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DIGITAL CHANNEL CODES WITH SUPPRESSED LOW FREQUENCY COMPONENTS

Розорінов Г.М., Фендрі Мохамед Аймен. Цифрові канальні коди з пригніченими низькочастотними складовими. Показано, що при цифровій передачі придушення низькочастотних складових досягається обмеженням дисбалансу передаваних позитивних і негативних імпульсів. Розраховані ефективність і спектральні характеристики дисбалансних двійкових кодів обмеженої довжини.

Ключові слова: блоковий код, система передачі, дисбалансний двійковий код, надмірність, поточна цифрова сума, енергетичний спектр

Розоринов Г.Н., Фендри Мохамед Аймен. Цифровые канальные коды с подавленными низкочастотными составляющими. Показано, что при цифровой передаче подавление низкочастотных составляющих достигается ограничением дисбаланса передаваемых положительных и отрицательных импульсов. Рассчитаны эффективность и спектральные характеристики дисбалансных двоичных кодов ограниченной длины.

Ключевые слова: блочный код, система передачи, дисбалансный двоичный код, избыточность, текущая цифровая сумма, энергетический спектр

Rozorynov G.N., Fendri Mohamed Aymen. Digital channel codes with suppressed low frequency components. In digital transmission suppression of the low-frequency components is achieved by constraining the unbalance of the transmitted positive and negative pulses. The efficiency and spectral properties of unbalance constrained codes with binary symbols are calculated.

Key words: block code, transmission system, unbalance binary code, redundancy, running digital sum, power spectrum

1. Introduction. The field of application of digital channel codes with suppressed low frequency components is quite broad [1…3]. Transmission systems designed to achieve DC-suppression are mostly based on so-called block codes, where the source digits are grouped in source words of m digits; the source words are translated using a code book into blocks of n digits called codewords. The disparity of a codeword is defined as the difference of the number of ones and the number of zeros in the codeword. In the simplest code type the source words have two alternative translations (modes) being of opposite disparity polarity. The choice of a particular codeword polarity is made in such a way that the so-called running digital sum (RDS) of the sequence after transmission of the new codeword is as close to zero as possible, where the RDS is defined for a binary stream as the accumulated sum of ones and zeros (a zero counted as – 1).

An analytic expression of the power spectral density function of zero-disparity codeword systems was derived, for example, in [4]. Authors applied the matrix analysis procedure. The procedure is straightforward and very well suited to machine computation. For large codeword length and large number of encoder states the memory requirements of the procedure become prohibitive.

Below the power density function of low-disparity based channel codes is computed. The variance of the RDS (in short sum variance) of the channel codes, adopted here as a criterion of the suppression of the energy near DC, is calculated.

2. Main part. A designer will often be confronted with the question of how good his system is with respect to the redundancy of the code and the resulting suppression of low-frequency components. There is a need for a yardstick to measure the performance of DC-suppressed channel codes in an absolute way. To that end asymptotic properties of sequences so constrained that the RDS of the sequence will take a limited number of values are discussed in this part. The results will be used to derive a figure of merit that takes into account both the redundancy of the code and the resulting frequency range of the sequence spectrum with suppressed components.

Consider binary sequences $x=(x_1,\dots,x_i),\ x_i\in[-1,+1]$. The RDS of a sequence plays a significant role in the analysis and synthesis of codes of which the spectrum vanishes at the low-frequency end. The RDS z_i is defined as:

$$z_i=\sum_{j=1}^{i}x_j .$$

Sequences x containing a finite number of RDS values, called z–constrained sequences. The information capacity of z sequences is a function of the number of allowed RDS values. The maximum number of RDS values a sequence takes is often called digital sum variation (DSV). The maximum entropy of a Markov information source with N allowed RDS values is given by:

$$C(N)=1+\log_2\cos\left(\frac{\pi}{N+1}\right). \qquad \textbf{(1)}$$

In the work [5] developed a useful time-domain measure of the low-frequency properties of DC-constrained sequences. It defined ω_0 as the (low-frequency) cut-off frequency according to: $\frac{1}{T}H(\omega_0T)=\frac{1}{2}$, where $H(\omega T)$ is the power density function versus frequency and T – the time duration of a channel symbol. Then:

$$\omega_0T\approx\frac{1}{2s^2}, \qquad \textbf{(2)}$$

where s^2 is the sum variance of the sequence.

This motivated us to use the sum variance as a criterion of the channel code's low-frequency properties (2). This is of practical importance because the sum variance of a sequence is often easier to calculate than the complete spectrum. The sum variance of a maxentropic z sequence is given in [5]:

$$\sigma^2(N)=\frac{2}{N+1}\sum_{k=1}^{N}\left[\frac{1}{2}(N+1)-k\right]^2\sin^2\left(\frac{\pi k}{N+1}\right). \qquad \textbf{(3)}$$

Table 1 lists the capacity (1) and sum variance (3) versus the DSV N.

Capacity and sum variance of maxentropic *z* sequences with DSV *N* **Table 1**

N	$C(N)$	$\Sigma^2(N)$
3	0,5	0,5
4	0,694	0,80
5	0,792	1,17
6	0,85	1,59
7	0,886	2,09
8	0,91	2,64
9	0,93	3,26

The asymptotic behaviour of the capacity and the sum variance for large DSV N can be derived:

$$C(N)\sim1-\frac{\pi^2}{(2\ln2)(N+1)^2}, \qquad \textbf{(4)}$$

and

$$\sigma^2(N)\sim\left(\frac{1}{12}-\frac{1}{2\pi^2}\right)(N+1)^2. \qquad \textbf{(5)}$$

With (4) and (5) the following important bound between the redundancy $1-C(N)$ and the sum variance of maxentropic z sequences is derived:

$$0,25 \geq \left[1-C(N)\right]\sigma^2(N) > \frac{\frac{\pi^2}{6}-1}{4\ln 2} = 0,2326. \qquad \textbf{(6)}$$

Actually the right-hand bound is within 1% accuracy for $N > 9$. Combining (2) and (6) yields:

$$\omega_0 T \approx 2,15\left[1-C(N)\right].$$

This expression clearly shows the linear trade-off between the redundancy and the cut-off frequency of maxentropic z sequences.

First some properties of coders based on codewords with an equal number of positive and negative pulses (zero-disparity) are discussed.

The number N_0 of zero-disparity codewords with n binary channel symbols (n even) is given by the binomial coefficient $N_0 = \binom{n}{0,5n}$. The code rate R is defined according to $R = \frac{1}{n}\log_2 N_0$.

The zero-disparity codewords are concatenated without information about the history and a fixed relationship exists between codewords and source words. Practical coding schemes demand the number of codewords to be a power of two, so that a subset of the N_0 available codewords should be used, which effectively lowers the code rate *R*. Here only 'full set' coding schemes are considered.

A generalization of the coding principle using zero-disparity codewords leads to the so-called alternate or low-disparity coding. Besides the set of codewords having zero-disparity sets of codewords with nonzero-disparity are used. The simplest code type has two alternate representations (modes) of the source words. The two alternate representations have opposite disparity, the choice of the positive or negative representation is determined by the polarity of the RDS just before transmission of the new codeword. The choice is made in such a way that the absolute value of the RDS after transmission of the new codeword is minimized, i.e. as close to zero as possible. Zero-disparity codewords can in principle be used in both modes. For ease of implementation zero-disparity codewords are sometimes divided into two sets to be used in both modes. It is clear if more subsets of codewords are used that the number of codewords is larger than in the case of zero-disparity encoding. Consequently this allows a larger maximum code rate for a given codeword length. Unfortunately the power in the low-frequency range will also increase if more subsets are used so that a trade-off between code rate and low-frequency content has to be found. In the following some properties of low-disparity coding are derived.

Let a codeword with length *n* (*n* even) consist of binary symbols x_i $1 \leq i \leq n$, $x_i \in [-1,+1]$. The disparity *d* of a codeword is defined by $d = \sum_{i=1}^{n} x_i$.

Assume further that a set of codewords S_+ is used with zero and positive disparity and a set S_- with elements of zero and negative disparity. Set S_+ consists of $K+1$ subsets $S_0, S_1, \ldots, S_K \left(K \leq 0,5n\right)$; the elements of the subsets S_j are all codewords with disparity $2j\left(0 \leq j \leq K\right)$. The codewords in S_- can be found by inversion of all n symbols of the codewords in set S_+ and vice versa. The cardinality N_j of the subset S_j is simply given by the binominal coefficient $N_j = \begin{pmatrix} n \\ 0,5n+j \end{pmatrix}$, $0 \le j \le K$. The total available number of codewords in S_+ is $M = \sum_{j=0}^{K} N_j$, so that the code rate is $R = \frac{1}{n}\log_2 M$.

As the disparity of the codewords is chosen such that the RDS after transmission of the codeword is minimized, it is not difficult to see that during transmission the RDS takes on a finite number of values. Without loss of generality it can be assumed that the sum values are symmetrically centered around zero. The set of values (states) the RDS assumes at the end (or start) of a codeword, the so-called terminal or principal states, is a subset of the RDS values the sequence can take.

Let the terminal digital sum of the k-th codeword be D_k. The sum after transmission of the $(k+1)$-th codeword is $D_{k+1} = D_k \pm d$, where d is the disparity of the $(k+1)$-th codeword. The sign of the disparity (if $d \neq 0$) of the codeword is chosen to minimize the accumulated sum D_{k+1}. A code with this property is said to be balanced. We find by inspection that D_k can take on one of the $2K$ values $\pm1, \pm3, \dots, \pm(2K-1)$. It can easily be found that the total number of RDS values the sequence can take within codewords, i.e. the DSV is given by

$$N = 2(2K - 1 + 0,5n) + 1 = 4K + n - 1. \quad \textbf{(7)}$$

As an illustration the RDS as a function of symbol time interval, the so-called unbalance trellis diagram, is shown in Fig. 1.

The thick curve shows the path of the codeword "$+ - - - + -$" starting in state 3.

The code has codeword length $n = 6$ and it uses the maximum number $K + 1 = 0,5n + 1 = 4$ subsets. Note the $2K = 6$ possible sum values at the end of each codeword and also the $N = 4K + n - 1 = 17$ values that the RDS can take within a codeword.

In the computation of the power density function and the sum variance of the encoded stream we need the stationary probability of being in a certain terminal state.

Assume the source blocks to be generated by a random independent process then the signal process $D^{(k)}$ is a simple stationary Markov process. The value that $D^{(k)}$ can take is related to one of the $2K$ states of the Markov process.

The state transition matrix P, with entries $P(i,j)$, where $P(i,j)$ is the probability that the next codeword will take it to terminal state j given that the encoder is currently in state i, can easily be found. As an illustration we have written down the general matrix P for $2K = 6$ terminal states:

$$P = \begin{array}{c} \begin{array}{cccccc} -5 & -3 & -1 & 1 & 3 & 5 \end{array} \\ \begin{bmatrix} p_0 & p_1 & p_2 & p_3 & 0 & 0 \\ 0 & p_0 & p_1 & p_2 & p_3 & 0 \\ 0 & 0 & p_0 & p_1 & p_2 & p_3 \\ p_3 & p_2 & p_1 & p_0 & 0 & 0 \\ 0 & p_3 & p_2 & p_1 & p_0 & 0 \\ 0 & 0 & p_3 & p_2 & p_1 & p_0 \end{bmatrix} \end{array} \begin{array}{c} \\ -5 \\ -3 \\ -1 \\ 1 \\ 3 \\ 5 \end{array}$$

The codeword symbols are, in general, transmitted on some standard pulse shape $g(t)$ at intervals of duration T. The determination of the power spectral density, then, can be done as in [4].

The comparison of DC-balanced channel codes with maxentropic z sequences should take into account both the sum variance and the rate. We come to the following definition of encoder efficiency

$$E = \frac{\left[1 - C(N)\right]\sigma^2(N)}{(1-R)s^2} \quad \textbf{(8)}$$

The efficiency E of various codes versus codeword length is plotted in Fig. 2. The polarity bit encoding principle has a simple implementation, but as we can notice from Fig. 2 it is far from optimum in the depicted range. Quite a different conclusion can be drawn from Fig. 2, showing the superiority of zero-disparity codes with respect to "polarity bit" encoding in the most practical $(1-R)$ interval. A calculation shows that for an unpractically large codeword length $n > 160$ the polarity bit encoding principle outperforms the zero-disparity encoding.

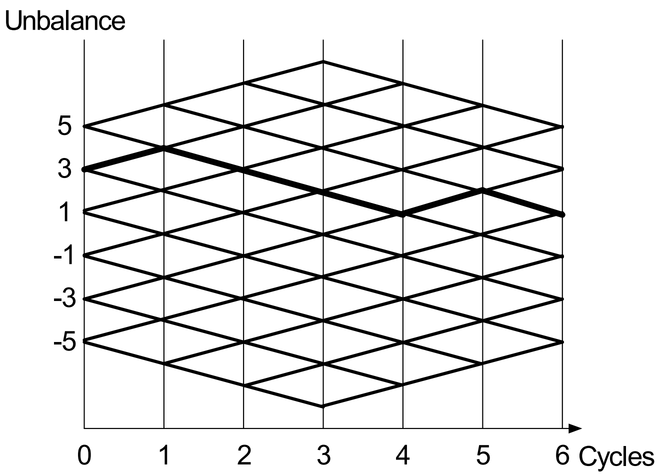

Fig. 1. Unbalance trellis diagram

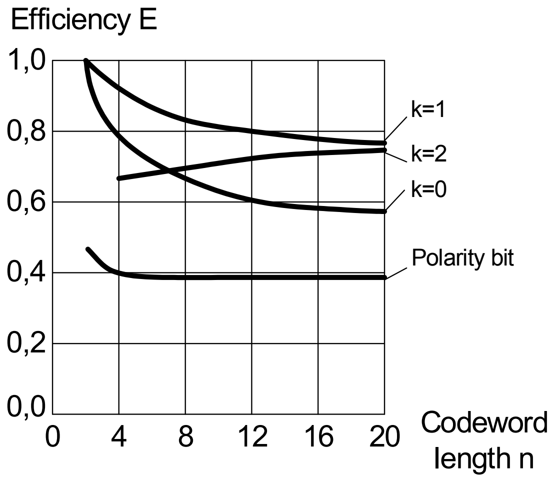

Fig. 2. Efficiency of simple alternate channel codes

3. Conclusion. It is calculated sum variance of simple DC-suppressing block channel codes and the power density function. It is compared the rate and sum variance of the channel sequence with maxentropic unbalance constrained sequences. There were shown that codes with a small rate have a good efficiency. Large rate codes become asymptotically bad with growing codeword length.

Codes based on the polarity bit principle have a simple implementation, but the efficiency is bad in the practical range of codeword length.

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