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THE TUMBLING MILL ROTATION STABILITY

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СТІЙКІСТЬ ОБЕРТАННЯ БАРАБАННОГО МЛИНА

Purpose. Creation of a mathematical model of conditions and factors of stability of the established motion of a machine unit whose working machine is a drum, permanently rotating around the horizontal axis, with a fluid fill of the chamber.

Methodology. A filled drum is considered to be a system with permanent composition with variable inertial parameters whose variability is due to the redistribution at relative motion of masses of fluid fill of the chamber on the body of the drum. Based on the application of the principle of hardening of a mechanical system, the drum rotation equations take into account the whole mass of the fill, regardless of the nature of its interaction with the surface of the chamber. We employed methods of the mechanics of relative motion for the mathematical description of the undisturbed motion of the system. The principle of establishing a hierarchy of variables was applied. A hypothesis is accepted about proximity of the system's motion to the rotation with a slowly changing angular velocity. It is believed that dynamic and inertial parameters of the filled drum are determined by the quasi-static dependences on the rotation velocity. We used a second-order Lagrange equation for a system with variable inertial parameters in order to determine the transitional motion. A direct Lyapunov method is applied to search for conditions of the motion stability.

Findings. Dynamic and inertial parameters of the filled drum are formalized using differential equations of undisturbed steady and transitional rotation. We obtained a condition of asymptotic stability of the established motion of a tumbling mill machine unit. Instability factors of the system's motion are determined.

Originality. Regularities of steady rotation of a drum mill are identified. It was established that factors of instability in the established, rotational around the horizontal axis, motion of the drum filled with a fluid medium, are the variations of variable inertial parameters – the axial moment of inertia and the moment of resistance of the fill to rotation. It is shown that the failure to comply with conditions of the motion stability could be caused by achieving the extreme negative values of derivatives from the inertial parameters of the in-chamber fill by the angular velocity of drum rotation.

Practical value. The developed mathematical model allows us to qualitatively determine conditions of steady rotational motion of the drum with a fluid fill of the chamber. Conditions for the occurrence of unstable motion have considerable practical significance because they cause self-excitation of auto-oscillations of the filled drum and determine improvement of effectiveness of grinding process in tumbling mills with traditional design solutions.

Keywords: *steady rotation, drum with a fluid fill, equation of undisturbed motion, equation of transitional motion, condition of asymptotic stability, instability factors, variable inertial parameters*

Introduction. Dispersing of solid bodies, implemented by grinding them to particles of small size, is carried out in order to improve the rate of various technological processes. An increase in the degree of dispersion of a substance leads to its increased physical and chemical activity. By changing the degree of grinding of materials, it is possible to influence their technological parameters.

The easiest, most cost-effective and, consequently, the most common technique to obtain solid bodies in dispersed state is mechanical grinding. The process of disintegration of materials in mills is one of the most energy-intensive, time-consuming and costly in the mining, metallurgical, chemical and many other industries.

Unresolved aspects of the problem. A wide range of different types of mills is exploited for fine grinding of hard materials, which differ by the principle of action and design. Efficient grinding of hard abrasive materials, however, could be achieved only by the slow-motion types of mills with a low speed of movement of the working bodies. Low-speed grinders, compared with medium-speed and fast-speed grinders, are characterized by low operating costs and high reliability of performance.

The most common type of slow-speed grinders are the tumbling mills that remain the basic equipment for large-tonnage fine grinding of various solid materials. Such mills make it possible to reach high unit performance and retain, simultaneously, its functionality at considerable abrasive wear of the working bodies.

At the same time, the main disadvantage of such grinders is the low mechanical performance efficiency coefficient, due to the high specific consumption of energy. This is predetermined by a relatively low intensity of circulation of the grinding load in a working chamber of rotating drum, since much of it is passive and is not involved in shredding. In this case, the process of grinding by impact action, abrasion and crushing is implemented at non-free fall followed by shifting of the active part of the fill whose specific part constitutes only 30–45 %.

The existing traditional trends and directions of the development of design and technological parameters of tumbling mills approached the limit of implementation, they have essentially exhausted their possibilities and cannot serve as a basis for further radical improvement in the grinding efficiency. At the same time, one of the new technological trends in order to substantially increase the extremely low energy efficiency of tumbling mills is to activate the circulation of the fill by setting it into oscillatory motion in a chamber. This may considerably strengthen the intensity of interaction between grinding bodies and particles of the ground material.

Identification of part of the general problem, which was not resolved previously. Recent decades have seen a rather active development of the direction related to improving the tumbling mills implying creation of forced pulsed motion of the fill in a chamber. Such an oscillating motion can increase the intensity of circulation of the fill in a rotating chamber and result in the destruction of the solid-body zone with a decrease in its specific mass part.

In order to implement a pulsed motion of the fill in a chamber along the transverse or longitudinal axial direction, it was proposed to use the in-chamber energy-exchanging units, profile lining of the working chamber, drum drives with a variable speed value and action by a magnetic field.

However, the proposed devices for setting the fill forcefully into pulsed motion were not widely employed in production due to low reliability predetermined by the accelerated abrasive wear of working bodies, as well as complexity in operation.

At the same time, it appears a very promising direction for practical application to improve grinding processes on the basis of effect of self-excitation of auto-oscillations of the in-chamber fill, which is based on traditional design solutions for tumbling mills with a working chamber without additional protruding elements exposed to accelerated abrasive wear. The application of such an approach implies the need for predicting the unstable rotation modes of the filled drum.

Analysis of the recent research. Under established operational mode in tumbling mills, the authors of [1] registered the excitation of forced elastic oscillations caused by kinematic bias of gear transmissions and by the imbalance of rotating masses. Based on the obtained results, it was proposed to use an intermediate shaft of the tumbling mill drive as a bundle of individual elastic elements. It is shown that the rigidity and damping capacity of such an element can greatly reduce dynamic loads.

The model of frictional vibrations, at slipping of the whole fill relative to the surface of the chamber, proposed by Prof. A. V. Slanevsky, was applied to describe the oscillations of in-chamber fill of the rotating drum, which occur during operation of tumbling mills and rotary kilns. It was proposed for the auto-oscillations, which due to the accelerated axial displacement of granular fill in a chamber and the overload of the drive are considered unfavorable, to change the drum rotation speed in order to leave the region of waning characteristic of friction.

When employing the model of frictional vibrations, during slip of the whole fill relative to the surface of the chamber and elastic oscillations of the drive, proposed by Prof. D. K. Kriukov, it was accepted that the rotor of the drive motor rotates at a constant speed, and the in-chamber fill is a physical pendulum with a suspension point on the axis of the drum. Based on the obtained results, it was recommended to use elastic couplings in the drives of tumbling mills, of gear and compensation kind, to reduce fluctuations in the system.

The model of frictional oscillations, proposed by Prof. A. N. Mariuta, at slippage of the central, slow-moving, part relative to the rest of the fill, is based on the phenomenon of the occurrence of unsteady modes of work registered during operation of tumbling mills. To model the dynamics of friction oscillations of the fill, the Froud pendulum theory was applied. It was assumed feasible to control the process of grinding and to maintain friction oscillations of a slow-moving nucleus in the zone of parametric resonance in order to improve performance and reduce energy intensity of grinding.

Building a model of frictional vibrations of the inner layers of the fill, proposed by Prof. I. V. Novitsky, is based on solving the problems on the motion of a small body along the surface of a rotating drum chamber and the motion of fill of a segment cross-section on the surface of such a chamber. A concept was developed of the control system over oscillations of the center of mass of the fill for the purpose of their intensification.

However, the prognostic possibilities of the specified dynamic models relative to establishing the conditions for the occurrence and activization of the auto-oscillatory motion of the fill in a chamber of the rotating drum do not seem obvious. This is due to the practical absence of the predicted slippage of the fill of a chamber during work of tumbling mills, as well as low reliability of the proposed complicated systems for automatic control over the processes of such equipment.

Recent studies into auto-oscillatory processes of the drum-type machines dealt with studying conditions for self-excitation of pulsations of the fill in a rotating chamber due to the loss of stability. Such oscillations result in a change in the internal structure and position of free surface of the granular fill.

In papers [2–6], the authors consider redistribution processes of the particles of the fill due to segregation associated with the complication of structure and the emergence of pattern formations with a loss of the motion stability.

In [2], the authors studied experimentally the occurrence of unusual structures of the granular fill in the form of a central nucleus in the cross-section of a rotating chamber. Redistributions of particles were registered by using a radioactive tracking method.

The influence of the chamber surface roughness on the formation of symmetric and asymmetric structures of the fill was experimentally examined in [3]. The registered phenomenon of the formation of convective thickened zones in the fill is regarded as an analogy to the volatility of Rayleigh.

In [4], the authors experimentally examined the effect of the emergence of a sinusoidal modulation of segregation zones in the cross-section of a chamber. They identified factors that influence the frequency and distribution of separation zones and mergers of groups of different particles.

Axial microsegregation of the granular fill using x-ray tomography is experimentally considered in [5]. The authors established dependence of the stability of prolonged one-directional formation of axial rollers on polydispersity of the fill.

In [6], the authors experimentally studied self-excitation of axial segregation of the fill caused by the independent relative rotation of the end walls of the chamber. They showed the effect of friction properties of the surface and the rate of shifting rotation of end walls on the noticeable change in the rate of particle displacement with the formation of elevations and hollows. They received spatial-temporal diagrams of structures of redistribution of particles in the form of bands.

In papers [7–14], the authors considered processes of self-excitation of pulsations of free surface of the granular fill in a rotating chamber, caused by the loss of system stability.

In [7], the authors experimentally examined multidirectional instability of the fill in a slowly rotating chamber. They registered a periodic increase, in a certain direction, in the volume of part of the fill with subsequent plastic deformation and destruction of the free surface. The empirical conditions for the occurrence of in the fill elements due to low perturbations of stresses were obtained.

Geometrical parameters of self-exciting pulsations of free surface of the granular fill with wet particles in a slowly rotating chamber were reviewed in [8]. The pulsations were caused by the periodic destruction of free surface as a result of collapse. The authors noted a good convergence of the performed numerical modeling of the process using the finite element method with the obtained experimental data.

Paper [9] addresses experimental research into velocity field fluctuations during self-excitation of pulsations of the granular fill in a slowly rotating chamber. A speckle spectroscopy of the fill was employed. The authors estimated velocity field distribution over different sections of the free surface. They revealed an increase in the frequency and amplitude of particle displacement rate with an increase in the rotational speed of the chamber.

In [10], the authors experimentally studied dynamics of the self-excitation of pulsations of the bound granular fill in a slowly rotating chamber. A speckle spectroscope and the system of synchronized vector measurements of forces were applied. A linear increase in the specific dissipation of energy of the fill's pulsations was revealed at increasing rotational speed of the chamber.

Transient dynamic processes during self-excitation of the fill's pulsations with particles of irregular shape in a slow rotating chamber were experimentally studied in [11]. A speckle spectroscope was used. The authors estimated parameters of the velocity field fluctuations and pseudo-temperature of the granular fill in the pulsation process. They revealed significant influence of the shape of particles in the fill on plastic deformations and dynamics of transient processes during pulsations.

In [12], the authors experimentally investigated fluctuations that occur during periodic collapse of free surface of the granular fill in a rotating chamber. A strict correlation was established between the amplitude of fluctuations in the particle velocity and the spatial and temporal characteristics of the auto-oscillatory process. They also found a strict relation between the amplitude of velocity fluctuations and stress fluctuations in the system. The task was set to construct a strict model that would describe dynamics of auto-oscillations at self-excitation of the fill's collapses.

Spatial-temporal stochastic dynamics of self-oscillatory processes of compaction and loosening at self-excitation of a periodic collapse of the granular fill in a rotating chamber was experimentally and analytically studied in [13]. It was proposed to divide the period of auto-oscillations into three stages. During the first stage of the exponential increase in speed, the particles of granular fill were losing stability. The second inertial stage was characterized by the achievement of maximum acceleration of particles at self-excitation of fluctuations due to random interactions. The third frictional stage was characterized by the loss of kinetic energy by particles and proved to be the most unstable, with considerable fluctuations. The authors obtained a simplified condition for the stability of motion of the examined system. Such a condition is based on measuring and estimating the magnitude of a generalized Lyapunov vector.

In [14], the authors conducted experimental and analytical studies into nonlinear dynamics of self-excitation of auto-oscillations of the fill by cylindrical disks in a slowly rotating chamber. The dynamics of chaotic periodic collapse is regarded as the alternation of compaction and loosening of the medium. It is shown that the instability of motion of the granular fill is caused by the excitations from periodic collapses. A strong correlation is revealed between general dynamic properties of the system and its instability. To assess motion stability of the filled chambers, it is proposed to apply the Lyapunov exponent.

At the same time, recent results of research into stability of operational modes of drum machines are related only to the description of behavior of separate parts of the granular fill near the free surface, primarily at low rotation of the chamber. Complexity of hardware control over the motion of fill and the difficulties of analytical modeling of motion stability made it impossible to define strict conditions for the self-excitation of auto-oscillations in the system.

Objectives of the article. Of considerable applied significance is the construction of mathematical model for steady rotation of the tumbling mill. Of particular interest is the task on finding the conditions of steady established motion of the drum-type machine unit, as well as instability factors of such a motion.

Presentation of the main research. Further analytical solution to the problem on determining steady rotation around the horizontal axis of the drum with the fluid in-chamber fill is searched for based on the application of provisions of nonlinear dynamics of mechanical systems with variable inertial parameters.

Such a rotor, which is the system of constant composition with the relative motion of masses, is considered as a body of variable mass. This implies not a change in the body weight, but rather variability in the geometry of masses predetermined by the redistribution with the relative motion of masses of the fill on the body of the rotor. Variability in the geometry of masses is caused by the changing inertial parameters of the filled chamber – the magnitude of the axial moment of inertia and the position of the center of masses.

In order to determine parameters of interaction between drum machines and the in-chamber fill, a solid shell of the drum and its fluid fill are treated as one mechanical system.

We considered rotation with angular velocity ω around its own horizontal axis on supports A and B of the solid shell with a cylindrical chamber of radius R and length L with end walls, which is partially filled with a fluid medium of density ρ and volume w. We introduce for consideration the absolutely fixed (inertial) $OXYZ$ coordinate system, and the relatively movable, rigidly coupled with the body shell, (non-inertial) $O'X'Y'Z'$ coordinate system.

The Z and Z' axes of coordinate systems are aligned with the rotation axis. Centers O and O' are aligned and placed at the same distance $L/2$ from the end walls. The X axis is directed horizontally, the Y axis – vertically.

In the examined case, the radius-vectors of the point in the system of mass m_v relative to the immobile O ($\bar{r}_v$) and movable O' ($\bar{r}_v'$) points coincide

$$\bar{r}_v = \bar{r}_v' .$$

The motion of a mechanical system relative to the $OXYZ$ coordinate system is considered absolute, in relation to the $O'X'Y'Z'$ system – relative.

The position of the $O'X'Y'Z'$ system is set by angle φ of the shell rotating around the Z and Z' axes.

Absolute velocity vector of the point is

$$\bar{V}_a = \bar{V}_e + \bar{V}_r , \qquad (1)$$

where $\bar{V}_a = \bar{V} = \frac{d\bar{r}_v}{dt}$; $\bar{V}_e = \bar{\omega} \times \bar{r}_v$ is the vector of portable speed; $\bar{V}_r = \bar{u}$ is the relative velocity vector; $\bar{u} = \left(\frac{d\bar{r}_v}{dt}\right)'$ is the derivative taken in the mobile coordinate system.

The main moment of the quantities of system motion relative to point O considering (1) is

$$\bar{G}_O = \bar{I}\bar{\omega} + \rho\int_w \left(\bar{r}_v \times \bar{u}\right) dw , \qquad (2)$$

where $\bar{I}$ is the inertia tensor of the system relative to point O.

Projection $\bar{G}_O$ onto the Z axis considering quasi-velocities is

$$G_{OZ} = I_Z \omega + \rho\int_w \left(r_{vx} u_y - r_{vy} u_x\right) dw ,$$

where $I_z = I_{z1} = I_{z2}$ is the moment of inertia of the system, I_{z1} is the shell inertia moment (index 1), $I_{z2} = \rho\int_w {}_{(2)}\left(r_{vx}^2 + r_{vy}^2\right) dw$ is the moment of inertia of the fill (index 2) relative to the Z axis.

Based on the application of the principle of solidification for a system of constant composition with variable inertial parameters, absolute time derivative t from $\bar{G}_O$ considering (2) is

$$\frac{d\bar{G}_o}{dt} = \bar{I}\frac{d\bar{\omega}}{dt} + \bar{\omega} \times \bar{I}\bar{\omega} + \frac{d\bar{I}}{dt}\bar{\omega} + $$
$$+ \rho\int_w \bar{\omega} \times \left(\bar{r}_v \times \bar{u}\right) dw + \rho\left(\bar{r}_v \times \frac{d\bar{u}}{dt}\right) dw ,$$

or following the transforms

$$\frac{d\bar{G}_o}{dt} = \bar{I}\frac{d\bar{\omega}}{dt} + \bar{\omega} \times \bar{I}\bar{\omega} + $$
$$+ 2\rho\int_w \bar{r}_v \times \left(\bar{\omega} \times \bar{u}\right) dw + \rho\int_w \left(\bar{r}_v \times \frac{d\bar{u}}{dt}\right) dw. \qquad (3)$$

The main moment of the forces of bond reactions relative to point *O,* applied to the shell, is

$$\bar{M}_{oR1}=\frac{d\bar{G}_o}{dt}-\rho\int_w\left(\bar{r}_\nu\times\bar{F}_M\right)dw,$$

where $\rho\int_w\left(\bar{r}_\nu\times\bar{F}_M\right)dw$ is the main moment of mass forces relative to point *O,* applied to the fill, $\bar{F}_M$ is the specific mass force of the fill.

One can accept

$$\bar{M}_{oR1}=\bar{M}_d,$$

where $\bar{M}_d$ is the main driving torque, applied to the shell.

Then, with regard to (3), the expression for the driving torque of the shell takes the form

$$\begin{aligned}\bar{M}_d&=\bar{I}\frac{d\bar{\omega}}{dt}+\bar{\omega}\times\bar{I}\bar{\omega}+2\rho\int_w\bar{r}_\nu\times\left(\bar{\omega}\times\bar{u}\right)dw+\\&+\rho\int_w\left(\bar{r}_\nu\times\frac{d\bar{u}}{dt}\right)dw-\rho\int_w\left(\bar{r}_\nu\times\bar{F}_M\right)dw.\end{aligned}\qquad(4)$$

Considering that mass forces are gravitational, projection (4) onto the *Z* axis is

$$\begin{aligned}M_d&=I_z\frac{d\omega}{dt}+2\rho\omega\int_w\left(r_{\nu x}u_x+r_{\nu y}u_y\right)dw+\\&+\rho\int_w\left(r_{\nu x}\frac{du_y}{dt}-r_{\nu y}\frac{du_x}{dt}\right)dw+\rho g\int_w r_{\nu x}dw.\end{aligned}$$

The main moment of influence of the fill on the shell is

$$\bar{N}_{o'}=\bar{M}_r=-\bar{M}_d,$$

where $\bar{M}_r$ is the main moment of resistance of the fill to the shell rotation.

At stationary shell rotation, the expression for the moment of resistance takes the form

$$M_r=m_z+M_{az},$$

where $m_z=m_{kz}=m_{\nu z}$ is the main moment of reactive forces of the fill relative to the Z axis, $m_{kz}=2\rho\omega\int_w\left(r_{\nu x}u_x+r_{\nu y}u_y\right)dw$ is the main moment of reactive Coriolis forces, $m_{\nu z}=\rho\int_w\left(r_{\nu x}\frac{du_y}{dt}-r_{\nu y}\frac{du_x}{dt}\right)dw$ is the main moment of reactive variational forces, $M_{az}=\rho g\int_w r_{\nu x}dw$ is the main moment of active mass forces of the fill relative to the *Z* axis.

In order to determine transition mode of rotation of the filled shell, we consider a mechanical system of points m_ν with variable inertial parameters with perfect holonomic relations, which has *k* degrees of freedom. Mass of the point may vary in a function of generalized coordinates q_i, velocity $\dot{q}_i$, and time *t*

$$m_\nu=f\left(q_i,\dot{q}_i,t\right),\quad\left(i=1,2,\ldots,k\right).$$

Lagrange *f* equation of the second kind for a system with variable inertial parameters employing the principle of solidification takes the form

$$\frac{d^*}{dt}\frac{\partial^*T}{\partial\dot{q}_i}-\frac{\partial^*T}{\partial q_i}-Q_i-R_i=0,\quad\left(i=1,2,\ldots,k\right),\qquad(5)$$

where *T* is the kinetic energy; Q_i is the generalized external power; R_i is the generalized reactive force; $\frac{d^*}{dt}$ is a special derivative from the magnitudes that depend on the motion of the fill relative to the shell, which makes it possible to remove the variable inertial parameters of the system beyond the sign of this derivative, or enter as a constant magnitude under this sign.

In the case of a two-dimensional motion of the system with variable inertial parameters as a generalized coordinate, one may adopt the body rotation angle φ, as a generalized velocity – angular speed ω of this body, and time *t*. Then Lagrange equation of the second kind (5) for a system with one degree of freedom with variable masses takes the form

$$\frac{d^*}{dt}\frac{\partial^*T}{\partial\omega}-\frac{\partial^*T}{\partial\varphi}=M_e-m,\qquad(6)$$

where $T=\frac{I\omega^2}{2}$; *I* is the generalized (reduced) axial moment of inertia of the system; M_e is the generalized (reduced) moment of external forces; *m* is the generalized (reduced) moment of reactive forces.

By applying the principle of establishing a hierarchy of variables, in particular, direct division of motion in nonlinear mechanics into quick (non-core) and slow (basic) components, we can assume that the motion of the filled shell is close to the rotation with a slow-changing angular velocity. Then the dynamic and inertial parameters of the filled shell are the quasi-static type dependences of its angular velocity

$$M_e=f(\omega);\ m=f(\omega);\ I=f(\omega);\ T=\frac{I\left(\omega\right)\omega^2}{2}.$$

After transformations (6), it is possible to obtain a motion equation of machine unit with the filled shell with variable inertial parameters in the form of moments

$$\begin{aligned}&\left[\frac{\omega^2}{2}\frac{d^2I\left(\omega\right)}{d\omega^2}+2\omega\frac{dI\left(\omega\right)}{d\omega}+I\left(\omega\right)\right]\frac{d\omega}{dt}=\\&=M_e\left(\omega\right)-m\left(\omega\right);\\&M_e\left(\omega\right)=M_m\left(\omega\right)-M_a\left(\omega\right),\end{aligned}\qquad(7)$$

where M_m is the generalized moment of driving forces; M_a is the generalized moment of active resistance forces.

In a general case, such a machine unit (Figure) consists of motor *M*, transmission *T*, shell *C* with the fill *F,* which rotates in supports *A* and *B*.

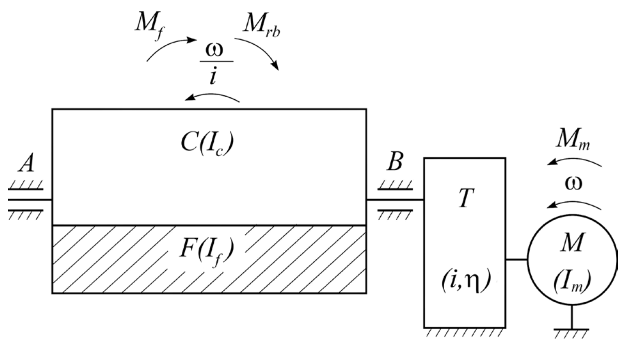

Fig. Calculation scheme of the drum-type machine unit: M – drive motor; I_m – axial moment of inertia of rotating parts of the motor; T – drive transmission; I – gear ratio of transmission; η – efficiency of transmission; C – shell of the drum; A and B – supports of the drum rotation; I_c – axial moment of inertia of the drum shell; F – the fill of drum chamber; I_f – axial moment of inertia of the fill; ω – angular velocity of motor rotation; ω/i – angular velocity of drum rotation; M_m – moment of drive motor; M_f – moment of resistance of the fill to the rotation of drum; M_{rb} – moment of resistance in the drum's supports

Equation (7) for the motor shaft, adopted as the body of reduction, takes the form

$$\left\{\left[\frac{\omega^2}{2}\frac{d^2 I_f(\omega)}{d\omega^2}+2\omega\frac{dI_f(\omega)}{d\omega}+I_f(\omega)+ +I_c\right]\frac{1}{i^2}+I_m\right\}\frac{d\omega}{dt}=M_m(\omega)-\frac{M_f(\omega)+M_{rb}}{i\eta}, \quad (8)$$

where $I_f = I_{z2} = \rho\int_w \left(r_{vx}^2 + r_{vy}^2\right)dw$ is the axial moment of inertia of the fill, $I_c = I_{z1}$ is the axial moment of inertia of the shell, I_m is the axial moment of inertia of the rotating parts of the motor, M_m is the moment of the drive motor, $M_f = M_{az} + m_z$ is the moment of resistance of the fill to the rotation of the shell, M_{az} is the moment of active mass forces of the fill, $m_z = m_{kz} = m_{vz}$ is the moment of reactive forces of the fill, m_{kz} is the moment of reactive Coriolis forces of the fill, m_{vz} is the moment of reactive variational forces of the fill, M_{rb} is the moment of resistance in the supports of the rotation of the shell, i is the gear ratio of transmission, η is the efficiency of transmission.

To determine conditions for stability of rotation of the filled shell, when considering (8), it is possible to introduce notations

$$M(\omega)=M_m(\omega)-\frac{M_f(\omega)+M_{rb}}{i\eta}; \quad (9)$$

$$I(\omega)=\left[\frac{\omega^2}{2}\frac{d^2 I_f(\omega)}{d\omega^2}+2\omega\frac{dI_f(\omega)}{d\omega}+ +I_f(\omega)+I_c\right]\frac{1}{i^2}+I_m. \quad (10)$$

Accepting $\omega = \omega_0 = const$, it is possible from (8), considering (9), to obtain equations for determining the values of ω_0 that match the established motion modes of the unit

$$M(\omega_0)=0. \quad (11)$$

Solution (8), relative to $d\omega/dt$, considering (9) and (10) takes the form

$$\frac{d\omega}{dt}=\frac{M(\omega)}{I(\omega)}. \quad (12)$$

We can accept as the undisturbed motion the established motion corresponding to $\omega = \omega_0$. Denoting the value of velocity at disturbed motion as $\omega = \omega_0 + x$, and introducing it to (12), it is possible to obtain

$$\frac{dx}{dt}=\frac{M(\omega_0+x)}{I(\omega_0+x)}.$$

It is possible to denote

$$F(\omega)=\frac{M(\omega)}{I(\omega)}.$$

Then

$$\frac{dF(\omega)}{d\omega}=\frac{1}{I(\omega)^2}\left[I(\omega)\frac{dM(\omega)}{d\omega}-M(\omega)\frac{dI(\omega)}{d\omega}\right].$$

Considering (11), it is possible to obtain

$$\frac{dF(\omega_0)}{d\omega}=\frac{1}{I(\omega_0)}\frac{dM(\omega_0)}{d\omega};$$

$$F(\omega_0) = 0.$$

By expanding $F(\omega)$ into a series and confining ourselves to the first two terms, it is possible to find

$$F(\omega_0+x)=\frac{1}{I(\omega_0)}\frac{dM(\omega_0)}{d\omega}x.$$

Then the equation of the disturbed motion takes the form

$$\frac{dx}{dt}=\frac{1}{I(\omega_0)}\frac{dM(\omega_0)}{d\omega}x. \quad (13)$$

By multiplying both parts of equation (13) by x and introducing function $\nu = x^2/2$, following the transformations it is possible to obtain

$$\frac{d\nu}{dt}=\frac{1}{I(\omega_0)}\frac{dM(\omega_0)}{d\omega}x^2. \quad (14)$$

Because function ν is definitely positive relative to x and its time derivative, calculated from the equation of disturbed motion (13) is determined by the right side of equality (14), based on Lyapunov's direct method, the established motion will be asymptotically stable if the right side of (14) is less than zero, and unstable if the right side is greater than zero. Then the condition of asymptotic stability for the established motion of a tumbling mill machine unit takes the form

$$\left[\frac{dM_f(\omega_o)}{d\omega}\frac{1}{i\eta}-\frac{dM_m(\omega_o)}{d\omega}\right]\Bigg/ \Bigg/\left\{\left[\frac{\omega_0^2}{2}\frac{d^2I_f(\omega_0)}{d\omega^2}+2\omega_0\frac{dI_f(\omega_0)}{d\omega}+ +I_f(\omega_o)+I_c\right]\frac{1}{i^2}+I_m\right\}>0. \quad (15)$$

Condition (15) differs from a similar condition at $I_f = const$ by the expression in the denominator.

An extreme negative value of the second derivative $d^2I_f(\omega_0)/d\omega^2$ can cause a negative value of the entire denominator while the extreme negative value of derivative $dM_f(\omega_0)/d\omega$ – negative value of the entire numerator (15). This can lead to the impossibility of meeting condition (15) and to the loss of motion stability.

Fulfilling the stability condition (15) greatly depends on the variation of dependences $I_f(\omega)$ and $M_f(\omega)$. The magnitudes of I_f and M_f are determined by the distribution of the fill in the cross-section of a rotating chamber.

A visualization method was applied in order to experimentally qualitatively assess the impact of instability factors on motion of the system. We received images of steady motion of the granular fill in the section of a permanently rotating chamber. Computational grids were used when processing the images. The values of the axial moment of inertia I_f were determined using the grid with a circular arrangement of cells; the moment of resistance M_f was determined using the grid with in-line arrangement of cells.

The analysis and generalization of the obtained results allowed us to establish dependences $I_f(\omega)$ and $M_f(\omega)$, similar to the data in paper [15]. The differentiation of these dependences was also performed.

It turned out that the axial moment of inertia I_f, with an increase in the speed ω, changes from a minimum value at rest that corresponds to the segment form of the fill's section, to the maximum value that corresponds to a near-wall layer under the mode of centrifugation. In this case, derivative $d^2I_f(\omega)/d\omega^2$ assumes extreme negative value close to the fill's mode of motion with full tossing of elements of the fill.

It was also revealed that the dependence of resistance moment M_f on velocity ω has the largest rigidity near a mode with full tossing. In this case, derivative $dM_f(\omega)/d\omega$ also assumes extreme negative value near this mode of the fill motion.

Conclusions and recommendations for further research. The factors of stability loss of the established rotation of a tumbling mill are variations in the variable values of the axial moment of inertia and the moment of resistance of the in-chamber fill to the rotation of the drum. Qualitative dependences of the conditions for implementation of such instability come down to reaching the extreme negative values of derivatives from the inertial parameters of the fill by the rotation velocity.

In the future, it might be expedient, based on an analysis of the modes and conditions for stability motion of the fill in chamber, to determine quantitative patterns in the occurrence of self-excitation of rotating auto-oscillations in the tumbling mills.

Refrences.

1. Vynohradov, B. V., 2016. *Statics and dynamics of drum mill drive*. Dnipropetrovsk: DVNZ UDKhTU.

2. Dube, O., Alizadeh, E., Chaouki, J. and Bertrand, F., 2013. Dynamics of non-spherical particles in a rotating drum. *Chemical Engineering Science* [e-journal], 101, pp. 486–502. DOI: 10.1016/j.ces.2013.07.011.

3. Ching-Fang Lee, Hsien-Ter Chou and Capart, H., 2013. Granular segregation in narrow rotational drums with different wall roughness: Symmetrical and asymmetrical patterns. *Powder Technology* [e-journal], 233, pp. 103–115. DOI: 10.1016/j.powtec.2012.08.034.

4. East, R. D. P., McGuinness, P., Box, F. and Mullin, T., 2014. Granular segregation in a thin drum rotating with periodic modulation. *Physical Review E* [e-journal], 90(5). DOI: 10.1103/PhysRevE.90.052205.

5. Finger, T., Schroter, M. and Stannarius, R., 2015. The mechanism of long-term coarsening of granular mixtures in rotating drums. *New Journal of Physics*, 17. DOI: 10.1088/1367-2630/17/9/093023.

6. Hsiu-Po Kuo, Wei-Ting Tseng and An-Ni Huang, 2016. Controlling of segregation in rotating drums by independent end wall rotations. *KONA Powder and Particle Journal* [e-journal], 33, pp. 239–248. DOI: 10.14356/kona.2016004.

7. Zimber, F., Kollmer, J. E. and Poschel, T., 2013. Polydirectional stability of granular matter. *Physical Review Letters* [e-journal], 111(16). DOI: 10.1103/PhysRevLett.111.168003.

8. Liu, P. Y., Yang, R. Y. and Yu, A. B., 2013. Particle scale investigation of flow and mixing of wet particles in rotating drums. *AIP Conference Proceedings* [e-journal], 1542(1), pp. 963–966. DOI: 10.1063/1.4812093.

9. Yang, H., Li, R., Kong, P., Sun, Q. C., Biggs, M. J. and Zivkovic, V., 2015. Avalanche dynamics of granular materials under the slumping regime in a rotating drum as revealed by speckle visibility spectroscopy. *Physical Review E* [e-journal], 91(4). DOI: 10.1103/PhysRevE.91.042206.

10. Yang, H., Jiang, G. L., Saw, H. Y., Davies, C., Biggs, M. J. and Zivkovic, V., 2016. Granular dynamics of cohesive powders in a rotating drum as revealed by speckle visibility spectroscopy and synchronous measurement of forces due to avalanching. *Chemical Engineering Science* [e-journal], 146, pp. 1–9. DOI: 10.1016/j.ces.2016.02.023.

11. Yang, H., Zhang, B. F., Li, R., Zheng, G. and Zivkovic, V., 2017. Particle dynamics in avalanche flow of irregular sand particles in the slumping regime of a rotating drum. *Powder Technology* [e-journal], 311, pp. 439–448. DOI: 10.1016/j.powtec.2017.01.064.

12. Wang, Z. and Zhang, J., 2015. Fluctuations of particle motion in granular avalanches – from the microscopic to the macroscopic scales. *Soft Matter* [e-journal], 11(27), pp. 5408–5416. DOI: 10.1039/c5sm00643k.

13. Wang, Z. and Zhang, J., 2015. Spatiotemporal chaotic unjamming and jamming in granular avalanches. *Scientific Reports* [e-journal], 5. DOI: 10.1038/srep 08128.

14. Maghsoodi, H. and Luijten, E., 2016. Chaotic dynamics in a slowly rotating drum. *Fevista Cubana de Fisica*, 33(1), pp. 50–54.

15. Sack, A. and Poschel, T., 2016. Dissipation of energy by dry granular matter in a rotating cylinder. *Scientific Reports* [e-journal], 6. DOI: 10.1038/srep26833.

Мета. Створення математичної моделі умов і факторів стійкості усталеного руху машинного агрегату, робочою машиною якого є стаціонарно обертовий навколо горизонтальної осі барабан із текучим заповненням камери.

Методика. Заповнений барабан розглядається як система сталого складу зі змінними інерційними параметрами, змінність яких обумовлена перерозподілом із відносним рухом мас текучого заповнення камери на тілі барабана. На підставі застосування принципу твердіння механічної системи, у рівняннях обертання барабана враховується вся маса заповнення, незалежно від характеру взаємодії її з поверхнею камери. Використані методи механіки відносного руху для математичного опису незбуреного руху системи. Застосовано принцип встановлення ієрархії змінних. Прийнята гіпотеза про близькість руху системи до обертання з кутовою швидкістю, що повільно змінюється. Вважається, що динамічні та інерційні параметри заповненого барабана є детермінованими від швидкості обертання квазістатичними залежностями. Використано рівняння Лагранжа другого роду для системи зі змінними інерційними параметрами для визначення перехідного руху. Застосовано прямий метод Ляпунова для відшукання умов стійкості руху.

Результати. Формалізовані динамічні та інерційні параметри заповненого барабана за допомогою диференціальних рівнянь незбуреного усталеного й перехідного обертання. Отримані умову асимптотичної стійкості усталеного руху машинного агрегату барабанного млина. Визначені фактори нестійкості руху системи.

Наукова новизна. Виявлені закономірності стійкого обертання барабанного млина. Встановлено, що факторами нестійкості усталеного обертового навколо горизонтальній осі руху барабана, що заповнений текучим середовищем, є варіації змінних інерційних параметрів – осьового моменту інерції й моменту опору заповнення обертанню. Показано, що невиконання умов стійкості руху може бути викликано досягненням екстремальних від'ємних значень похідних від інерційних параметрів внутрішньокамерного заповнення по кутовій швидкості обертання барабана.

Практична значимість. Розроблена математична модель дозволяє якісно визначати умови сталого обертового руху барабана з текучим заповненням камери. Умови виникнення нестійкого руху мають істотне прикладне значення, оскільки викликають самозбудження автоколивань заповненого барабана й визначають підвищення ефективності процесу подрібнення в барабанних млинах традиційних конструктивних рішень.

Ключові слова: *стійке обертання, барабан із текучим заповненням, рівняння незбуреного обертання, рівняння перехідного руху, умова асимптотичної стійкості, фактори нестійкості, змінні інерційні параметри*

Цель. Создание математической модели условий и факторов устойчивости установившегося движения машинного агрегата, рабочей машиной которого является стационарно вращающийся вокруг горизонтальной оси барабан с текучим заполнением камеры.

Методика. Заполненный барабан рассматривается как система постоянного состава с переменными инерционными параметрами, переменность которых обусловлена перераспределением с относительным движением масс текучего заполнения камеры на теле барабана. На основании применения принципа затвердевания механической системы, в уравнениях вращения барабана учитывается вся масса заполнения, независимо от характера взаимодействия ее с поверхностью камеры. Использованы методы механики относительного движения для математического описания невозмущенного движения системы. Применен принцип установления иерархии переменных. Принята гипотеза о близости движения системы к вращению с медленно изменяющейся угловой скоростью. Считается, что динамические и инерционные параметры заполненного барабана являются детерминированными от скорости вращения квазистатическими зависимостями. Использовано уравнение Лагранжа второго рода для системы с переменными инерционными параметрами для определения переходного движения. Применен прямой метод Ляпунова для отыскания условий устойчивости движения.

Результаты. Формализованы динамические и инерционные параметры заполненного барабана с помощью дифференциальных уравнений невозмущенного установившегося и переходного вращения. Получено условие асимптотической устойчивости установившегося движения машинного агрегата барабанной мельницы. Определены факторы неустойчивости движения системы.

Научная новизна. Выявлены закономерности устойчивого вращения барабанной мельницы. Установлено, что факторами неустойчивости установившегося вращательного вокруг горизонтальной оси движения барабана, заполненного текучей средой, являются вариации переменных инерционных параметров – осевого момента инерции и момента сопротивления заполнения вращению. Показано, что невыполнение условий устойчивости движения может быть вызвано достижением экстремальных отрицательных значений производных от инерционных параметров внутрикамерного заполнения по угловой скорости вращения барабана.

Практическая значимость. Разработанная математическая модель позволяет качественно определять условия устойчивого вращательного движения барабана с текучим заполнением камеры. Условия возникновения неустойчивого движения имеют существенное прикладное значение, поскольку вызывают самовозбуждение автоколебаний заполненного барабана и определяют повышение эффективности процесса измельчения в барабанных мельницах традиционных конструктивных решений.

Ключевые слова: *устойчивое вращение, барабан с текучим заполнением, уравнение невозмущенного вращения, уравнение переходного движения, условие асимптотической устойчивости, факторы неустойчивости, переменные инерционные параметры*

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ASYMPTOTIC METHOD FOR INVESTIGATING RESONANT REGIMES OF NONLINEAR BENDING VIBRATIONS OF ELASTIC SHAFT

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АСИМПТОТИЧНИЙ МЕТОД ДОСЛІДЖЕННЯ РЕЗОНАНСНИХ РЕЖИМІВ НЕЛІНІЙНИХ ЗГИНАЛЬНИХ КОЛИВАНЬ ПРУЖНОГО ВАЛА

Purpose. To develop a method for determining resonant modes of industrial equipment of elastic shaft type, which is widely used in the mining industry, through the study of mathematical model of nonlinear oscillations. Mathematical models of oscillatory systems previously were studied in the literature mainly based on the numerical and experimental approaches. This paper proposes using a combination of the wave theory of motion and asymptotic methods of nonlinear mechanics using special apparatus of periodic functions to investigate the vibrational dynamics of the system and conditions of resonance phenomena in it, as well as to describe the method for determining the resonance curves to increase the margin of safety of industrial equipment.

Methodology. Methods for studying resonance amplitudes and frequencies, determining the strength characteristics of equipment are based on the use of asymptotic methods of nonlinear mechanics, wave motion theory and theory of special Ateb-functions.

Findings. In this work the conditions of resonance amplitude and frequency depending on the system parameters were obtained analytically for these nonlinear vibrational systems of elastic shaft and the overall method for determining the resonance curves was described.

Originality. For the first time a complete analysis of the impact of physical, mechanical and geometrical factors of the dynamic pricess on the resonant frequency and amplitude in systems such as elastic shaft was conducted on the