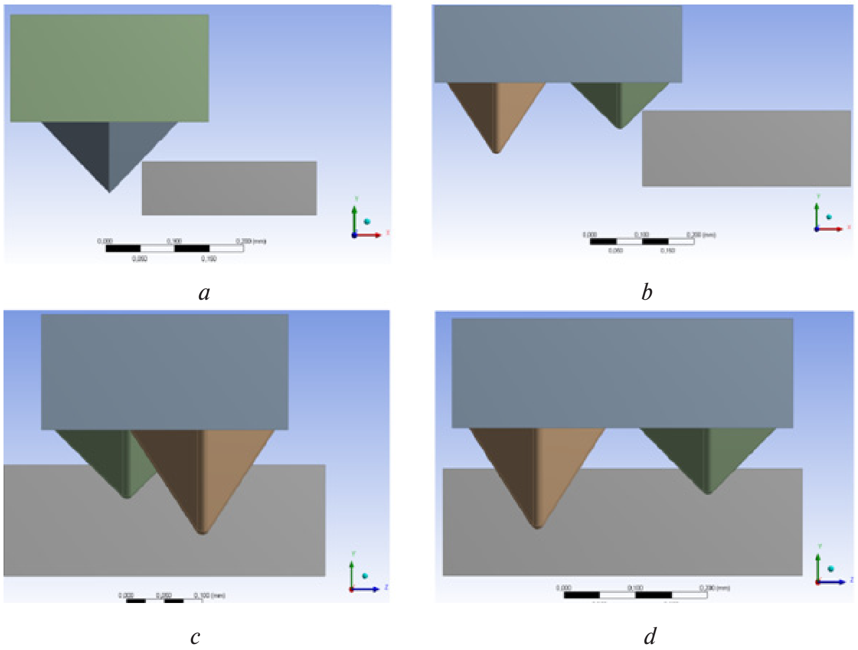
a *b* *c* *d*

Fig. 2. 3D-model of the "tie-grain/grain-workpiece" system:
a – single grain; b – grains are located consistently; c – grains overlap each other; d – grains are located nearby

According to [1] a finite-element mesh with eight nodes (elements-bars) is the most suitable for blanking because a smaller number of units are provided at one and the same precision calculations resulting in less computation time. Unit sizes of 0.005 mm make an optimum size for the power used by a computer.

As for tie and grains, in this case a tetrahedral mesh is chosen, because it is very difficult to perform discretization of a geometric model of abrasive tools by means of elements-bars. Since this paper mostly focuses on blanking, then to reduce the time of calculations it is advisable to increase the size of the grid for ties and grains to 0.03 mm.

When you enter the initial data for the calculation model, one of the most important steps is to choose the mathematical models of materials for the elements of the system and select the type of contact interaction.

Since the cutting tool has hardness by 2–3 times greater than the material being processed, it is advisable to describe it as a completely elastic body [3]; it is appropriate to choose a more simple mathematical model, such as *MAT_ELASTIC. For a workpiece it is necessary to choose a more complex model – *MAT_JOHNSON_COOK. This model considers influence of plastic deformation and temperature at the liquid limit, can be applied for the simulation of processing at cutting speeds when conditions are realized for adiabatic warming and reducing the strength of the material.

For contact interaction *CONTACT_ERODING_NODES_TO_SURFACE algorithm was chosen, because it suits the best to solve the problem of modeling the cutting process in terms of the formation of swarf and, consequently, the formation of new surfaces. Moreover, this algorithm provides the most stable solution of contact problem without "slippage" of nodes through the line contact.

After the calculation, the user can browse the received data in LS-PREPOST. This pre-postprocessor provides a lot of diagrams and charts, including: distribution of plastic deformation, temperature, strain displacement, velocity, strain, internal tensions, graphic of cutting forces, kinetic energy and so on. Some examples of the results are shown in Figs. 3–6.

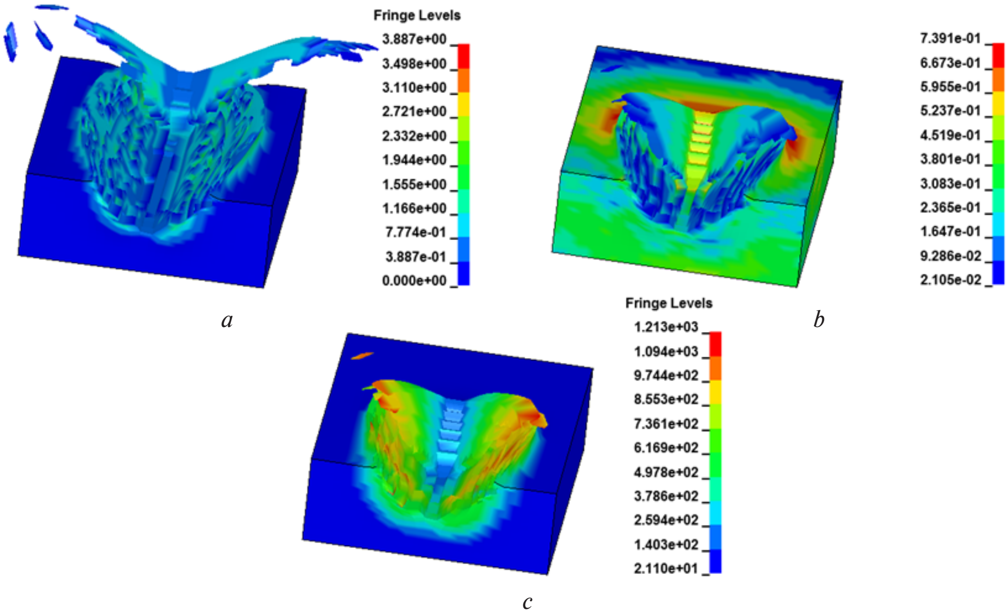

Fig. 3. Diagrams of the results for the workpiece while processing by a single grain: a – plastic deformation; b – shear strain; c – temperature

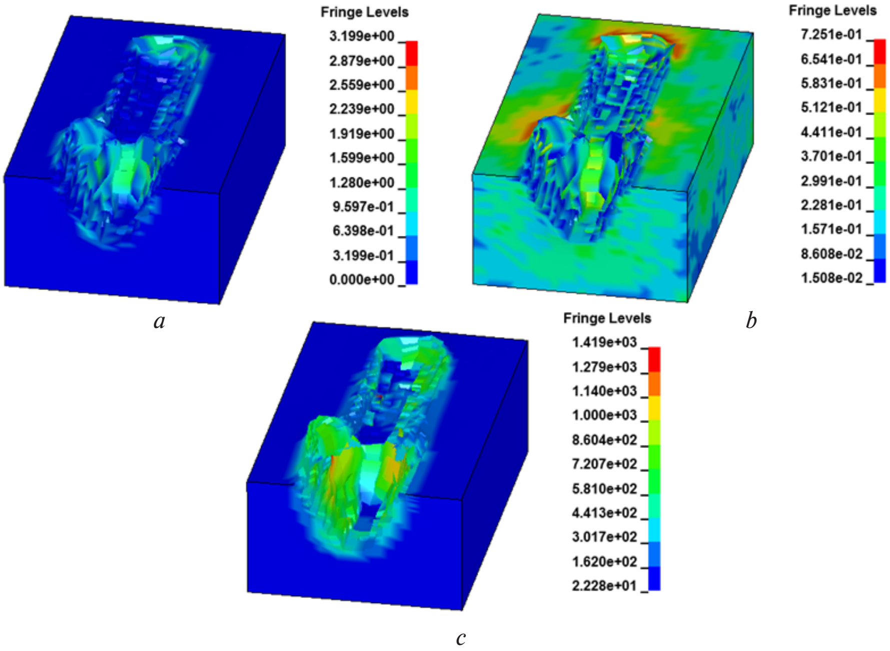

Fig. 4. Diagrams of the results for the workpiece while processing by two separate grains in their sequential arrangement: a – plastic deformation; b – shear strain; c – temperature

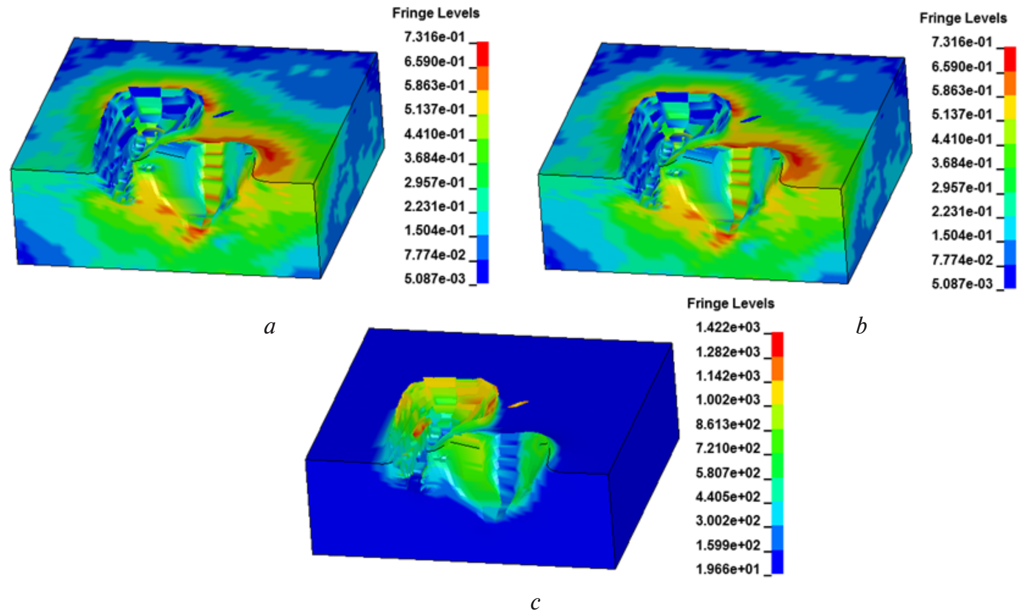

Fig. 5. Diagrams of the results for the workpiece while processing by two separate grains when they overlap each other: a – plastic deformation; b – shear strain; c – temperature

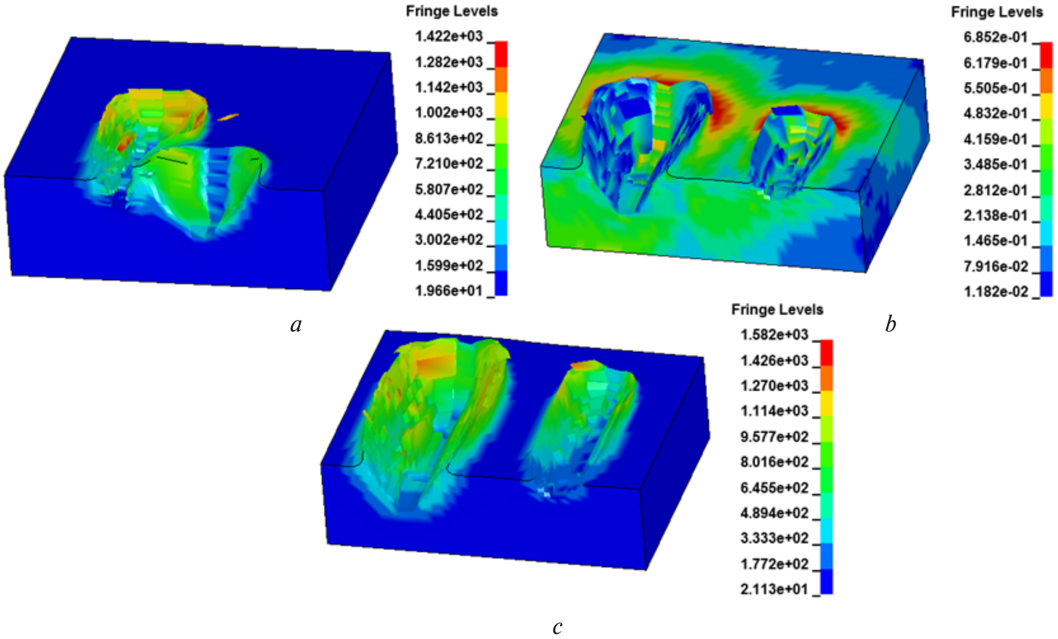

Fig. 6. Diagrams of the results for the workpiece while processing by two separate grains when they are close to each other: a – plastic deformation; b – shear strain; c – temperature

Apart from diagrams, the results of the calculation program LS-PREPOST displays graphs of dependence of resultant cutting force and the total energy from the time. Such diagrams for cutting by a single grain of the grinding wheel are shown in Figs. 7 and 8.

The graph shows that the cutting force on the grain sharply increases to 11 N during the metal cutting by the grain, and then gradually decreases and after 0.004 sec levels off at 1 H, this is due to the beginning of a sustainable process of cutting with removing the micro-chip. Plastic deformation diagram for this case is shown in Fig. 3, *a*.

The total energy dependence on time (Fig. 8) is fully consistent with graphic of cutting force. During the cutting (0–0.004 sec) energy increases sharply and ranges from 10–15 J. After the chip removal (0.004–0.008 sec) energy is gradually increasing to 16–17 J and stabilized at this level throughout the process of cutting by a single grain.

Conclusions and recommendations for further research. FEM is the most relevant method for modeling workflows of grinding, because the results most adequately correspond to experimental research.

For surface modeling of the tool coordinate transformation matrix method proposed by Professor Portman

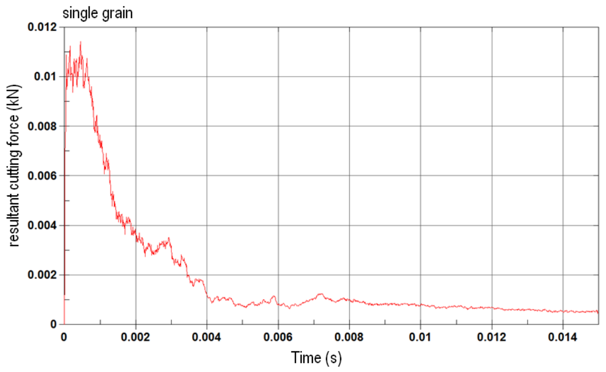

Fig. 7. The resultant cutting force by a single grain

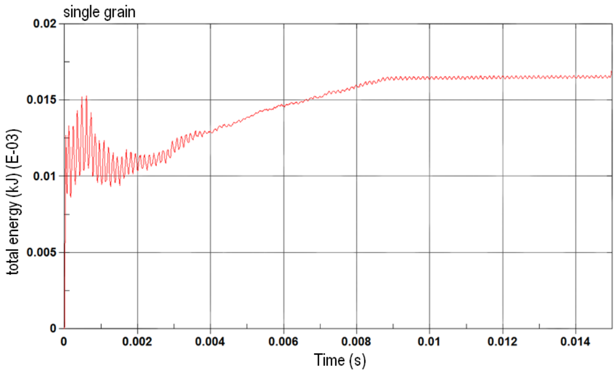

Fig. 8. Total energy while cutting by a single grain

was used. The resulting general model of tool surface, calculation of this model in program MathCAD allowed obtaining a graphical representation of various tool surfaces.

The problem of contact interaction of tool grain and workpiece was solved. The problem of modeling the cutting process in terms of the formation of swarf and, consequently, the formation of new surfaces was solved. The distribution of plastic deformation, temperature, strain displacement, velocity, strain, internal tensions, graphic of cutting forces and kinetic energy has been determined.

As a result of mathematical modeling of grinding by individual abrasive grains with different versions of overlapping grains graphic dependences for cutting forces and other indicators of workflow were obtained.

The results obtained can be the basis of new technologies of processing structural steel, designing methodology workflow management for abrasive grinding.

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Мета. 3D-моделювання робочого процесу абразивного шліфування конструкційних сталей одиничним зерном і окремими зернами за різних умов перекриття й висотності.

Методика. Представлене математичне моделювання методом скінчених елементів робочих процесів абразивного шліфування конструкційних сталей на прикладі сталі 45. В якості абразивного інструмента обрано електрокорундовий круг з керамічною зв'язкою. Для моделювання інструментальної поверхні застосований матричний метод перетворення координат.

Результати. Була вирішена контактна задача взаємодії інструментального зерна й заготовки. Розв'язана задача моделювання процесу різання в умовах утворення стружки та, як наслідок, утворення нових поверхонь. Визначено розподіл пластичних деформацій, температури, деформацій зсуву, швидкості деформацій, внутрішнього напруження, отримані графіки сил різання й кінетичної енергії.

Наукова новизна. Отримана загальна модель інструментальної поверхні, обрахунок якої в середовищі MathCAD дозволив отримати графічне представлення різноманітних інструментальних поверхонь. У результаті математичного моделювання робочого процесу абразивного шліфування окремими зернами з різними варіантами перекриття зерен отримані графічні залежності для сил різання та інших показників робочого процесу.

Практична значимість. Отримані результати можуть лягти в основу новітніх технологій обробки конструкційних сталей, розробки методології управління робочим процесом абразивного шліфування.

Ключові слова: *метод скінченних елементів, абразивне шліфування, конструкційні сталі, абразивний інструмент, електрокорунд, керамічна зв'язка*

Цель. 3D-моделирование рабочего процесса абразивного шлифования конструкционных сталей единичным зерном и отдельными зернами при различных условиях перекрытия и высотности.

Методика. Выполнено математическое моделирование методом конечных элементов рабочих процессов абразивного шлифования конструкционных сталей на примере стали 45. В качестве абразивного инструмента использован электрокорундовый круг с керамической связкой. Для моделирования инструментальной поверхности применен матричный метод преобразования координат.

Результаты. Была решена контактная задача взаимодействия инструментального зерна и заготовки. Решена задача моделирования процесса резки в условиях образования стружки и, как следствие, образование новых поверхностей. Определено распределение пластических деформаций, температуры, деформаций сдвига, скорости деформации, внутреннего напряжения, получены графики сил резания и кинетической энергии.

Научная новизна. Получена общая модель инструментальной поверхности, расчет которой в среде MathCAD позволил получить графическое представление различных инструментальных поверхностей. В результате математического моделирования рабочего процесса абразивного шлифования отдельными зернами с различными вариантами перекрытия зерен получены графические зависимости для сил резания и других показателей рабочего процесса.

Практическая значимость. Полученные результаты могут лечь в основу новых технологий обработки конструкционных сталей, разработки методологии управления рабочим процессом абразивного шлифования.

Ключевые слова: *метод конечных элементов, абразивное шлифование, конструкционные стали, абразивный инструмент, электрокорунд, керамическая связи*

Рекомендовано до публікації докт. техн. наук С. Д. Цибулею. Дата надходження рукопису 23.09.16.

UDC 621.833.6

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RESEARCH OF KINEMATICS OF CONTACT INTERACTION OF CYCLOIDAL PROFILES IN GEROTOR GEARING

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ДОСЛІДЖЕННЯ КІНЕМАТИКИ КОНТАКТНОЇ ВЗАЄМОДІЇ ЦИКЛОЇДАЛЬНИХ ПРОФІЛІВ У ЗАЧЕПЛЕННІ ГЕРОТОРНОЇ ПАРИ

Purpose. Determination of laws of movement of contact points of the conjugate profiles of the wheels of a gerotor gearing in the process of relative motion, analytical determination of motion velocities, identification of the nature of friction which results in gearing, identification of profile zones with the highest degree of friction.

Methodology. The research is based on the theory of non-centroidal cycloidal gearings, statements of differential geometry, and the laws of motion of the material point according to the trajectory which is defined by the function.

Findings. The result of the research is the identification of the accurate mathematical relations of the relative velocities of the contact point of two conjugate profiles in gerotor gearing. It was identified that there are two simultaneous velocities of contact points along the cycloidal profile and along the profile of the lantern gear. In this case the velocity vectors are located on the common tangent at the point of conjugation. They are different by module and their index depends on the phase of operation cycle.

Originality. The laws of movement of contact points in gerotor gearing were determined and the changing velocity mode in gearing was reliably established. In the process of research the universal formula of the relationship between the motion velocity of the points along the curved trajectory and its equidistant curve which takes account of trajectory curvature was obtained. It was established that in the interaction of the convex area of the equidistant curve with the cycloidal curve with conjugated lantern there is a sliding friction, while in the generating process of the concave area there is a rolling friction. The linearization functions which allow stabilizing the changing velocity mode were obtained.

Practical value. The research results enabled us to assess the impact of the velocity mode on the degree of friction in the points of conjugation of the profile surfaces, to localize the areas with the highest degree of friction and to find the ways of reducing the destructive impact of friction and to increase the stability of the profiles while gearing. The results may also be used in the process of development of the processing technology of the cycloidal profiles in the conditions of generating process with the linearization function of the velocity of relative generating process of the tool profiles and the workpiece. The linearization will stabilize the load and increase the processing accuracy.

Keywords: *contact point, velocity, gerotor gearing, cycloidal profile*
