ных сталей на примере стали 45. В качестве абразивного инструмента использован электрокорундовый круг с керамической связкой. Для моделирования инструментальной поверхности применен матричный метод преобразования координат.

Результаты. Была решена контактная задача взаимодействия инструментального зерна и заготовки. Решена задача моделирования процесса резки в условиях образования стружки и, как следствие, образование новых поверхностей. Определено распределение пластических деформаций, температуры, деформаций сдвига, скорости деформации, внутреннего напряжения, получены графики сил резания и кинетической энергии.

Научная новизна. Получена общая модель инструментальной поверхности, расчет которой в среде MathCAD позволил получить графическое представление различных инструментальных поверхностей. В результате математического моделирования рабочего процесса абразивного шлифования отдельными зернами с различными вариантами перекрытия зерен получены графические зависимости для сил резания и других показателей рабочего процесса.

Практическая значимость. Полученные результаты могут лечь в основу новых технологий обработки конструкционных сталей, разработки методологии управления рабочим процессом абразивного шлифования.

Ключевые слова: *метод конечных элементов, абразивное шлифование, конструкционные стали, абразивный инструмент, электрокорунд, керамическая связи*

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RESEARCH ON KINEMATICS OF CONTACT INTERACTION OF CYCLOIDAL PROFILES IN GEROTOR GEARING

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ДОСЛІДЖЕННЯ КІНЕМАТИКИ КОНТАКТНОЇ ВЗАЄМОДІЇ ЦИКЛОЇДАЛЬНИХ ПРОФІЛІВ У ЗАЧЕПЛЕННІ ГЕРОТОРНОЇ ПАРИ

Purpose. Determination of laws of movement of contact points of the conjugate profiles of the wheels of a gerotor gearing in the process of relative motion, analytical determination of motion velocities, identification of the nature of friction which results in gearing, identification of profile zones with the highest degree of friction.

Methodology. The research is based on the theory of non-centroidal cycloidal gearings, statements of differential geometry, and the laws of motion of the material point according to the trajectory which is defined by the function.

Findings. The result of the research is the identification of the accurate mathematical relations of the relative velocities of the contact point of two conjugate profiles in gerotor gearing. It was identified that there are two simultaneous velocities of contact points along the cycloidal profile and along the profile of the lantern gear. In this case the velocity vectors are located on the common tangent at the point of conjugation. They are different by module and their index depends on the phase of operation cycle.

Originality. The laws of movement of contact points in gerotor gearing were determined and the changing velocity mode in gearing was reliably established. In the process of research the universal formula of the relationship between the motion velocity of the points along the curved trajectory and its equidistant curve which takes account of trajectory curvature was obtained. It was established that in the interaction of the convex area of the equidistant curve with the cycloidal curve with conjugated lantern there is a sliding friction, while in the generating process of the concave area there is a rolling friction. The linearization functions which allow stabilizing the changing velocity mode were obtained.

Practical value. The research results enabled us to assess the impact of the velocity mode on the degree of friction in the points of conjugation of the profile surfaces, to localize the areas with the highest degree of friction and to find the ways of reducing the destructive impact of friction and to increase the stability of the profiles while gearing. The results may also be used in the process of development of the processing technology of the cycloidal profiles in the conditions of generating process with the linearization function of the velocity of relative generating process of the tool profiles and the workpiece. The linearization will stabilize the load and increase the processing accuracy.

Keywords: *contact point, velocity, gerotor gearing, cycloidal profile*

Introduction. Gerotor gearing based on the non-centroidal epi- or hypocycloidal gearing is the main component of a separate class of hydraulic machines which are gerotor pumps, motors, dosing pumps and bottom-hole motors for deep-hole drilling. Research studies relating to the gerotor gearing, are displayed in the list of papers [1–4]. Papers [1, 4] present the design of the gearing and synthesis of job profiles. In paper [2] force interaction, occurring during the working of gearing is presented. Work [3] is devoted to research on the operating characteristics of gerotor huidraulic machine. Despite wide expansion and well-known design methods the detailed study of the kinematics of gearing has got little attention. Particularly, the kinematics of contact points of the conjugate cycloidal profiles arouses a considerable interest. It is apparent that understanding of the processes at the points of contact interaction will allow disclosing the nature of negative processes connected with friction and weariness. At the same time the results of the research may become of interest in the process of developing the technology for producing the profiles of the gears of gerotor gearing in the conditions of generating process.

Analysis of the recent research. Analysis of the kinematics of the contact points in the gearing of gerotor pump is given in work [5]. The authors state that there is a specific sliding in gearing which is conditioned by the changing velocity mode and unequal curves of the profiles. There is also research on the impact of the design factors of the trochoidal (epicycloidal) gearing on sliding velocity. Mathematical models of sliding velocity are given only for trochoidal profile as the most wearable geometric element of gearing.

Unsolved aspects of the problem. The results presented in the work [5] do not disclose the mechanics of the contact interaction to the full extent, in particular:

- the analysis of velocity mode in gearing is given only for the profile surface outlined by the equidistant to the trochoidal curve as the most compliant for the negative impact of sliding friction;
- this research is related only to the gearings on the basis of epitrochoid (shortened epicycloid).

The objective of the article is to make a detailed analysis of the kinematics of contact interaction in the gearings of the gerotor pair based on the shortened hyper- and epicycloids; to determine all velocity constituents that exist at the contact point; to determine the dependence between a velocity mode and separate constituents of the velocity of the contact point.

Presentation of the main research. At the initial stage we consider the gearing scheme of the gerotor pair and single out the main constructive parameters for further calculations. The gearing schemes based on different cycloidal curves are presented in Figure 1. The gerotor pair is a type of internal gearing in which one of the gear profiles is outlined by equidistant to cycloidal curve – shortened epicycloid (Fig. 1, *a*) or hypocycloid (Fig. 1, *b*). The profile of the conjugate gear is outlined by the curve of the constant radius. The initial data for calculation and constructional design is the number of teeth of the internal satellite z_1, eccentricity of the gearing *e*, radius

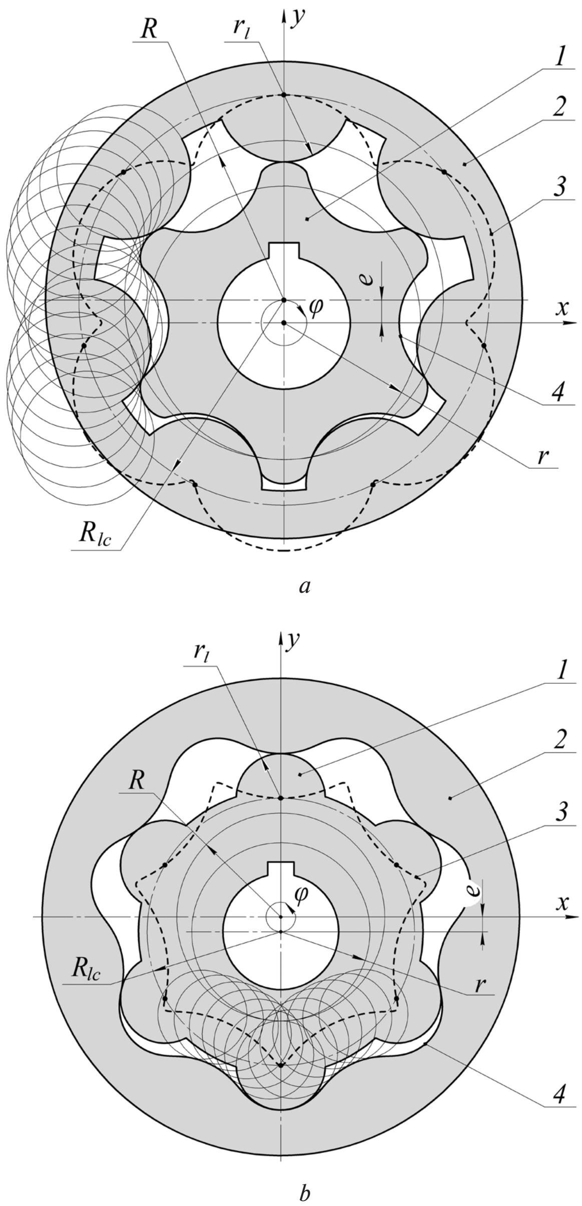


Fig. 1. The scheme of gearing of the gerotor pair with the description of the main design specifications:
a – on the basis of the epicycloid; b – on the basis of hypercycloid; 1 – internal satellite; 2 – external gear with the internal tooth rim; 3 – cycloidal curve; 4 – equidistant to the cycloidal curve; e – eccentricity of the gearing; R, r – radius of the centroid wheels; R_{lc} – radius of lantern centres; r_l – radius of the lantern; φ – angle of rotation of the movable centroid

of the lantern centres R_{lc}, radius of the lantern r_l. The radii of centroid wheels *r* and *R*, are determined by certain formulae and connected by the relation

$$r = e \cdot z_1;\; R = e \cdot z_2;\; \frac{r}{R} = \frac{z_1}{z_2},$$

where z_2 is the number of teeth of the external wheel, $z_2 = z_1 + 1$.

According to the theory of non-centroidal gearing [4], the cycloidal curves are described by the parametric equations:

- for the shortened epicycloid

$$x(\varphi) = R_{lc} \cdot \cos(\varphi / z_2) - e \cdot \cos\varphi ;$$
$$y(\varphi) = R_{lc} \cdot \sin(\varphi / z_2) - e \cdot \sin\varphi ;$$

- for the shortened hypocycloid

$$x(\varphi) = R_{lc} \cdot \cos(\varphi / z_1) + e \cdot \cos\varphi ;$$
$$y(\varphi) = R_{lc} \cdot \sin(\varphi / z_1) - e \cdot \sin\varphi .$$

The factual profile outlined by the equidistant to the cycloidal curve which is remoted from it by the value of lantern radius r_l. The equidistant curve is described by the equations [1]

$$\xi(\varphi) = x(\varphi) \mp \frac{r_l \cdot y'(\varphi)}{\sqrt{x'(\varphi)^2 + y'(\varphi)^2}} ; \quad (1)$$

$$\eta(\varphi) = y(\varphi) \pm \frac{r_l \cdot x'(\varphi)}{\sqrt{x'(\varphi)^2 + y'(\varphi)^2}} , \quad (2)$$

where $x'(\varphi)$ and $y'(\varphi)$ are the first derivatives of the coordinates $x(\varphi)$ and $y(\varphi)$.

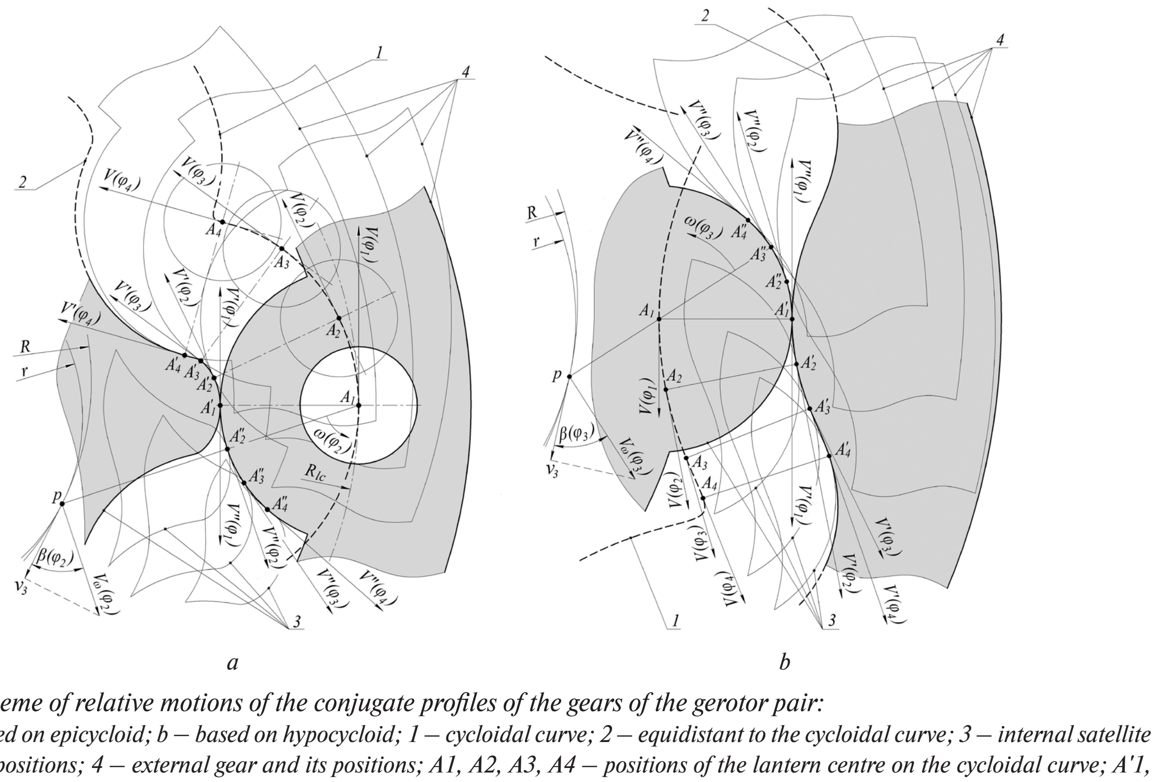
The change of signs corresponds to the external and internal equidistant.

The gearing based on epicycloid (Fig. 1, *a*) is specific because the lantern gear in case is the external gear with an internal tooth rim. The gearing based on hypocycloid (Fig. 1, *b*) is opposite where the lantern gear is the internal satellite.

Another stage of research is the analysis of the roll motion in the process of gearing in operation. It is clear that the character of the interworking is a point interaction and the kinematics of the contact point is the main object for the research.

Let us consider the scheme of relative motions of the conjugate profiles of the gears of the gerotor pair (Fig. 2) and determine the character of the contact point movement. In this case the motions of the gears in meshing are presented in the form of separate fixed positions which are defined by the phase of the working cycle. The contact point changes its position on the profiles of both gears at every phase. If the motion of the lantern is considered relating to the unmoved cycloidal profile, then the contact point moves along equidistant to cycloidal curve with the velocity $V'(\varphi)$. But in relation to the unmoved lantern the same point moves in the opposite direction along its arc with the velocity $V''(\varphi)$, and in this case $V'(\varphi) \neq V''(\varphi)$. The contact point moves simultaneously along equidistant to the cycloidal curve and along the arc of the lantern with different velocities. The directions $V'(\varphi)$ and $V''(\varphi)$ always coincide with the tangent line at the contact point of the equidistant and the lantern. Let us determine $V'(\varphi)$ and $V''(\varphi)$.

Firstly, let us determine the translational velocity of the motion of the lantern centre along the cycloidal curve



a *b*

Fig. 2. Scheme of relative motions of the conjugate profiles of the gears of the gerotor pair:
a – based on epicycloid; b – based on hypocycloid; 1 – cycloidal curve; 2 – equidistant to the cycloidal curve; 3 – internal satellite and its positions; 4 – external gear and its positions; A1, A2, A3, A4 – positions of the lantern centre on the cycloidal curve; A'1, A'2, A'3, A'4 – positions of contact point along equidistant to the cycloidal curve; A"1, A"2, A"3, A"4 – positions of contact point along the arc of the constant radius; V(φ$_i$) – instantaneous velocity of the motion of the lantern centre along the cycloidal curve; V'(φ$_i$) – instantaneous velocity of the motion of contact point along equidistant to the cycloidal curve; V"(φ$_i$) – instantaneous velocity of the motion of contact point along the arc of the constant radius; p – the pitch point of the centroids; v_3 – translational velocity of the pitch point motions; V$_ω$(φ$_i$) – projection of velocity v; β(φ$_i$) – projective angle between the directions of the velocities v and Vω(φ$_i$); ω(φ$_i$) – angular velocity of the rotation of the section (between the arc centre and pitch point) around the arc centre

$V(\varphi)$. We will form the calculation scheme to determine the velocity and introduce corresponding identifications (Fig. 3). The velocity of the working cycle is given by the input frequency of rotation (the frequency of rotation of the input shaft which makes the gearing operate). If we consider the motion of the lantern gear relatively to the unmoved equidistant profile, then this gear makes translational motion around the circle with radius e, with the input frequency and rotational motion around the instantaneous centre with the output frequency. The centres of the lanterns move along cycloidal trajectory and the vector of their velocity coincides with the tangent line to the trajectory at the instantaneous points.

Let us determine angular velocities ω_1 and ω_2 of the rotational motions in relation to the centres O_1 and O_2

$$\omega_1 = 2 \cdot \pi \cdot \frac{n_{in}}{60}; \quad (3)$$

- for epicycloid

$$\omega_2^E = 2 \cdot \pi \cdot n_{out} = 2 \cdot \pi \frac{n_{in}}{60 \cdot z_2};$$

- for hypocycloid

$$\omega_2^H = 2 \cdot \pi \cdot n_{out} = 2 \cdot \pi \frac{n_{in}}{60 \cdot z_1}.$$

The superscripts "E" and "H" show the type of the cycloidal curve which is the basis of the gearing – epicycloid or hypocycloid correspondingly.

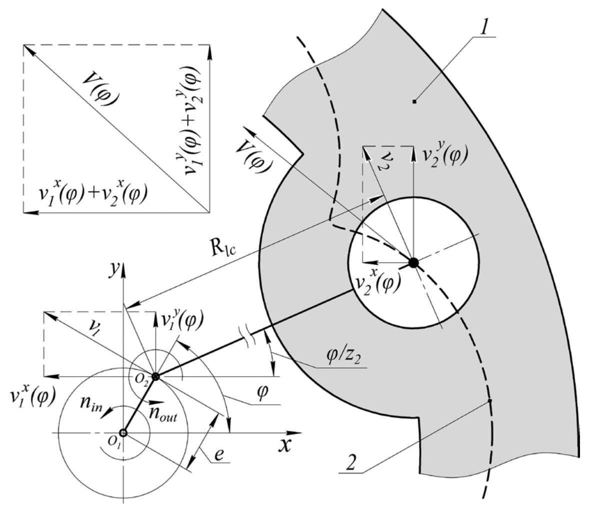


Fig. 3. The scheme for determining the velocity V(φ) of the motion of the lantern centre along the cycloidal trajectory:

1 – lantern gear; 2 – cycloidal trajectory; n_{in} – frequency of rotation at the input shaft; n_{out} – frequency of rotation of the lantern gear around the instantaneous centre; v_1 – translational velocity of the motion of the lantern gear around with the radius e relatively to the centre O_1; v_2 – translational velocity of the motion of the lantern centre relatively to the centre of the instantaneous rotation O_2; $v_1^x(\varphi)$, $v_{1c}^y(\varphi)$, $v_2^x(\varphi)$, $v_2^y(\varphi)$ – projections of the velocity vectors v_1 and v_2 onto the coordinate axes

Applying the formula of the connection of the angular and linear velocities we can determine translational velocities of the motion of the centre of the lantern gear around a circle e and the point of the circle

$$v_1 = e \cdot \omega_1;$$

$$v_2^E = R_{lc} \cdot \omega_2^E;$$

$$v_2^H = R_{lc} \cdot \omega_2^H.$$

Depending on the phase of the generating process φ, the values of the determined velocities stay constant by measure and different by directions, which influences their resultant. That is why the squared absolute value of the resultant velocity should be determined as the sum of the squires if the projections of the velocities on the coordinate axis.

Let us determine the projections of the velocities v_1 and v_2 taking into account the phase of the generating process φ

$$v_1^x(\varphi) = e \cdot \omega_1 \cdot \cos\varphi;$$

$$v_1^y(\varphi) = e \cdot \omega_1 \cdot \sin\varphi;$$

$$v_2^x(\varphi) = v_2^E \cdot \cos\left(\frac{\varphi}{z_2}\right) = R_{lc}\frac{\omega_1}{z_2} \cdot \cos\left(\frac{\varphi}{z_2}\right); \quad (4)$$

$$v_2^y(\varphi) = v_2^E \cdot \sin\left(\frac{\varphi}{z_2}\right) = R_{lc}\frac{\omega_1}{z_2} \cdot \sin\left(\frac{\varphi}{z_2}\right). \quad (5)$$

The direction of the determined projections coincides and, therefore, we can determine their sums by the coordinate axes

$$V_x(\varphi) = v_1^x(\varphi) + v_2^x(\varphi);$$

$$V_y(\varphi) = v_1^y(\varphi) + v_2^y(\varphi).$$

For the hypocycloidal gearing in the formulae (4) and (5), instead of v_2^E it is necessary to use v_2^H. The resultant $V(\varphi)$ will be determined by the formula

$$V(\varphi) = \sqrt{(V_x(\varphi))^2 + (V_y(\varphi))^2}. \quad (6)$$

After assigning the constituents the expression (6) takes on the form

$$V(\varphi) = \sqrt{\frac{4\pi^2 \cdot n_{in}^2 \cdot w(\varphi)}{z_2^2}},$$

where $w(\varphi)$ is a variable value which characterises the square of the length of the line segment pA_i which connects the pitch point of the centroids p and the current position of the lantern centre A_i. Depending on the type of the curve, the value $w(\varphi)$ is determined by the following formulae

$$w^E(\varphi) = R_{lc}^2 + R^2 - 2 \cdot R_{lc} \cdot R \cdot cos(\varphi \cdot z_1/z_2);$$

$$w^H(\varphi) = R_{lc}^2 + r^2 - 2 \cdot R_{lc} \cdot r \cdot cos(\varphi \cdot z_2 / z_1).$$

To determine the first required velocity $V'(\varphi)$, we may use formula [1]

$$V'(\varphi) = \sqrt{(\xi'(\varphi))^2 + (\eta'(\varphi))^2},$$

where $\xi'(\varphi)$ and $\eta'(\varphi)$ are the first derivatives from the coordinates $\xi(\varphi)$ and $\eta(\varphi)$ (1, 2).

For a more rational determination of $V'(\varphi)$, we use the formula of the connection of the translational and angular velocities. Suppose that the radii of the curvature $\rho(\varphi)$ at points A_1, A_2, A_3, A_4 are instantaneous radii of the rotation of the points (Fig. 2). Then the angular velocities of the rotation of these points and points A'_1, A'_2, A'_3, A'_4 which have common radii, will be equal and will be determined by the formula

$$\omega_\rho(\varphi) = \frac{V(\varphi)}{\rho(\varphi)} = \frac{V'(\varphi)}{\rho(\varphi) - r_l}.$$

Radius of curvature $\rho(\varphi)$ is determined by the well-known formula [4]

$$\rho(\varphi) = \frac{\left(x'(\varphi)^2 - y'(\varphi)^2\right)^{\frac{3}{2}}}{x'(\varphi) \cdot y''(\varphi) - x''(\varphi) \cdot y'(\varphi)},$$

where $x''(\varphi)$ and $y''(\varphi)$ are the second derivatives from the coordinates $x(\varphi)$ and $y(\varphi)$.

Now we can determine the first velocity $V'(\varphi)$

$$V'(\varphi) = \frac{V(\varphi) \cdot (\rho(\varphi) - r_l)}{\rho(\varphi)}. \quad (7)$$

The second velocity $V''(\varphi)$ is determined by using the conformity that the normal at the contact point always goes through the pitch point p of the centroids r and R (Willis theorem). The linear velocity v_3 of the pitch point p along the centroid r will be determined as $r \cdot \omega_1$ (for hypocycloid $v_3 = R \cdot \omega_1$), and the velocity $V_\omega(\varphi)$ will be determined as the product $v_3 \cdot \cos(\beta(\varphi))$.The angle $\beta(\varphi)$ between the direction of the velocity v_3 and perpendicular to the line segment pA_1 can be found with the help of the sine and cosine theorem

$$\beta^E(\varphi) = arcsin\left(\frac{R_{lc} \cdot sin(\varphi \cdot z_1 / z_2)}{\sqrt{w^E(\varphi)}}\right);$$

$$\beta^H(\varphi) = arcsin\left(\frac{R_{lc} \cdot sin(\varphi \cdot z_2 / z_1)}{\sqrt{w^H(\varphi)}}\right).$$

The expression in the denominator shows the length of the line segment pA_1 which is dependently on the position of the pitch point p changes its value. Let us rearrange the formula to determine the velocity $V_\omega(\varphi)$

$$V_\omega^E(\varphi) = r \cdot \omega_1 \cdot \cos(\beta^E(\varphi));$$

$$V_\omega^H(\varphi) = R \cdot \omega_1 \cdot \cos(\beta^H(\varphi)).$$

Having determined the velocity $V_2(\varphi)$ and knowing the length of the line segment pA_1 at the corresponding phase of the generating process we can determine the angular velocity $\omega(\varphi)$ of the turn of the line segment pA_1 relatively to the lantern centre A_1

$$\omega^E(\varphi) = \frac{V_\omega^E(\varphi)}{\sqrt{w^E(\varphi)}};$$

$$\omega^H(\varphi) = \frac{V_\omega^H(\varphi)}{\sqrt{w^H(\varphi)}}.$$

The absolute value of the second velocity $|V''(\varphi)|$ will be calculated by the formula

$$\left|V''(\varphi)\right|^E = \omega^E(\varphi) \cdot r_l; \quad (8)$$

$$\left|V''(\varphi)\right|^\Gamma = \omega^\Gamma(\varphi) \cdot r_l. \quad (9)$$

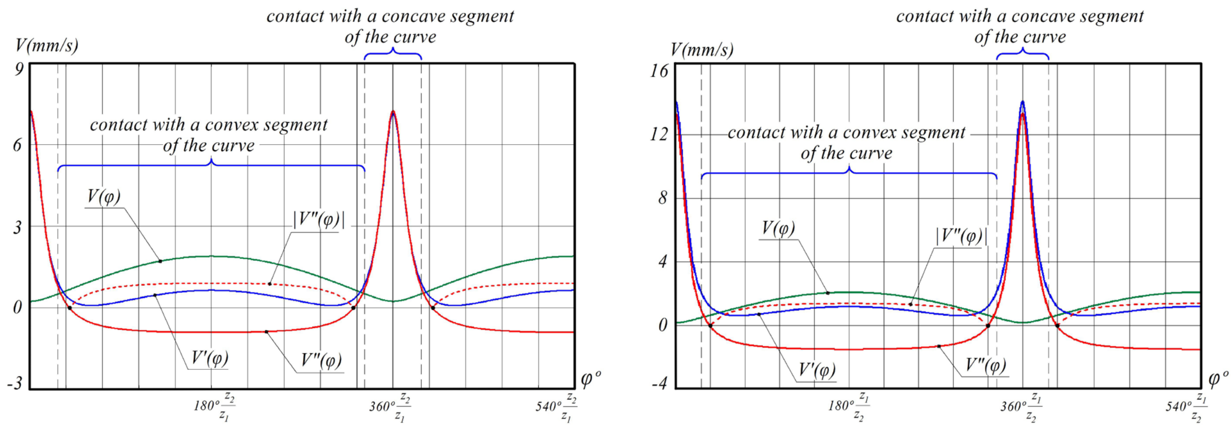
Because of the special features of the functions of instantaneous angular velocities $\omega^E(\varphi)$ and $\omega^H(\varphi)$, the velocities determined by the formulae (8) and (9) always have absolute positive value which do not take into consideration the vector direction.

In order to understand the friction nature of the conjugate profiles, it is necessary to determine relative values of the vectors $V'(\varphi)$ and $V''(\varphi)$. If these velocities have equal value then it is a rolling friction, if the values are different then it is a sliding friction. Velocity $V'(\varphi)$ has a relative value as formula (7) takes into account the value of curvature. The relative value $V''(\varphi)$ will be determined by the following formula

$$V''(\varphi) = V'(\varphi) - V(\varphi) - \delta;$$

$$\delta = V'(\varphi_0) - \left|V''(\varphi_0)\right| - V(\varphi_0); \ \varphi_0 = 0.$$

According to the formulae we build the dependency graphs of the velocities $V(\varphi)$, $V'(\varphi)$ and $V''(\varphi)$ of the working cycle phase shown in Fig. 4. The graphs are systematic cyclic curves with clear extreme points, which means a sharp acceleration of the contact point at certain phases. The analysis of the graphs shows that the maximum velocity is achieved at the moment of the contact of the central symmetrical section of the lantern arc with the central point of the concave section of equidistant to the cycloidal curve ($\varphi = 0$). It should also be noted that the velocities $V'(\varphi)$ and $V''(\varphi)$, under the interaction of the convex section of cycloidal curve with lantern, have opposite values, which means that there is sliding friction and more excessive wear. Moreover, both types of curves are of the same nature. In case of the contact with concave section the situation is better. Not considering the extreme point and considerable acceleration, the values of velocities are equal, which means there is rolling friction. The data we received are proved by the intensive wear of the satellite teeth edges in real conditions.



a *b*

Fig. 4. Nature of changes of translational velocities V(φ), V'(φ) and V''(φ) depending on the phase of the working cycle of gearing:
a – based on epicycloid; b – based on hypocycloid

Velocities $V'(\varphi)$ and $V''(\varphi)$ can be linearized specifying the frequency on the input shaft in the form of dependence. But it is impossible to linearize two velocities at the same time. So if $V'(\varphi) = const$, then $V''(\varphi) \neq const$. To determine the function of linearization of the velocity $V'(\varphi)$, we should take the value n_{in} out of the formula (7) and after that the value V' will be given as constant and independent of the parameter φ. In return the frequency will take the form of dependence $n_{in}(\varphi)$. The functions of the frequencies for linearization of the velocities $V'(\varphi)$ and $V''(\varphi)$ for the gearing based on epicycloids will be the following

$$n'_{in}(\varphi) = \frac{30 \cdot V' \cdot z_2}{\pi \cdot k(\varphi) \cdot \sqrt{w^E(\varphi)}};$$

$$n''_{in}(\varphi) = \frac{30 \cdot V'' \cdot z_2 \cdot \sqrt{w^E(\varphi)}}{\pi \cdot R \cdot r_l \cdot z_1 \cdot \sqrt{1 - \frac{R_{lc}^2 \cdot \sin\left(\frac{\varphi \cdot z_1}{z_2}\right)^2}{w^E(\varphi)}}},$$

where $k(\varphi) = \frac{\rho(\varphi) - r_l}{\rho(\varphi)}$.

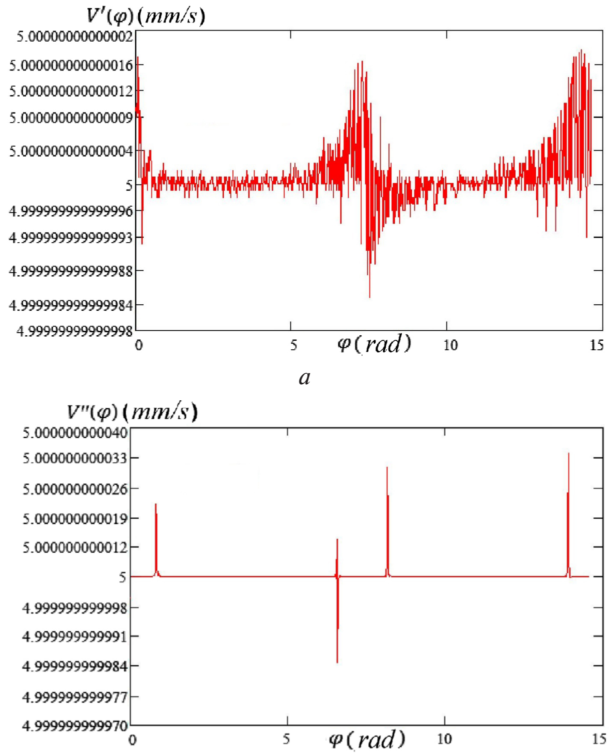
The obtained functions ($n'_{in}(\varphi)$ or $n''_{in}(\varphi)$) are substituted in formula (3), instead of n_{in}. Dependencies $V'(\varphi)$ and $V''(\varphi)$ in this case will take into account the functions of linearization. The linearized functions are shown in Fig. 5. The graphs were built for the value of 5 mm/s.

The visual graphs are different from the linear graphs because of the disturbances caused by the scale of the vertical axe. Taking into account the fact that the range of deviations of the nominal velocity of 5 mm/s does not exceed $2 \cdot 10^{-14}$ mm/s for $V'(\varphi)$ and $4 \cdot 10^{-12}$ for $V''(\varphi)$, these infinitesimal deviations prove the linearity of the given functions. For the gearing based on hypocycloid the function of linearization will be different and be the following

$$n'_{in}(\varphi) = \frac{30 \cdot V' \cdot z_1}{\pi \cdot k(\varphi) \cdot \sqrt{w^H(\varphi)}};$$

$$n''_{in}(\varphi) = \frac{30 \cdot V'' \cdot z_1 \cdot \sqrt{w^H(\varphi)}}{\pi \cdot r \cdot r_l \cdot z_2 \cdot \sqrt{1 - \frac{R_{lc}^2 \cdot \sin\left(\frac{\varphi \cdot z_2}{z_1}\right)^2}{w^H(\varphi)}}}.$$

From the point of view of operation, the found functions of linearization have little importance. It is explained by the fact that machines and mechanisms



a *b*

Fig. 5. Linearized velocities:
a – linearization V'(φ) ; b – linearization V''(φ)

which contain the gerotor pair are driven by the motors with constant frequency of rotation and the frequency cannot be changed depending on the angle of the shaft turn. However, the necessity of linearization may appear in the process of production of the profile surfaces of the gears when it is necessary to reach the velocity stability of the generating process to avoid extra loading. There is also a need in the functions of linearization while creating a gerotor hydraulic machine with the improved dynamic characteristics and durability.

Conclusions. The kinematic analysis of the contact interaction in the gearing of the gerotor pair was made. The analysis includes the following results:

- the accurate mathematic dependences to determine the velocities of the motion of the contact point of the conjugate profiles along the arc which are the functions of the working cycle phase of the gearing were obtained;
- the dependence that shows the interrelation between the velocities of the instantaneous motion of the points by two trajectories – a curve and an equidistant to it, taking into account the common centre of curvature of the two trajectories was obtained;
- it was found that the convex section of the equidistant to the cycloidal curve (during the contact with conjugate curve of the constant radius) has the sliding friction and, correspondently, the wear on this section will be more intensive. The concave sections have rolling friction which does not change the geometry much;
- the functions of linearization we obtained allow stabilizing the velocity mode and decreasing the degree of wear, but the practical realization demands special programmed drivers with a changeable frequency of rotation of the shaft according to the function of the linearization.

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Мета. Визначення законів руху точок контакту спряжених профілів коліс геротороної пари у процесі взаємного переміщення, аналітичне визначення швидкостей переміщення, встановлення природи тертя, що має місце в зачепленні, ідентифікація ділянок профілів з найбільшою інтенсивністю тертя.

Методика. Дослідження базуються на теорії позацентроїдних циклоїдальних зачеплень, положеннях диференціальної геометрії, законах руху матеріальної точки за траєкторією, що задана функцією.

Результати. Встановлення точних математичних залежностей відносних швидкостей точки контакту двох спряжених профілів у зачепленні геротороної пари. Встановлено, що мають місце дві одночасні швидкості точки контакту – уздовж циклоїдального профілю та уздовж профілю цівкового колеса, причому вектори швидкостей лежать на спільній дотичній у точці спряження, різні за модулем, а їх знак залежить від фази робочого циклу.

Наукова новизна. Визначені закони руху контактних точок у зачепленні геротороної пари, достовірно встановлено змінний швидкісний режим у зачепленні. У ході досліджень отримана універсальна формула зв'язку швидкостей переміщення точок за криволінійною траєкторією та еквідистанті до неї, що враховує кривизну траєкторії. Встановлено, що при взаємодії випуклої ділянки еквідистанти до циклоїдальної кривої зі спряженою цівкою, має місце тертя ковзання, а при обкаті увігнутої ділянки – тертя кочення. Отримані функції лінеаризації, що дозволяють стабілізувати змінний швидкісний режим.

Практична значимість. Отримані в ході дослідження результати дають змогу оцінити вплив швидкісного режиму на інтенсивність тертя в точках спряження профільних поверхонь, локалізувати ділянки з найбільшою інтенсивністю тертя, знайти шляхи зменшення шкідливого впливу тертя та підвищити стійкість профілів у зачепленні. Результати також можуть бути використані при розробці технології обробки циклоїдальних профілів в умовах обкату, з функцією лінеаризації швидкості взаємного обкату профілів інструмента й деталі. Лінеаризація дасть змогу стабілізувати навантаження й підвищити точність обробки.

Ключові слова: *точка контакту, швидкість, геротороа пара, циклоїдальний профіль*

Цель. Определение законов движения точек контакта сопряженных профилей колес геротороной пары в процессе взаимного перемещения, аналитическое определение скоростей перемещения, определение природы трения, которое имеет место в зацеплении, идентификация участков профилей с наибольшей интенсивностью трения.

Методика. Исследования базируются на теории внецентроидних циклоидальных зацеплений, положениях дифференциальной геометрии, законах движения материальной точки по траектории, заданной функцией.

Результаты. Результатом исследования является получение точных математических зависимостей относительных скоростей точки контакта двух сопряженных профилей в зацеплении геротороной пары. Установлено, что имеют место две одновре-

менные скорости точки контакта – вдоль циклоидального профиля и вдоль профиля цевочного колеса, причем векторы скоростей лежат на общей касательной в точке сопряжения, различные по модулю, а их знак зависит от фазы рабочего цикла.

Научная новизна. Определены законы движения контактных точек в зацеплении героторной пары, достоверно установлен переменный скоростной режим в зацеплении. В ходе исследований получена универсальная формула связи скоростей перемещения точек по криволинейной траектории и эквидистанте к ней, которая учитывает кривизну траектории. Установлено, что при взаимодействии выпуклого участка эквидистанты к циклоидальной кривой с сопряженной цевкой, имеет место трение скольжения, а при обкатке вогнутого участка – трение качения. Получены функции линеаризации, которые позволяют стабилизировать переменный скоростной режим.

Практическая значимость. Полученные в ходе исследования результаты позволяют оценить влияние скоростного режима на интенсивность трения в точках сопряжения профильных поверхностей, локализовать участки с наибольшей интенсивностью трения и найти пути уменьшения вредного влияния трения, тем самым повысить износоустойчивость профилей в зацеплении. Результаты также могут быть использованы при разработке технологии обработки циклоидальных профилей с функцией линеаризации скорости взаимной обкатки профилей инструмента и детали. Линеаризация позволит стабилизировать нагрузку и повысить точность обработки.

Ключевые слова: *точка контакта, скорость, героторная пара, циклоидальный профиль*

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THE INFLUENCE OF IMPLEMENTATION OF CIRCULAR PIPES IN LOAD-BEARING STRUCTURES OF BODIES OF FREIGHT CARS ON THEIR PHYSICO-MECHANICAL PROPERTIES

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ВПЛИВ УПРОВАДЖЕННЯ КРУГЛИХ ТРУБ ДО НЕСУЧИХ КОНСТРУКЦІЙ КУЗОВІВ ВАНТАЖНИХ ВАГОНІВ НА ЇХ ФІЗИКО-МЕХАНІЧНІ ВЛАСТИВОСТІ

Purpose. Implementation of the innovative draft system for automatic couplers of railway cars. Such a system is alternative to an existing one, where energy of longitudinal loads are absorbed by draw gears, the main working elements of which are absorbing devices.

Methodology. The research presented used the modern methods of the car dynamics and the theory of vibrations for designing a mathematical model to define accelerations in the supporting structure of the car body with viscous materials, the theoretical and applied mechanics for modelling dynamic processes in a new draft system, designing in modern engineering software applications for creating an adequate spatial virtual model of the supporting system of an railway cars, finite elements for calculations of accelerations occurring in the supporting structure of railway cars under longitudinal loading, and the F-test for the model adequacy. Generally, the algorithm of the research con-