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THE MATHEMATIC MODEL OF AND METHOD FOR SOLVING THE NEUMANN GENERALIZED HEAT-EXCHANGE PROBLEM FOR EMPTY ISOTROPIC ROTARY BODY

Purpose. To build a new generalized mathematic 3D model for calculating temperature fields in the empty isotropic rotary body, which rotates with constant angular velocity, with the known equations of generating lines, which takes into account finite velocity of the heat conductivity and is made in the form of boundary problem of mathematical physics for hyperbolic equations of the heat conductivity, as well as to find solutions for the obtained boundary problem.

Methodology. Usage of known integral transforms of Laplace and Fourier and developed new integral transformation for 2D finite space.

Findings. A non-stationary temperature field was found in the empty isotopic body, which rotates with constant angular velocity around the axis OZ, taking into account finite velocity of the heat conductivity under condition that heat-transfer properties of the body are constant, and no internal sources of the heat are available. At the initial moment of time, the body temperature is constant, and values of heat flows on the outside surface of the body are known.

Originality. It is the first generalized 3D mathematic model, which is created for calculating temperature fields in the empty isotropic rotary body, which is restricted by end surfaces and lateral surface of rotation and rotates with constant angular velocity around the axis OZ, with taking into account finite velocity of the heat conductivity in the form of the Neumann problem. In this work, an integral transformation was formulated for the 2D finite space, with the help of which a temperature field in the empty isotropic rotary body was determined in the form of convergence series by the Fourier functions.

Practical value. A non-stationary temperature field was found in the empty isotopic body taking into account finite velocity of the heat conductivity. The obtained solution can be used for modeling temperature fields, which occur in different technical systems (satellites, forming rolls, turbines, and others).

Keywords: *the Neumann boundary problem, generalized energy-transport equation, the Laplace integral transformation, relaxation time*

The problem statement, analysis of the recent research and publications. Rotating discs with irregular shape are the most critical elements in many machines. Possibility to obtain high level of machine performance mainly depends upon solidity and durability of their discs. In most turbomachines, discs operate under the high loads, hence leading to plastic deformations. In this case, it is necessary to take into consideration effect of high temperatures and occurrence of temperature stresses. The law of temperature variation along the disc radius has a great effect on the rate of temperature stresses. It is known that when the square law of temperature variation along radius is employed instead of the linear law, a significant increase in circumferential stress is observed. Therefore, high accuracy in determining the temperature field is critically important for calculation of strength. Besides, under the high rotary speed, the effect of finite velocity of the heat conduction on the heat exchange becomes appreciable [1–3]. That is why a number of problems, which present great theoretical and practical interest, include studying the temperature field in the rotary bodies which rotate around their axis, taking in account the finite velocity of the heat conduction.

As the review of scientific papers shows, heat exchange in the rotating bodies has not been studied fully yet [1]. It is shown that numerical methods are not always effective for studying nonstationary non-axis-symmetrical problems of heat-exchange of the cylinders which rotate at the high rates.

Thus, it is stated in [1] that conditions for reliability of calculations by the finite element method and finite difference method, which are used for calculating non-stationary non-axis-symmetrical temperature fields of the rotating cylinders, are described by the same characteristics and can be expressed in the following way

$$1-\frac{\Delta F_0}{\Delta\varphi^2}\geq 0 \quad \text{and} \quad \frac{1}{\Delta\varphi}-\frac{Pd}{2}\geq 0.$$

If $Pd = 10^5$ and corresponds to the angular velocity $\omega = 1.671\ \text{sec}^{-1}$ of rotation of metal cylinder with the radius of 100 mm, then changes of $\Delta\varphi$ and ΔF_o should comply with the following conditions

$$\Delta\varphi \le 2\cdot 10^{-5} \quad \text{and} \quad \Delta F_o \le 2\cdot 10^{-10}.$$

In case of a uniformly cooled cylinder when Bi = 5 time period needed for the temperature to reach 90 % of stationary state is equal to $Fo \approx 0.025$ [1]. It means that within this period of time, at least $1.3\cdot 10^8$ operations should be fulfilled in order to reach the stationary temperature distribution.

Moreover, it should be mentioned that it would be necessary to make $3.14\cdot 10^5$ calculations within one cycle of computation as the inside state of the ring should be characterized by $3.14\cdot 10^5$ points. It is obvious that this number of calculations needed for getting a numerical result is unrealistic.

Therefore, we will employ integral transformations for solving boundary problems, which occur during mathematic modeling of 3D non-stationary heat-exchange processes in rotating bodies.

The problem statement. Let us consider calculation of the temperature field in the rotary body (Fig. 1) with generating lines L_3, L_6, whose equations are $r = \xi(z)$, $r = \xi_1(z)$, correspondingly, in cylindrical coordinate system (ρ, φ, z). The rotary body is restricted by two end faces S_1 $(z = 0)$ and S_2 $(z = \mathrm{h})$ and by lateral surfaces S_3 and S_4. The lateral surfaces of rotation S_3 and S_4 cross the surface S_j along the lines L_j, $j = 1, 2$ and L_k, $k = 4, 5$, correspondingly.

The body rotates with the constant angular velocity ω around the axis OZ, and the heat-conduction velocity is known. The heat-conduction properties of the body do not depend on temperature, and no internal sources of the heat are available. At the initial moment of time, the body temperature is constant G_0, and values of heat flows on the outside and inside lateral surfaces of the body $V(\varphi, z)$ and $V_1(\varphi, z)$ are known correspondingly. On the end faces, values of heat flow $G_1(r, \varphi)$ and $G_2(r, \varphi)$ at $z = 0$ and $z = h$ are known.

Solving the problem. In the [1], a generalized heat-transfer equation is presented for the moving element of solid medium taking into account finiteness of the heat-conduction velocity value. According to [1], a generalized equation for the energy balance of a solid body which rotates with constant angular velocity ω around axis OZ, and whose heat-transfer properties do not depend on temperature, and no internal sources of the heat are available, can be written in the following way

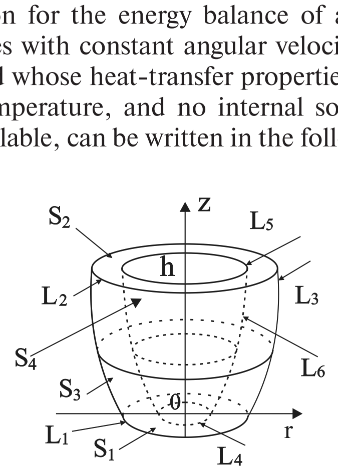

Fig. 1. Rotary body with generating lines $r = \xi(z)$, $r = \xi_1(z)$

$$\gamma\, c\left\{\frac{\partial T}{\partial t}+\omega\frac{\partial T}{\partial\varphi}+\tau_r\left[\frac{\partial^2 T}{\partial t^2}+\omega\frac{\partial^2 T}{\partial\varphi\partial t}\right]\right\}= =\lambda\left[\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial\varphi^2}+\frac{\partial^2 T}{\partial z^2}\right], \qquad (1)$$

where γ is density of the medium; c is specific heat capacity; λ is the heat conductivity coefficient; $T(\rho, \varphi, z, t)$ is the temperature of the medium; t is time; τ_r is relaxation time.

Mathematically, the problem of defining cylinder temperature field consists of integration of differential equation of heat conduction (1) into domains

$$D = \{(\rho, \varphi, z, t)\,|\,\rho\in(\xi_1(z), \xi(z));$$
$$\varphi\in(0, 2\pi);\quad z\in(0, 1),\ t\in(0, \infty)\},$$

which, taking into consideration the accepted assumptions, can be written as

$$\frac{\partial\theta}{\partial t}+\omega\frac{\partial\theta}{\partial\varphi}+\tau_r\frac{\partial^2\theta}{\partial t^2}+\tau_r\omega\frac{\partial^2\theta}{\partial\varphi\partial t}= =\frac{a}{R^2}\left[\frac{\partial^2\theta}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial\theta}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2\theta}{\partial\varphi^2}+\chi\frac{\partial^2\theta}{\partial z^2}\right], \qquad (2)$$

for initial conditions

$$\theta(\rho,\varphi,z,0)=0;\qquad \frac{\partial\theta(\rho,\varphi,z,0)}{\partial t}=0, \qquad (3)$$

and boundary conditions

$$\begin{cases}\displaystyle\int_0^t \left.\frac{\partial\theta}{\partial\rho}\right|_{\rho=\varsigma_1(z)} e^{\frac{\zeta-t}{\tau_r}}\,d\zeta=\Psi(\varphi,z)\\ \displaystyle\int_0^t \left.\frac{\partial\theta}{\partial\rho}\right|_{\rho=\varsigma(z)} e^{\frac{\zeta-t}{\tau_r}}\,d\zeta=G(\varphi,z)\end{cases}; \qquad (4)$$

$$\begin{cases}\displaystyle\int_0^t \left.\frac{\partial\theta}{\partial z}\right|_{z=0} e^{\frac{\zeta-t}{\tau_r}}\,d\zeta=\Theta(\rho,\varphi)\\ \displaystyle\int_0^t \left.\frac{\partial\theta}{\partial z}\right|_{z=1} e^{\frac{\zeta-t}{\tau_r}}\,d\zeta=\Lambda(\rho,\varphi)\end{cases}, \qquad (5)$$

where $\theta=\dfrac{T(\rho,\varphi,z,t)-G_0}{T_{\max}-G_0}$ is the relative temperature of the body; $a=\dfrac{\lambda}{c\gamma}$ is thermal diffusivity; $R=\max\limits_z\{\xi(z)\}$;

$$\chi=\left(\frac{R}{h}\right)^2;\quad \rho=\frac{r}{R};\quad z=\frac{z}{h};\quad \varsigma(z)=\frac{\xi(z)}{R};\quad \varsigma_1(z)=\frac{\xi_1(z)}{R};$$

$$G(\varphi,z)=\frac{V(\varphi,z)\,\tau_r R}{\lambda(T_{\max}-G_0)};\qquad \Psi(\varphi,z)=\frac{V_1(\varphi,z)\,\tau_r R}{\lambda(T_{\max}-G_0)};$$

$$\Theta(\rho,\varphi)=\frac{G_1(\rho,\varphi)\tau_r h}{\lambda(T_{\max}-G_0)};\quad \Lambda(\rho,\varphi)=\frac{G_2(\rho,\varphi)\tau_r h}{\lambda(T_{\max}-G_0)};$$

$G(\varphi, z)$, $\Theta(\rho, \varphi)$, $L(\rho, \varphi) \in C(0, 2\pi)$.

In this case, solution of the boundary problem (2–5) $\theta(\rho, \varphi, z, t)$ is twice continuously differentiated by ρ and φ, z, once – by t in the domain D and continuous on the $\bar{D}$ [4], i.e. $\theta(\rho,\varphi,z,t)\in C^{2,1}(D)\cap C(\bar{D})$, and functions $G(\varphi, z)$, $\Psi(\varphi, z)$, $\Theta(\rho, \varphi)$, $\Lambda(\rho, \varphi)$, $\theta(\rho, \varphi, z, t)$ can be decomposed into the Fourier complex series [4]

$$\left\{\begin{matrix}\theta(\rho,\varphi,z,t)\\ G(\varphi,z)\\ \Psi(\varphi,z)\\ \Theta(\rho,\varphi)\\ \Lambda(\rho,\varphi)\end{matrix}\right\} = \sum_{n=-\infty}^{+\infty}\left\{\begin{matrix}\theta_n(\rho,z,t)\\ G_n(z)\\ \Psi_n(z)\\ \Theta_n(\rho)\\ \Lambda_n(\rho)\end{matrix}\right\}\cdot\exp(in\varphi), \quad (6)$$

where

$$\left\{\begin{matrix}\theta_n(\rho,z,t)\\ G_n(z)\\ \Psi_n(z)\\ \Theta_n(\rho)\\ \Lambda_n(\rho)\end{matrix}\right\} = \frac{1}{2\pi}\int_0^{2\pi}\left\{\begin{matrix}\theta(\rho,\varphi,z,t)\\ G(\varphi,z)\\ \Psi(\varphi,z)\\ \Theta(\rho,\varphi)\\ \Lambda(\rho,\varphi)\end{matrix}\right\}\cdot\exp(-in\varphi)d\varphi;$$

$$\theta_n(\rho,z,t)=\theta_n^{(1)}(\rho,z,t)+i\theta_n^{(2)}(\rho,z,t);$$

$$G_n(z)=G_n^{(1)}(z)+iG_n^{(2)}(z);\quad \Psi_n(z)=\Psi_n^{(1)}(z)+i\Psi_n^{(2)}(z);$$

$$\Theta_n(\rho)=\Theta_n^{(1)}(\rho)+i\Theta_n^{(2)}(\rho);\quad \Lambda_n(\rho)=\Lambda_n^{(1)}(\rho)+i\Lambda_n^{(2)}(\rho).$$

In view of the fact that $\theta(\rho, \varphi, z, t)$ is a real-valued function, let us confine ourselves by considering only $\theta_n(\rho, z, t)$ for $n = 0, 1, 2, \ldots$, because $\theta_n(\rho, z, t)$ and $\theta_{-n}(\rho, z, t)$ are complexly conjugate [4]. By putting values of functions from (6) into (2–5) we can obtain the following system of differential equations

$$\frac{\partial\theta_n^{(i)}}{\partial t}+\vartheta_n^{(i)}\theta_n^{(m_i)}+\tau_r\frac{\partial^2\theta_n^{(i)}}{\partial t^2}+\tau_r\vartheta_n^{(i)}\frac{\partial\theta_n^{(m_i)}}{\partial t}= =\frac{a}{R^2}\left[\frac{\partial^2\theta_n^{(i)}}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial\theta_n^{(i)}}{\partial\rho}-\frac{n^2}{\rho^2}\theta_n^{(i)}+\chi\frac{\partial^2\theta_n^{(i)}}{\partial z^2}\right], \quad (7)$$

for initial conditions

$$\theta_n^{(i)}(\rho,z,0)=0;\qquad \frac{\partial\theta_n^{(i)}(\rho,z,0)}{\partial t}=0, \quad (8)$$

and boundary conditions

$$\left\{\begin{matrix}\int_0^t\left.\frac{\partial\theta_n^{(i)}}{\partial\rho}\right|_{\rho=\varsigma_1(z)}e^{\frac{\zeta-t}{\tau_r}}d\zeta=\Psi_n^{(i)}(z)\\ \int_0^t\left.\frac{\partial\theta_n^{(i)}}{\partial\rho}\right|_{\rho=\varsigma(z)}e^{\frac{\zeta-t}{\tau_r}}d\zeta=G_n^{(i)}(z)\end{matrix}\right. ; \quad (9)$$

$$\left\{\begin{matrix}\int_0^t\left.\frac{\partial\theta_n^{(i)}}{\partial\rho}\right|_{z=0}e^{\frac{\zeta-t}{\tau_r}}d\zeta=\Theta_n^{(i)}(\rho)\\ \int_0^t\left.\frac{\partial\theta_n^{(i)}}{\partial\rho}\right|_{z=1}e^{\frac{\zeta-t}{\tau_r}}d\zeta=\Lambda_n^{(i)}(\rho)\end{matrix}\right. , \quad (10)$$

where $\vartheta_n^{(1)}=-\omega n$; $\vartheta_n^{2}=\omega n$; $m_1=2$, $m_2=1$; $i=1, 2$.

Let us employ the Laplace integral transform [4] for the system of differential equations (7) with conditions (8–10)

$$\tilde{f}(s)=\int_0^\infty f(\tau)\,e^{-s\tau}\,\mathrm{d}\tau.$$

As a result, we obtain the following system of differential equations

$$s\tilde{\theta}_n^{(i)}+\vartheta_n^{(i)}\left(\tilde{\theta}_n^{(m_i)}+\tau_r s\tilde{\theta}_n^{(m_i)}\right)+\tau_r s^2\tilde{\theta}_n^{(i)}= =\frac{a}{R^2}\left[\frac{\partial^2\tilde{\theta}_n^{(i)}}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial\tilde{\theta}_n^{(i)}}{\partial\rho}-\frac{n^2}{\rho^2}\tilde{\theta}_n^{(i)}+\chi\frac{\partial^2\tilde{\theta}_n^{(i)}}{\partial z^2}\right], \quad (11)$$

with boundary conditions

$$\left.\frac{\partial\tilde{\theta}_n^{(i)}}{\partial\rho}\right|_{\rho=\varsigma_1(z)}=\tilde{\Psi}_n^{(i)}(z);\quad \left.\frac{\partial\tilde{\theta}_n^{(i)}}{\partial\rho}\right|_{\rho=\varsigma(z)}=\tilde{G}_n^{(i)}(z); \quad (12)$$

$$\left.\frac{\partial\tilde{\theta}_n^{(i)}}{\partial z}\right|_{z=0}=\tilde{\Theta}_n^{(i)}(\rho);\quad \left.\frac{\partial\tilde{\theta}_n^{(i)}}{\partial z}\right|_{z=1}=\tilde{\Lambda}_n^{(i)}(\rho), \quad (13)$$

where

$$\tilde{\Psi}_n^{(i)}(z)=\Psi_n^{(i)}(z)(z)\left(1+\frac{1}{s\tau_r}\right);\quad \tilde{G}_n^{(i)}(z)=G_n^{(i)}(z);$$

$$\left(1+\frac{1}{s\tau_r}\right);\quad \tilde{\Theta}_n^{(i)}(\rho)=\Theta_n^{(i)}(\rho)\left(1+\frac{1}{s\tau_r}\right);$$

$$\tilde{\Lambda}_n^{(i)}(z)=\Lambda_n^{(i)}(z)\cdot\left(1+\frac{1}{s\tau_r}\right),\quad (i=1,2).$$

In order to solve the boundary equation (11–13), let us apply the following integral transformation

$$\bar{f}(\mu_{n,k})=\iint_D Q(\mu_{n,k},\rho,z)\cdot\rho\cdot f(\rho,z)d\sigma. \quad (14)$$

Proper functions $Q(\mu_{n,k}, \rho, z)$ and proper values $\mu_{n,k}$ can be defined by solving the Sturm-Liouville problem

$$\frac{\partial^2 Q}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial Q}{\partial\rho}-\frac{n^2}{\rho^2}Q+\mu^2{}_{n,k}\cdot Q+\chi\frac{\partial^2 Q}{\partial z^2}=0; \quad (15)$$

$$\left\{\begin{matrix}\left.\frac{\partial Q}{\partial\rho}\right|_{\rho=\varsigma_1(z)}=0, & \left.\frac{\partial Q}{\partial\rho}\right|_{\rho=\varsigma(z)}=0\\ \left.\frac{\partial Q}{\partial z}\right|_{z=0}=0, & \left.\frac{\partial Q}{\partial z}\right|_{z=1}=0\end{matrix}\right. . \quad (16)$$

Proper functions $Q(\mu_{n,k}, \rho, z)$ and proper values $\mu_{n,k}$ of the Sturm-Liouville problem (15–16) can be defined by formulas, which are shown in [4], and formula of inverse transformation can be expressed in the following way

$$f(\rho,z)=\sum_{j=1}^{\infty}\frac{Q(\mu_{n,k},\rho,z)}{\left\|Q(\mu_{n,k},\rho,z)\right\|^2}\bar{f}(\mu_{n,k}). \quad (17)$$

Let us apply the integral transformation (14) to the system of integral equations (11) with boundary conditions (12–13), and, as a result, we obtain the following system of ordinary algebraic equations relatively to $\bar{\bar{\theta}}_n^{(i)}$

$$s\bar{\bar{\theta}}_n^{(i)}+\vartheta_n^{(i)}\left(\bar{\bar{\theta}}_n^{(m_i)}+\tau_r s\bar{\bar{\theta}}_n^{(m_i)}\right)+\tau_r s^2\bar{\bar{\theta}}_n^{(i)}=q_{n,k}\left(\frac{\tilde{\Omega}_{n,k}^{(i)}}{\mu^2_{n,k}}-\bar{\bar{\theta}}_n^{(i)}\right), \quad (18)$$

where

$$\tilde{\Omega}_{n,k}^{(i)}=\int_0^1\Big[\varsigma(z)\cdot Q\big(\mu_n,\varsigma(z),z\big)\cdot\tilde{G}_n^{(i)}(z)-\varsigma_1(z)\times$$
$$\times Q\big(\mu_n,\varsigma_1(z),z\big)\cdot\tilde{\Psi}_n^{(i)}(z)\Big]\cdot dz+$$
$$+\chi\oint_L\rho\left(Q\big(\mu_{n,k},\rho,z\big)\frac{\partial\bar{\theta}_n^{(i)}}{\partial z}-\bar{\theta}_n^{(i)}\frac{\partial Q\big(\mu_{n,k},\rho,z\big)}{\partial z}\right)d\rho;$$

$$q_{n,k}=\frac{a}{R^2}\mu^2{}_{n,k},\quad i=1,2.$$

Curvilinear integral is calculated by closed positively oriented contour *ADCB* (Fig. 2).

By solving the system of equations (18), we obtain

$$\bar{\bar{\theta}}_n^{(i)}=\alpha_{n,k}\frac{\tilde{\Omega}_{n,k}^{(i)}\cdot\sigma_{n,k}+(-1)^{i+1}\omega n\tilde{\Omega}_{n,k}^{(m_i)}(1+s\tau_r)}{(\sigma_{n,k})^2+\omega^2n^2(1+s\tau_r)^2}, \quad (19)$$

where $\alpha_{n,k}=\dfrac{a}{R^2}$; $\sigma_{n,k}=\tau_r s^2+s+q_{n,k}$, $i=1, 2$.

By applying formula of the Laplace inverse transformation [4] to the function (19), we obtain the real-valued functions

$$\bar{\theta}_n^{(1)}(\mu_{n,k},t)=\sum_{j=1}^{2}\zeta_{n,k}(s_j)\cdot\Big\{\tilde{\Omega}_{n,k}^{(1)}(s_j)\cdot\big[(2\tau_r s_j+1)+\tau_r\omega ni\big]+$$
$$+\tilde{\Omega}_{n,k}^{(2)}(s_j)\cdot\big[\tau_r\omega n-(2\tau_r s_j+1)i\big]\Big\}\cdot\left(e^{s_jt}-1\right)+\sum_{j=3}^{4}\zeta_{n,k}(s_j)\times \quad (20)$$
$$\times\Big\{\tilde{\Omega}_{n,k}^{(1)}(s_j)\cdot\big[(2\tau_r s_j+1)-\tau_r\omega ni\big]+\tilde{\Omega}_{n,k}^{(2)}(s_j)\times$$
$$\times\big[\tau_r\omega n+(2\tau_r s_j+1)i\big]\Big\}\cdot\left(e^{s_jt}-1\right);$$

$$\bar{\theta}_n^{(2)}(\mu_{n,k},t)=\sum_{j=1}^{2}\zeta_{n,k}(s_j)\Big\{\tilde{\Omega}_{n,k}^{(2)}(s_j)\cdot\big[(2\tau_r s_j+1)+\tau_r\omega ni\big]-$$
$$-\tilde{\Omega}_{n,k}^{(1)}(s_j)\cdot\big[\tau_r\omega n-(2\tau_r s_j+1)i\big]\Big\}\left(e^{s_jt}-1\right)+\sum_{j=3}^{4}\zeta_{n,k}(s_j)\times \quad (21)$$
$$\times\Big\{\tilde{\Omega}_{n,k}^{(2)}(s_j)\cdot\big[(2\tau_r s_j+1)-\tau_r\omega ni\big]-\tilde{\Omega}_{n,k}^{(1)}(s_j)\times$$
$$\times\big[\tau_r\omega n+(2\tau_r s_j+1)i\big]\Big\}\cdot\left(e^{s_jt}-1\right),$$

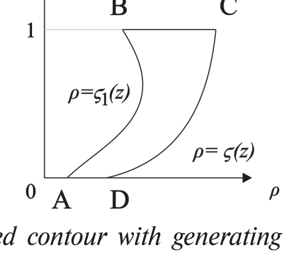

Fig. 2. Closed contour with generating lines $\rho=\varsigma_1(z)$, $\rho=\varsigma(z)$

where $\zeta_{n,k}(s_j)=\dfrac{0.5s_j^{-1}\alpha_{n,k}}{(2\tau_r s_j+1)^2+(\tau_r\omega n)^2}$, and values of s_j for $j=1, 2, 3, 4$ are defined by the following formulas

$$s_{1,2}=\frac{(\tau_r\omega ni-1)\pm\sqrt{(1+\tau_r\omega ni)^2-4\tau_r q_{n,k}}}{2\tau_r};$$

$$s_{3,4}=\frac{(\tau_r\omega ni+1)\pm\sqrt{(1-\tau_r\omega ni)^2-4\tau_r q_{n,k}}}{2\tau_r}.$$

Thus, by applying formulas of inverse transformations (6) and (17), we obtain the temperature field of the rotary body, which rotates with the constant angular velocity ω around the axis *OZ*, taking into account finite velocity of the heat conductivity

$$\theta(\rho,\varphi,z,t)=\sum_{n=-\infty}^{+\infty}\left\{\sum_{k=1}^{\infty}\Big[\bar{\theta}_n^{(1)}(\mu_{n,k},t)+i\cdot\bar{\theta}_n^{(2)}(\mu_{n,k},t)\Big]\times\right.$$
$$\left.\times\frac{Q(\mu_{n,k},\rho,z)}{\|Q(\mu_{n,k},\rho,z)\|^2}\right\}\cdot\exp(in\varphi),$$

where values $\bar{\theta}_n^{(1)}(\mu_{n,k},t)$ and $\bar{\theta}_n^{(2)}(\mu_{n,k},t)$ are defined by the formulas (20, 21).

Conclusions. It is the first mathematic model, which is created for calculating temperature fields in the *empty isotropic rotary body* taking into account finite velocity of the heat conductivity, and which is made in the form of a boundary problem of mathematical physics for hyperbolic equations of the heat conductivity with the Neumann boundary conditions. An integral transformation was formulated for the 2D finite space, with the help of which a temperature field in the empty isotropic rotary body was determined in the form of convergence series by the Fourier functions. The obtained solution of the generalized boundary problem of heat exchange in the isotropic rotary body, which rotates taking into account finite velocity of the heat conductivity, can be used for modeling temperature fields, which occur in different technical systems (satellites, forming rolls, turbines, and others).

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Математична модель і метод рішення узагальненої задачі Неймана теплообміну порожнього ізотропного тіла обертання

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Мета. Побудова нової узагальненої просторової математичної моделі розрахунку температурних полів у порожньому ізотропному тілі обертання з відомими рівняннями твірних ліній, що обертається з постійною кутовою швидкістю, з урахуванням кінцевої швидкості поширення тепла у вигляді крайової задачі математичної фізики для гіперболічного рівняння теплопровідності, а також знаходження рішень отриманої крайової задачі.

Методика. Використання відомих інтегральних перетворень Лапласа, Фур'є, а також розробленого нового інтегрального перетворення для двовимірного кінцевого простору.

Результати. Знайдено нестаціонарне температурне поле в порожньому ізотропному тілі, що обертається з постійною кутовою швидкістю навколо осі OZ, з урахуванням кінцевої швидкості поширення тепла за умови, що теплофізичні властивості тіла є постійними, а внутрішні джерела тепла відсутні. У початковий момент часу температура тіла є постійною, а на зовнішній поверхні тіла відомі значення теплових потоків.

Наукова новизна. Уперше побудована узагальнена просторова математична модель розрахунку температурних полів у порожньому ізотропному тілі обертання, обмеженому торцями й бічною поверхнею обертання, що обертається з постійною кутовою швидкістю навколо осі OZ, з урахуванням кінцевої швидкості розповсюдження тепла у вигляді крайової задачі Неймана. У роботі побудовано інтегральне перетворення для двовимірного кінцевого простору, із застосуванням якого знайдено температурне поле порожнього ізотропного тіла обертання у вигляді збіжних рядів по функціям Фур'є.

Практична значимість. Знайдено нестаціонарне температурне поле в порожньому ізотропному тілі обертання з урахуванням кінцевої швидкості поширення тепла, що може знайти застосування при моделюванні температурних полів, які виникають у багатьох технічних системах (у супутниках, сортопрокатних валках, роторах енергетичних агрегатів, дискових гальмах й ін.).

Ключові слова: *крайова задача Неймана, інтегральне перетворення, час релаксації, порожнє тіло обертання, температурне поле*

Математическая модель и метод решения обобщенной задачи Неймана теплообмена пустого изотропного тела вращения

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Цель. Построение новой обобщенной пространственной математической модели расчета температурных полей в пустом изотропном теле вращения с известными уравнениями образующих линий, вращающемся с постоянной угловой скоростью, с учетом конечной скорости распространения тепла в виде краевой задачи математической физики для гиперболического уравнения теплопроводности, а также нахождение решений полученной краевой задачи.

Методика. Использование известных интегральных преобразований Лапласа, Фурье, а также разработанного нового интегрального преобразования для двумерного конечного пространства.

Результаты. Найдено нестационарное температурное поле в пустом изотропном теле, которое вращается с постоянной угловой скоростью вокруг оси OZ, с учетом конечной скорости распространения тепла при условии, что теплофизические свойства тела являются постоянными, а внутренние источники тепла отсутствуют. В начальный момент времени температура тела является постоянной, а на наружной поверхности тела известны значения тепловых потоков.

Научная новизна. Впервые построена обобщенная пространственная математическая модель расчета температурных полей в пустом изотропном теле вращения, ограниченном торцами и боковой поверхностью вращения, которое вращается с постоянной угловой скоростью вокруг оси OZ, с учетом конечной скорости распространения тепла в виде краевой задачи Неймана. В работе построено интегральное преобразование для двумерного конечного пространства, с применением которого найдено температурное поле пустого изотропного тела вращения в виде сходящихся рядов по функциям Фурье.

Практическая значимость. Найдено нестационарное температурное поле в пустом изотропном теле вращения с учетом конечной скорости распространения тепла, что может найти применение при моделировании температурных полей, которые возникают во многих технических системах (в спутниках, сортопрокатных валках, роторах энергетических агрегатов, дисковых тормозах и др.).

Ключевые слова: *краевая задача Неймана, интегральное преобразование, время релаксации, пустое тело вращения, температурное поле*

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