THE $K^+ \to \pi^+ \nu \bar{\nu}$ DECAY

T.Szabo

Department of Theoretical Physics, Uzhhorod National University
Voloshyna St 32, Uzhhorod 88000, Ukraine

Rare K decay amplitudes such as $K^+ \to \pi^+ \nu \bar{\nu}$ are presumed to receive contributions from two separate sources: short-distance (SD) and long-distance (LD) effects. Long-distance contribution for $K^+ \to \pi^+ \nu \bar{\nu}$ is calculated. Its magnitude is by three orders smaller than the short-distance effects.

Rare K decay amplitudes such as $K^+ \to \pi^+ \nu \bar{\nu}$ are presumed to receive contributions from two separate sources, usually referred to as short-distance (SD) and long-distance (LD) effects. The former is an effective four-fermion interaction resulting from quark and lepton contributions to a box diagram while the latter is typically due to hadronic intermediate states.

Long-range effects have been considered by a series of authors. For instance, Rein and Seghal [1], Lu and Wise [2], and Geng *et al.* [3] have evaluated in a variety of ways the effective matrix element by modeling the $K-\pi-Z$ vertex and then having the Z couple to $\nu-\bar{\nu}$. Others, such as Marciano and Passa [4] or Buchalla and Buras [5] have evaluated gluonic contributions to penguin diagrams. Bigi and Gabbiani [6] have considered extensions of the standard model including those due to right-handed currents, nonminimal Higgs sectors, additional families, and supersymmetry. None of these effects significantly enhance or suppress the standard model estimate. There is also an excellent comprehensive review of rare K decays in the encyclopedic article by Buchalla, Buras, and Lautenbacher [7]. At the same time, we should add that there is continued interest in exploring this rare decay mode as a test of the standard model. For a recent experimental review of the situation see, e.g., the paper by Adler *et al.* [8]. Part of the interest in the $K^+ \to \pi^+ \nu \bar{\nu}$ decay is that the experiment, though very difficult, is feasible and the results give us some clear information about standard model parameters since the LD effects, with their hadronic unknown quantities, appear to be clearly some three orders of magnitude smaller than the SD effects. The latter contribution to the branching ratio [9] is of order 10^{-10}, while the LD effects are of order 10^{-13}. A second reason for interest in the decay is the connection to CP violation, which could be studied in $K_L \to \pi \nu \bar{\nu}$.

In this paper we plan to study a not as of yet calculated class of long-range interactions and show how their contribution to the decay amplitude can be evaluated. The result turns out to be surprisingly simple and potentially of interest for other applications as well. The relevant diagram, similar in some ways to one estimated for double β decay, is displayed in Fig.1. To complete the calculation we must calculate the amplitude for $\bar{u} s \to K^+$ and $u \bar{d} \to \pi^+$. This is easily done by using partial conservation of axial vector current; denoting by $f_{\pi,K}$ the pion and kaon decay constants, the matrix element for $K^+ \to \pi^+ \nu \bar{\nu}$ is evaluated with the result [10]

$$M = 2^4 \left(\frac{G_F}{\sqrt{2}} \right)^2 \frac{1}{2} f_\pi f_K q_1^\alpha q_2^\beta T_{\alpha\beta} \sin\theta_c \cos\theta_c.$$

In the above $T_{\alpha\beta}$ is the amplitude for $W^+W^- \to \nu \bar{\nu}$ by exchange of an electron, the central part of Fig. 1. The amplitude $T_{\alpha\beta}$ is reasonably complicated, but taking the double divergence reduces drastically the kinematic complexity, particularly in the limit of vanishing electron mass. We find for the diagram of Fig. 1 that

$$q_1^\alpha q_2^\beta T_{\alpha\beta} = -\bar{v}(p_1)\, \gamma_\sigma q_1^\sigma v(p_2) \left[1 + \frac{m_e^2}{(m_\pi^2 - 2q_1 \cdot p_1 - m_e^2} \right].$$

In the limit $m_e \to 0$, the total result for the amplitude is then

$$M = 4 \left(\frac{G_F}{\sqrt{2}} \right)^2 f_\pi f_K \bar{v}(p_1) \gamma_\sigma (q_1^\sigma - q_2^\sigma) v(p_2) \sin\theta_c \cos\theta_c,$$

and the comparison to the standard decay amplitude N for $K^+ \to \pi^0 e^+ \nu$ is very simple. We find

$$\frac{M}{N} = 2\cos\theta_c f_\pi f_K \left(\frac{G_F}{\sqrt{2}} \right),$$

so that the branching ratio for $K^+ \to \pi^+ \nu \bar{\nu}$ is approximately 2.5×10^{-14}. The muon contribution is also significant though smaller than the electron contribution since there is a partial cancellation in the expression for $q_1^\alpha q_2^\beta T_{\alpha\beta}$. Not surprisingly the cancellation is essentially complete for the τ lepton. To summarize, the LD effects for the diagrams of Fig. 1 are $\le 5\times10^{-14}$, comparable to other LD contributions, but certainly not larger.

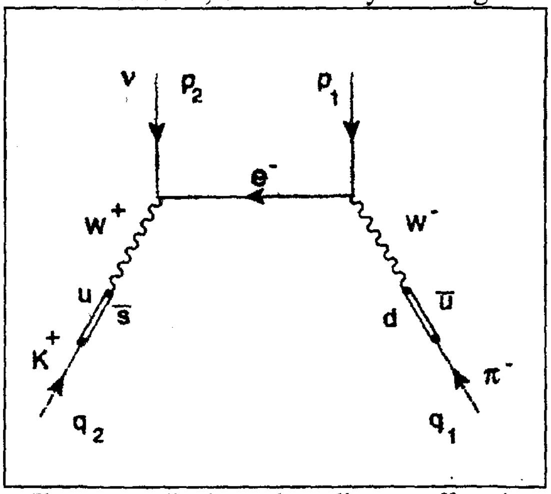

Fig. 1. Contribution to long distance effects in $K^+ \to \pi^+ \nu \bar{\nu}$.

We mentioned in the beginning that these diagrams bear some resemblance to those in neutrinoless double β decay. There, of course, both outgoing particles are leptons rather than one lepton and one antilepton, and the intermediate propagator is lepton number violating. The double divergence still simplifies the calculation considerably so that one could get a reasonably simple estimate of, e.g., the longrange pion exhange contribution to neutrinoless double β decay.

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$K^+ \to \pi^+ \nu \bar{\nu}$ РОЗПАД

Т.Сабо

Кафедра теоретичної фізики, Ужгородський національний університет,
вул. Волошина, 32, Ужгород 88000

В амплітуді рідкісних розпадів K-мезонів враховано вклади двох окремих джерел – ефекти малих та великих відстаней. Розраховано вклад великих відстаней в амплітуду розпаду $K^+ \to \pi^+ \nu \bar{\nu}$. Його величина на три порядки менша, ніж вклад малих відстаней. Розпад $K^+ \to \pi^+ \nu \bar{\nu}$ викликає інтерес також у зв'язку з CP-порушенням.