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ANALYSIS OF CALCULATION MODELS WHILE SOLVING GEOMECHANICAL PROBLEMS IN ELASTIC APPROACH

Purpose. To carry out comparative analysis of results concerning determination of stress-strain state (SSS) of shallow mine workings using calculation models for problems in elastic approach.

Methodology. Rock mechanics methods; methods of analysis of the results of theoretical calculations and numerical experiments using FEM.

Findings. Algorithm to calculate SSS of rock mass, involving extended mine working, has been substantiated. Numerical experiment has demonstrated that to determine displacements within a mine working boundary it is required to consider plane strain state. It has been identified that the use of a plane stress state results in misinterpreted values of mine working boundary displacements to compare with accurate ones. The following basic conclusion has been made: it is possible to use adequate three-dimensional (i.e. spatial) problem while arranging relations according to certain rules to solve the considered problem with a calculation model of limited length as for the research subject.

Originality. Divergence has been determined between extended mine working boundary displacements calculated using calculation models of a plane stress state (approximate analytical model), and those of a plane strain (accurate calculation model). It has been demonstrated that calculation of extended mine workings should involve analytical model of a plane deformation reflecting SSS of *rock mass-mine working* system adequately; otherwise, calculation errors may be more than 100 % for horizontal displacements, and 25 % for vertical ones.

Practical value. The obtained results make it possible to select reasonably calculation models for extended mine workings where support stiffness properties vary periodically. For instance, such mine workings are meant whose stability and deformability are provided by frame, anchor, combined frame-anchor, complete reinforced-concrete, reinforced equally distanced sequences of anchors, and similar support structures.

Keywords: *extended mine working, rock mass, stress state, deformation, geomechanical model*

Analysis of progress in the area. Currently, the majority of basic methods, aimed at determination of stress-strain state (SSS) of rock mass, rely upon the use of a calculation model corresponding to a “plane problem”.

In this context, it is often done as follows:

1. A 1 m width fragment is cut out of rock mass, containing a mine working.
2. SSS of rock *mass-mine working* geomechanical system [1] is calculated for the condition of a plane stress state.

The fact that rocks in the neighbourhood of the mine working are in the state of a plane deformation is not involved while solving such problems using traditional approaches of elasticity theory; the fact can be explained by their minor deformability.

Due to initially included simplifications to decrease labour costs, the majority of classic approaches of elasticity theory cannot solve numerous arising problems properly, and meet the requirements concerning accuracy, amounts, calculating speed, and expenditures. Thus, solving even relatively small local geomechanical problems [2] is connected with carrying out of volume complex research. Currently, such problems are solved more efficiently with the use of modern potential of computer technology as well as specialized software systems [3]. In this context, calculation involves such peculiarities as scale effect, and coefficients, reducing hardness of rocks [4]; thus, it helps carry out multivariant modelling of experimental object, phenomenon, or situation [5]. As a result, relatively small costs make it possible to forecast qualitatively both design and operational expenditures for specific mining and geological conditions [6]. Moreover, each result of analytical studies is tested in practice [7] aside from continuous monitoring of the studied object state [8].

In the large, it enables to represent adequately deformational processes progressing in the neighbourhood of geotechnical systems with a probability of 95 %. Among other things, it concerns the processes, resulting in the loss of elastoplastic stability of rock mass, loose by a system of underground cavities.

Objective of the research is to make comparative analysis of the results concerning determination of stress-strain state of shallow geotechnical systems with the use of calculation models for problems with elastic approach.

Materials and methods. Stage one of the research involved numerical experiment during which displacements of mine working boundaries were compared. The

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mine working had four-square 6 × 6 cross section; its initial depth was 30 meters (Fig. 1).

All the calculations were performed for rocks whose average values of physical and mechanical characteristics are presented in Fig. 1.

To minimize boundary effect within the central part of the model, where the mine working is located, dimensions of rock mass area were taken as those, being equal to 66 × 66 meters on the basis of

$$B = 11 \cdot b,$$

where B is width and height of the design area; b is width and height of the mine working.

Moreover, to interpret the obtained data conveniently, Poisson ratio and specific weight were specified as similar ones for all the rocks.

Rock mass weight was external load acting on the study area.

During calculation in the context of a plane stress state, SSS of a "plate" (i. e. cross section of the rock mass containing the mine working) with 1 m width was determined involving characteristics of rocks represented in Table 1.

SSS of similar "plate" for a plane deformation was determined using characteristics from Table 2.

In this process, reduced characteristics of rock mass, containing mine working, were used instead of actual ones. The characteristics were determined on the common formulas resulting from the generalized Hooke's law corresponding to a plane stress state, and plane deformation condition

$$\left.\begin{aligned} v^* &= \frac{v}{1-v} \\ E^* &= \frac{E}{1-v^2} \end{aligned}\right\}. \tag{1}$$

In this context, v^* and E^* are reduced values of Poisson ration and modulus of total rock deformation respectively; v and E are their actual values.

Equalities (1) were derived basing upon the following arguments:

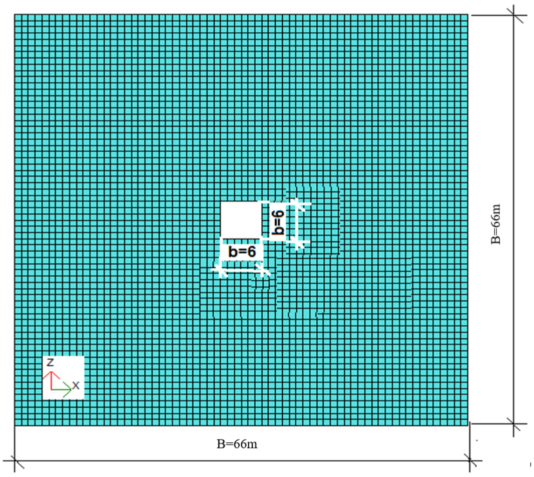

Fig. 1. Calculation model to determine displacements of the mine working boundary

Table 1

Specified characteristics of rocks

Rock	Specific weight γ, kN/m^3	Poisson ratio *v*, unit	Elasticity modulus *E*, MPa
Argillaceous shale	20.0	0.3	20 000
Sandstone and limestone	20.0	0.3	40 000
Marble and granite	20.0	0.3	60 000
Basalt and quartzite	20.0	0.3	100 000

Table 2

Reduced properties of rocks

Rock	Specific weight γ, kN/m^3	Poisson ratio *v*, unit	Elasticity modulus *E*, MPa
Argillaceous shale	20.0	0.43	21 978
Sandstone and limestone	20.0	0.43	43 956
Marble and granite	20.0	0.43	65 934
Basalt and quartzite	20.0	0.43	109 890

1. For the cases of a plate stress and a plane deformation within 0_{XYZ} coordinate system within 0_{XZ} (Fig. 1), equilibrium equations are completely identical, and look like

$$\left.\begin{aligned} \frac{\partial\sigma_X}{\partial_X} + \frac{\partial\tau_{XZ}}{\partial_Z} + X = 0 \\ \frac{\partial\sigma_Z}{\partial_Z} + \frac{\partial\tau_{Xz}}{\partial_X} + Z = 0 \end{aligned}\right\}. \tag{2}$$

In this context, σ_X and σ_Z are normal stresses; τ_{XZ} is the same, tangential; X and Z are projections on coordinate axes of a volume force whose dimension is (kN/m^3).

In such a case, normal stress σ_Y, acting towards axis 0_Y, is foregone (it is equal to zero for a plate stress state (i. e. $\sigma_Y = 0$); it is equal to $\sigma_Y = v \cdot (\sigma_X + \sigma_Z)$ if the deformation is a plane one. Moreover, despite a plane SSS type, tangential stresses τ_{XY} and τ_{YZ}, acting within 0_{xy} and 0_{xz} planes, are also equal to zero.

2. Equations of state (i. e. the generalized Hooke's law) for a case of a plane stress state (in this case, normal stresses towards 0_Y axis are equal to zero, i. e. $\sigma_Y = 0$) are

$$\left.\begin{aligned} \varepsilon_X &= \frac{1}{E} \cdot \left(\sigma_X - v \cdot \sigma_Z\right) \\ \varepsilon_Z &= \frac{1}{E} \cdot \left(\sigma_Z - v \cdot \sigma_X\right) \\ \varepsilon_{XZ} &= \frac{2 \cdot \left(1+v\right)}{E} \cdot \gamma_{XZ} \end{aligned}\right\}. \tag{3}$$

In this context, ε_X and ε_Z are normal relative deformations towards 0_x and 0_z axes respectively; γ_{XZ} is shear deformation within 0_{XZ} plane.

3. Equations of state (i. e. the generalized Hooke's law) for a case of a plane stress state (in this case, normal stresses towards 0_Y axis are equal to $\sigma_Y = v \cdot (\sigma_X + \sigma_Z)$ are

$$\left.\begin{aligned} \varepsilon_X &= \frac{1}{E}\cdot\left[\left(1-\nu^2\right)\cdot\sigma_X-\nu\cdot\left(1+\nu\right)\cdot\sigma_Z\right]\\ \varepsilon_Z &= \frac{1}{E}\cdot\left[\left(1-\nu^2\right)\cdot\sigma_Z-\nu\cdot\left(1+\nu\right)\cdot\sigma_X\right]\\ \tau_{XZ} &= \frac{2\cdot\left(1+\nu\right)}{E}\cdot\gamma_{XZ} \end{aligned}\right\}. \tag{4}$$

If in (4) we express actual deformational material constants ν and E using (1), we obtain an equation system which is absolutely identical to (3).

The stated helps conclude that it is quite sufficient to consider a structure being in a stress state (in this case, it is a fragment of rock mass containing a mine working) for accurate modelling of a plane deformation condition assuming reduced analogues of actual deformational constants calculated according to (1).

To determine displacements of mine working boundaries [9], a well-reputed world practice technique to solve geomechanical problems using FEM was applied in a software system *Lira*.

To determine divergence between displacement points within mine working boundaries for the considered cases, the following common formulas were used to determine a maximum quadratic error (on the modulus), and a square one

$$\left.\begin{aligned} \Delta &= \max\left|1-\frac{U_{pns}}{U_{pd}}\right|\cdot 100\,\%\\ \sigma &= \sqrt{\frac{1}{n}\cdot\sum_{i=1}^{n}\left(1-\frac{U_{pns}}{U_{pd}}\right)^2\cdot 100\,\%} \end{aligned}\right\}. \tag{5}$$

In this context, U_{pns} are displacements or forces, calculated in terms of analytical model of a plane stress state; U_{pd} are the same in terms of a plane deformation scheme; Δ is a maximum error; σ is a mean-square error.

Since it is quite important to know the ratio between the mine working boundary displacements in different directions while determining its deformations, formulas of the following type were used

$$\left.\begin{aligned} \delta_{pns} &= \left|\frac{U_{z,\,pns}}{U_{x,\,pns}}\right|\\ \delta_{pnd} &= \left|\frac{U_{z,\,pnd}}{U_{x,\,pnd}}\right| \end{aligned}\right\}, \tag{6}$$

where δ is absolute value of vertical displacement of a certain point within the mine working boundary ratio (Fig. 2) U_z to a horizontal displacement U_x. In (6) formula, "*pns*" index corresponds to analytical model of a plane stress state, and "*pnd*" index corresponds to analytical model of a plane deformation.

The obtained boundary displacements correspond to the conditions of the mine working, placed within argillaceous shale; actual values are $E = 20\,000$ MPa, $\nu = 0.3$; reduced values are $E = 21\,978$ MPa, $\nu = 0.43$ (Table 3).

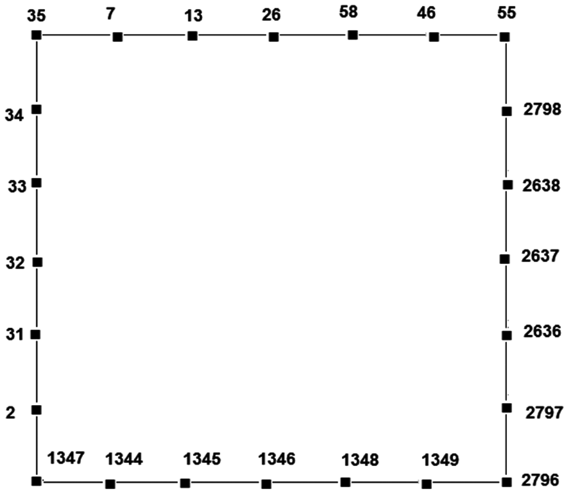

Fig. 2. Enumeration of nodes of the mine working boundary in which displacements were determined

Table 4 demonstrates divergence between the displacements, calculated by (5). The divergence concerns those, determined with the help of different analytical models, and those, determined with the help of (6).

Note: Table 4 should be considered together with Table 3.

Table 3

Scheduled displacements of the mine working boundary

Node	Displacements, mm			
	Plane stress state		Plane deformation	
	U_x	U_z	U_x	$U_{z,i}$
2	0.022	–1.383	0.054	–1.126
7	–0.017	–1.746	–0.002	–1.443
13	–0.011	–1.776	–0.003	–1.470
26	–	–1.786	–	–1.479
31	0.030	–1.440	0.064	–1.175
32	0.033	–1.489	0.067	–1.217
33	0.029	–1.538	0.062	–1.257
34	0.020	–1.590	0.050	–1.303
35	–0.003	–1.662	0.017	–1.366
46	0.017	–1.746	0.002	–1.443
55	0.003	–1.662	–0.017	–1.366
58	0.011	–1.776	0.003	–1.470
1345	–0.012	–1.172	–0.004	–0.936
1346	–	–1.161	–	–0.926
1347	–0.003	–1.303	0.019	–1.056
1348	0.012	–1.172	0.004	–0.936
1349	0.020	–1.207	0.002	–0.967
2636	–0.030	–1.440	–0.064	–1.175
2637	–0.033	–1.489	–0.067	–1.217
2638	–0.029	–1.538	–0.062	–1.257
2796	0.003	–1.303	–0.019	–1.056
2797	–0.022	–1.383	–0.054	–1.126
2798	–0.020	–1.590	–0.050	–1.303

Table 4

Ratios between the scheduled displacements

Node	Maximum relative divergence, %		Displacement ratios (times)	
	Δ_x	Δ_z	δ_{pns}	δ_{pnd}
2	59	23	63	21
7	750	21	103	722
13	267	21	161	490
26	–	21	–	–
31	53	23	48	18
32	51	22	45	18
33	53	22	53	20
34	60	22	80	26
35	118	22	554	80
46	750	21	103	722
55	118	22	554	80
58	267	21	161	490
1344	900	25	60	484
1345	200	25	98	234
1346	–	25	–	–
1347	116	23	434	56
1348	200	25	98	234
1349	900	25	60	484
2636	53	23	48	18
2637	51	22	45	18
2638	53	22	53	20
2796	116	23	434	56
2797	59	23	63	21
2798	60	22	80	26

The data, represented in Tables 3 and 4, make it possible to make following conclusions:

1. Values of vertical displacements of points within a boundary of a mine working, arranged within argillaceous shale, are almost 1–2 mm units. Horizontal displacements of the points are 20–700 times less than vertical ones.

2. Vertical displacements, calculated within a plane deformation model, are less than those, determined within a plane stress model. Relative divergence is 20–25 %.

3. It turned out to be impossible to determine similar regularity for horizontal displacements. Divergence between horizontal deformations, determined with the help of different calculation models, varies within 59–900 %. Table 5 demonstrates the results of displacement determinations of a mine working arranged within sandstones (or limestones) for such actual values as $E =$ $= 40\ 000$ MPa and $\nu = 0.3$, and such reduced values as $E = 43\ 956$ MPa and $\nu = 0.43$.

Table 6 demonstrates divergence between the displacements, determined with the help of different analytical models calculated by formula (5), and those, calculated by (6).

Note: Table 6 should be considered together with Table 5.

It follows from Tables 5 and 6 data that:

1. Vertical displacements within a boundary of a mine working, arranged in sandstones (or limestones) are of tenths of a millimetre. Horizontal displacements of the points are 20–650 times less than vertical ones.

2. Vertical displacements, calculated with the help of a plane deformation model are less than those, determined using a plane stress model; relative divergence is 21–25 %.

3. Divergence of values of horizontal deformations is 53–900 % if different calculation models are used.

Table 7 shows the displacements, obtained for a mine working, arranged within marble (granite). Actual values are $E = 60\ 000$ MPa and $\nu = 0.3$; reduced ones are $E = 65\ 934$ MPa and $\nu = 0.43$.

Table 8 demonstrates divergence between the displacements, determined with the help of different analytical models, and calculated by formula (5), and those, calculated by (6).

Table 5

Calculated displacements of the mine working boundary

Node	Displacements, mm			
	Plane stress state		Plain deformation	
	U_x	U_z	U_x	$U_{z,i}$
2	0.011	–0.692	0.027	–0.56
7	–0.009	–0.873	–0.001	–0.72
13	–0.005	–0.888	–0.002	–0.74
26	0	–0.893	0	–0.74
31	0.015	–0.720	0.032	–0.59
32	0.016	–0.745	0.034	–0.61
33	0.014	–0.769	0.031	–0.63
34	0,010	–0.795	0.025	–0.65
35	–0.002	–0.831	0.009	–0.68
46	0.009	–0.873	0.001	–0.72
55	0.002	–0.831	–0.009	–0.68
58	0.005	–0.888	0.002	–0.74
1344	–0.010	–0,603	–0.001	–0.48
1345	–0.006	–0.586	–0.002	–0.47
1346	0	–0.580	0	–0.46
1347	–0.001	–0.651	0.009	–0.53
1348	0.006	–0.586	0.002	–0.47
1349	0.010	–0.603	0.001	–0.48
2636	–0.015	–0.720	–0.032	–0.59
2637	–0.016	–0.745	–0.034	–0.61
2638	–0.014	–0.769	–0.031	–0.63
2796	0.001	–0.651	–0.009	–0.53
2797	–0.011	–0.692	–0.027	–0.56
2798	–0.010	–0.795	–0.025	–0.65

Table 6

Ratios between the calculated displacements

Node	Maximum relative divergence, %		Displacement ratios (times)	
	Δ_x	Δ_z	δ_{pns}	δ_{pnd}
2	59	23	63	21
7	800	21	63	21
13	150	21	97	722
26	–	21	178	368
31	53	23	–	–
32	53	23	48	18
33	55	22	47	18
34	60	22	55	20
35	122	22	80	26
46	800	21	416	76
55	122	22	97	722
58	150	21	416	76
1344	900	25	178	368
1345	200	25	60	484
1346	–	25	98	234
1347	111	23	–	–
1348	200	25	651	59
1349	900	25	98	234
2636	53	23	60	484
2637	53	23	48	18
2638	55	22	47	18
2796	111	23	55	20
2797	59	23	651	59
2798	60	22	63	21

Note: Table 8 should be considered together with Table 7.

Data, contained in Tables 7 and 8, makes it possible to conclude the following:

1. Vertical displacements of a boundary of a mine working, arranged in marble (granite) are of tenths of a millimetre. In this context, horizontal displacements of the check points are 20–550 times less than vertical ones.

2. Deformations, calculated with the help of a plane deformation model, are less than those, determined using a plane stress model (relative divergence is 21–25 %).

3. Divergence between horizontal deformations, determined using different analytical models, varies within 53–600 %.

Table 9 shows the displacements, obtained for a mine working, arranged within basalt (quartzite). Actual values are E = 100 000 MPa and ν = 0.3; reduced ones are E = 109 890 MPa and ν = 0.43.

Table 10 demonstrates divergence between the displacements, determined with the help of different analytical models, and calculated by formula (5), and those, calculated by (6).

Table 7

Calculated displacements of the mine working boundary

Node	Displacements, mm			
	Plane stress state		Plane deformation	
	U_x	U_z	U_x	$U_{z,i}$
2	0.007	–0.461	0.018	–0.375
7	–0.006	–0.582	–0.001	–0.481
13	–0.004	–0.592	–0.001	–0.490
26	0.000	–0.595	0.000	–0.493
31	0.010	–0.480	0.021	–0.392
32	0.011	–0.496	0.022	–0.406
33	0.010	–0.513	0.021	–0.419
34	0.007	–0.530	0.017	–0.434
35	–0.001	–0.554	0.006	–0.455
46	0.006	–0.582	0.001	–0.481
55	0.001	–0.554	–0.006	–0.455
58	0.004	–0.592	0.001	–0.490
1344	–0.007	–0.402	–0.001	–0.322
1345	–0.004	–0.391	–0.001	–0.312
1346	0.000	–0.387	0.000	–0.309
1347	–0.001	–0.434	0.006	–0.352
1348	0.004	–0.391	0.001	–0.312
1349	0.007	–0.402	0.001	–0.322
2636	–0.010	–0.480	–0.021	–0.392
2637	–0.011	–0.496	–0.022	–0.406
2638	–0.010	–0.513	–0.021	–0.419
2796	0.001	–0.434	–0.006	–0.352
2797	–0.007	–0.461	–0.018	–0.375
2798	–0.007	–0.530	–0.017	–0.434

Note: Table 10 should be considered together with Table 9.

It follows from Tables 9 and 10 that:

1. Vertical displacements of the considered points of a boundary of a mine working, arranged in marble (granite) are of tenths of a millimetre. In this context, horizontal displacements of the points are almost 18–332 times less than vertical ones.

2. Vertical displacements, calculated within a plane deformation model, are slightly less to compare with those, determined using a plane stress model (relative divergence is 21–25 %).

3. It turned out to be impossible to determine similar regularity for horizontal displacements. Divergence between horizontal deformations, determined with the help of different analytical models, is 50–133 %.

Figs. 3 and 4 demonstrate graphically the data from Tables 2–10 calculated with the help of lower equality (5).

Analysis of the data from Figs. 3, 4 helps conclude:

1. The use of the abovementioned calculation models (i. e. a plane stress model, and a plane deformation

Table 8

Ratios between the calculated displacements

Node	Maximum relative divergence, %		Displacement ratios (times)	
	Δ_x	Δ_z	δ_{pns}	δ_{pnd}
2	61	23	66	21
7	500	21	97	481
13	300	21	148	490
26	0	21	–	–
31	52	22	48	19
32	50	22	45	18
33	52	22	51	20
34	59	22	76	26
35	117	22	554	76
46	500	21	97	481
55	117	22	554	76
58	300	21	148	490
1344	600	25	57	322
1345	300	25	98	312
1346	–	25	–	–
1347	117	23	434	59
1348	300	25	98	312
1349	600	25	57	322
2636	52	22	48	19
2637	50	22	45	18
2638	52	22	51	20
2796	117	23	434	59
2797	61	23	66	21
2798	59	22	76	26

Table 9

Calculated displacements of the mine working boundary

Node	Displacements, mm			
	Plane stress state		Plane deformation	
	U_x	U_z	U_x	$U_{z,i}$
2	0.004	–0.277	0.011	–0.23
7	–0.003	–0.349	0.000	–0.29
13	–0.002	–0.355	–0.001	–0.29
26	0.000	–0.357	0.000	–0.30
31	0.006	–0.288	0.013	–0.24
32	0.007	–0.298	0.013	–0.24
33	0.006	–0.308	0.012	–0.25
34	0.004	–0.318	0.010	–0.26
35	–0.001	–0.332	0.003	–0.27
46	0.003	–0.349	0.000	–0.29
55	0.001	–0.332	–0.003	–0.27
58	0.002	–0.355	0.001	–0.29
1344	–0.004	–0.241	0.000	–0.19
1345	–0.002	–0.234	–0.001	–0.19
1346	0.000	–0.232	0.000	–0.19
1347	–0.001	–0.261	0.004	–0.21
1348	0.002	–0.234	0.001	–0.19
1349	0.004	–0.241	0.000	–0.19
2636	–0.006	–0.288	–0.013	–0.24
2637	–0.007	–0,298	–0.013	–0.24
2638	–0.006	–0.308	–0.012	–0.25
2796	0.001	–0.261	–0.004	–0.21
2797	–0.004	–0.277	–0.011	–0.23
2798	–0.004	–0.318	–0.010	–0.26

model) makes it possible to understand that maximum divergence of calculation results takes place for horizontal displacements (maximum absolute divergence is 900 %; mean square one is 370 %, Fig. 3). In this context, it should be noted that for the considered cases, horizontal boundary deformations are 18–600 times less than vertical ones. Thus, the divergence affects total deformation moderately.

2. Vertical deformations within a mine working boundary differ to a far lesser degree; however, even such divergence is quite essential (maximum absolute divergence is 25 %; mean square one is 23 %, Fig. 3).

3. In terms of vertical displacements of a mine working boundary, dependence of error upon rock deformation properties is almost non-available; it looks like a horizontal line being practically parallel to an absciss (Fig. 4).

4. Calculation of extended mine workings should involve analytical model of a plane deformation since the errors, obtained for horizontal displacements and vertical displacements of a mine working boundary are quite significant.

Further research involved consideration of analytical model of rock mass, containing extended (i.e. unlimited towards 0_Y axis) mine working with the use of spatial analytical model having finite length, height, and width. The research is important for adequate SSS modelling of objects which stiffness characteristics vary cyclically. For instance, such a situation takes place if metal (or reinforced concrete) frame, anchor or combined support is mounted uniformly in a mine working [10] as well as a combination of the listed support variations with reinforced concrete encasement [11].

In this case, central SSS of neighbouring “compartments” will be identical; moreover, SSS changes along its length (i.e. towards 0_Y axis) will take place within each of the “compartment”.

In this context, SSS will vary cyclically along the mine working at reasonable distance from its ends within sections being perpendicular to 0_Y axis.

To solve such problems, it is necessary to consider spatial elements of *rock mass-mine working* system which are in a plane deformation state. Hence, further research studied box units (Fig. 5).

Table 10

Ratios between the calculated displacements

Node	Maximum relative divergence, %		Displacement ratios (times)	
	Δ_x	Δ_z	δ_{pns}	δ_{pnd}
2	64	23	69	20
7	–	21	116	–
13	100	21	178	294
26	–	21	–	–
31	54	23	48	18
32	46	23	43	19
33	50	23	51	21
34	60	22	80	26
35	133	22	332	91
46	–	21	116	–
55	133	22	332	91
58	100	21	178	294
1344	–	25	60	–
1345	100	25	117	187
1346	–	25	–	–
1347	125	24	261	53
1348	100	25	117	187
1349	–	25	60	–
2636	54	23	48	18
2637	46	23	43	19
2638	50	23	51	21
2796	125	24	261	53
2797	64	23	69	20
2798	60	22	80	26

In the course of the numerical experiment, displacements within a boundary of a mine working were compared in terms of a plane deformation of rock mass (Fig. 1), and in the context of a spatial deformation.

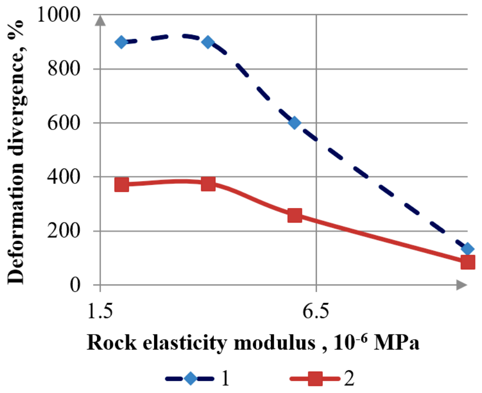

Fig. 3. Divergence between horizontal displacements calculated using different analytical models:
1 – maximum divergence; 2 – the same, mean square ones

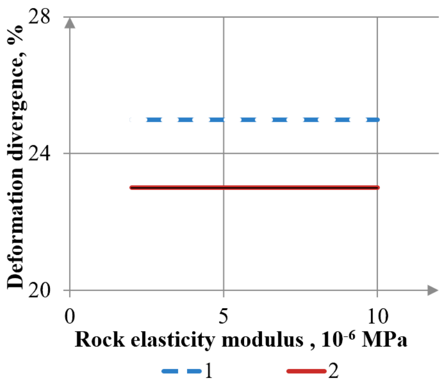

Fig. 4. Divergence between vertical displacements calculated using different analytical models:
1 – maximum divergence; 2 – the same, mean square ones

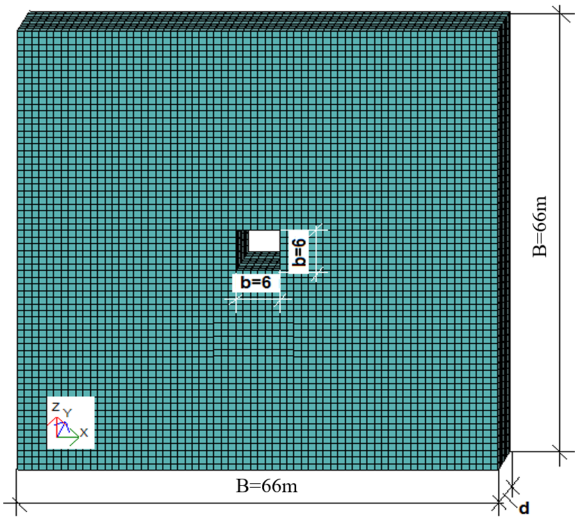

Fig. 5. Analytical model to determine SSS of an extended mine working "compartment"

To achieve close to a plane deformation state in case two (Fig. 5), the following connections were applied to the rock mass fragment surface within nodes of volume elements:

- along a lower edge (0_{XY} plane) – prohibition against displacement towards 0_Z axis;
- along the side edges within 0_{ZX} plane – prohibition against displacement towards 0_Y axis;
- along the side edges within 0_{ZX} plane – prohibition against displacement towards 0_X axis.

Understructure units with 36 × 36 m dimensions were considered.

Thickness of the units was 0.33, 1.0, and 4.0 m (sizes one and two correspond to minimum and maximum spacing of metal frame support mounting; the last one corresponds to maximum spacing of anchor support rows).

Width and height of finite elements (i.e. their dimensions towards 0_X and 0_Y axes) were taken as those being equal to a metre. Maximum divergence and average divergence of boundary displacements between corresponding analytical models, represented in Figs. 1, 5, and calculated by (3), are represented in Figs. 6, 7.

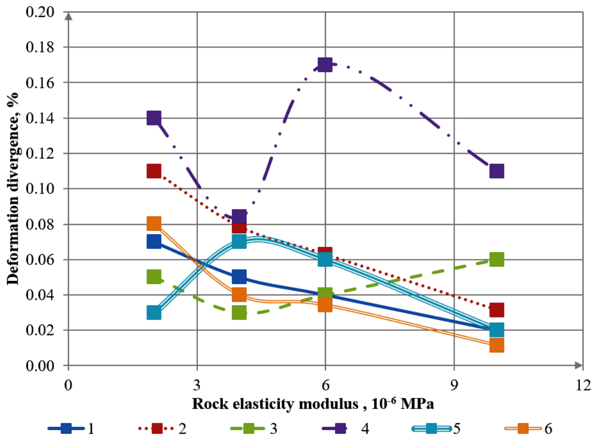

Fig. 6. Divergence between horizontal displacements, calculated with the help of different analytical models:

1, 3, and 5 – maximum divergence; Rows 2, 4, and 6 – the same, mean square ones

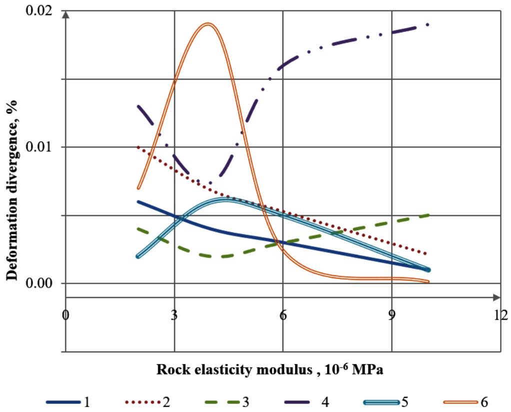

Fig. 7. Divergence between vertical displacements, calculated with the help of different analytical models:

1, 3, and 5 – maximum divergence; 2, 4, and 6 – the same, mean square ones

Analysis of curves in Figs. 6, 7 helped conclude that divergence between initial data, calculated with the help of Table 1 and analytical model, represented in Fig. 5, as well as the initial data, calculated with the help of Table 2 and analytical model, represented in Fig. 1, is less than 1 %.

The abovementioned helped conclude that the analytical model, represented in Fig. 5 and the proposed system to arrange connections makes it possible to model SSS under the conditions of a plane deformation using a spatial fragment of infinite structure. It has become possible owing to the fact that displacements of nodes of rock block were excluded towards 0_y axis while introducing additional connections; hence, a state of a spatial fragment of mine working, close to SSS of a plane deformation, has been achieved (Fig. 5).

Conclusions. The approach, stated in the paper, helps adopt an adequate analytical model being adequate to calculate SSS of extended underground mining objects whose stability is provided with the help of frame, anchor, combined frame-anchor, complete reinforced-concrete, reinforced equally distanced sequences of anchors, and combined reinforced- concrete and anchor supports.

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Аналіз розрахункових схем при вирішенні задач геомеханіки у пружній постановці

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Мета. Провести порівняльний аналіз результатів визначення напружено-деформованого стану (НДС) гірничих виробок неглибокого закладення з використанням розрахункових схем для задач у пружній постановці.

Методика. Методи механіки гірських порід. Методи аналізу результатів теоретичних розрахунків і чисельних експериментів із застосуванням МСЕ.

Результати. Обґрунтовано алгоритм розрахунку НДС породного масиву, що вміщує протяжну гірничу виробку. У ході чисельного експерименту показано, що для визначення переміщень на контурі виробки слід розглядати стан плоскої деформації. Встановлено, що використання із цією метою плоского напруженого стану призводить до спотворення значень переміщень контуру виробки в порівнянні з їх точними значеннями. Зроблено основний висновок про те, що шляхом розстановки зв'язків за певними правилами для вирішення даної задачі з розрахунковою схемою обмеженої довжини стосовно об'єкту досліджень допустиме використання відповідної тривимірної (просторової) задачі.

Наукова новизна. Встановлені розбіжності між переміщеннями контуру протяжної виробки, розрахованими в рамках розрахункових схем плоского напруженого стану (наближена розрахункова схема) і плоскої деформації (точна розрахункова схема). Показано, що при розрахунку протяжних гірничих виробок слід використовувати розрахункову схему плоскої деформації, що адекватно відображає НДС системи „породний масив – виробка“, в іншому випадку похибки розрахунків можуть становити більше 100 % для горизонтальних і 25 % для вертикальних переміщень.

Практична значимість. Отримані результати дозволяють обґрунтовано призначати розрахункові схеми для протяжних виробок, жорсткісні властивості кріплення яких періодично змінюються. Це можуть бути, наприклад, гірничі виробки, стійкість і деформативність яких забезпечується рамним, анкерним, комбінованим рамно-анкерним, суцільним залізобетонним, посиленими розставленими через рівні відстані рядами анкерами і тому подібними конструкціями кріплення.

Ключові слова: *протяжна виробка, породний масив, напружений стан, деформація, геомеханічна модель*

Анализ расчетных схем при решении задач геомеханики в упругой постановке

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Цель. Провести сравнительный анализ результатов определения напряженно-деформированного состояния (НДС) горных выработок неглубокого заложения с использованием расчетных схем для задач в упругой постановке.

Методика. Методы механики горных пород. Методы анализа результатов теоретических расчетов и численных экспериментов с применением МКЕ.

Результаты. Обоснован алгоритм расчета НДС породного массива, вмещающего протяженную выработку. В ходе численного эксперимента показано, что для определения перемещений на контуре выработки следует рассматривать состояние плоской деформации. Установлено, что использование для данной цели плоского напряженного состояния приводит к искажению значений перемещений контура выработки по сравнению с их точными значениями. Сделан основной вывод о том, что путем расстановки связей по определенным правилам для решения рассматриваемой задачи с расчетной схемой ограниченной длины применительно к объекту исследований допустимо использование соответствующей трехмерной (пространственной) задачи.

Научная новизна. Установлены расхождения между перемещениями контура протяженной выработки, рассчитанными в рамках расчетных схем плоского напряженного состояния (приближенная расчетная схема) и плоской деформации (точная расчетная схема). Показано, что при расчете протяженных горных выработок следует использовать расчетную схему плоской деформации, которая адекватно отображает НДС системы „породный массив – выработка“, в противном случае погрешности расчетов могут составлять более 100 % для горизонтальных и 25 % для вертикальных перемещений.

Практическая значимость. Полученные результаты позволяют обоснованно назначать расчетные схемы для протяженных выработок, жесткостные свойства крепи которых периодически изменяются. Это могут быть, например, горные выработки, устойчивость и деформативность которых обеспечивается рамными, анкерными, комбинированными рамно-анкерными, сплошными железобетонными, усиленными расставленными через равные расстояния рядами анкеров и им подобными конструкциями крепей.

Ключевые слова: *протяженная выработка, породный массив, напряжённое состояние, деформация, геомеханическая модель*

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