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ROTATING ELECTRIC FIELD IN A MULTI-PHASE ELECTRODE SYSTEM

Зіньковський Ю. Ф., Сидорук Ю. К., Туровський А. О. Електричне обертове поле у багатофазній електродній системі. Запропоновано математичну модель провідникової багатоелектродної циліндричної структури, що забезпечує рівномірний розподіл напруженості електричного поля між електродами при значно нижчій їх ємності порівняно із плоскими електродами. Відповідно до моделі виконано аналіз розподілу напруженості електричного поля всередині простору, обмеженого багатозв'язною границею на основі теорії інтегральних сингулярних рівнянь у вигляді задачі спряження поля. Розрахунок розподілу поля зведено до розгляду часткової задачі, коли лише один електрод знаходиться під напругою відмінною від нуля. Показано, що при гармонічному живленні електродної системи, що складається з однакових за розмірами рівномірно розташованих дугоподібних електродів, напругами однакової амплітуди, фаза кожного з яких дорівнює його кутовому положенню, забезпечується існування у структурі зони обертового рівномірного електричного поля. Наведено розрахункові співвідношення для визначення напруженості електричного поля між електродами.

***Ключові слова*:** електричне поле, висока частота, сингулярні рівняння, комплексна площина

Зиньковский Ю. Ф., Сидорук Ю. К., Туровский А. А. Электрическое вращающееся поле в многофазной электродной системе. Предложена математическая модель проводящей многоэлектродной цилиндрической структуры, которая обеспечивает равномерное распределение напряженности электрического поля между электродами при значительно низшей их емкости по сравнению с плоскими электродами. В соответствии с моделью выполнен анализ распределения напряженности электрического поля внутри пространства, ограниченного многосвязной границей на основе теории интегральных сингулярных уравнений в виде задачи сопряжения поля. Расчет распределения поля сведен к рассмотрению частичной задачи, когда только один электрод находится под напряжением отличным от нуля. Показано, что при гармоничном питании электродной системы, состоящей из одинаковых по размерам равномерно расположенных дугообразных электродов, напряжениями одинаковой амплитуды, фаза каждого из которых равна его угловому положению, обеспечивается существование зоны вращающегося равномерного электрического поля в структуре. Приведены расчетные соотношения для определения напряженности электрического поля между электродами.

***Ключевые слова*:** электрическое поле, высокая частота, сингулярные уравнения, комплексная плоскость

Zinkovskiy Yu. F., Sydoruk Yu. K., Turovskiy A. O. Rotating electric field in a multi-phase electrode system. In our study we have proposed a mathematical model of conductive multielectrode cylindrical structure in order to provide both a homogeneous distribution of electric field and lower capacitance with respect to plane electrodes. Based on the model the analysis of the electric field distribution inside the space restricted with multiple connected boundary on the basis of the boundary value coupling approach of the theory of singular integral equations is performed. The calculation of the electric field distribution is reduced to the consideration of partial problem where only one electrode possesses a non-zero voltage. It is shown that providing the harmonic feeding of the electrode system consisting of identically sized equally spaced arc-shaped electrodes with voltages of the same amplitudes and phases equal to the angular position of the electrode on the circle will ensure the feasibility of the rotating and homogeneous field in the area. The expressions for determination of the electric field strength in the area are given.

***Keywords*:** electric field, high frequency, singular integral equations, complex plane.

1. Introduction. Multiple connected electrode structures are often used in electron flow control systems, in radio frequency dielectric heating apparatus or for hypothermia in medicine. However, in listed cases the important problem arises of providing the field controlling and field distribution homogeneity within a given area between the electrodes.

A common way for providing of homogeneous electric field distribution is the use of two parallel metal plates [1]. In order to reduce the inhomogeneity caused by edge effects and so ensure the uniformity of the field between the plates it is necessary to increase the size of the electrodes, which simultaneously will cause significant increase of their capacity.

When creating such systems it is also necessary to take into account the fact that the load on the generator is predominantly of capacitive nature, and significant increase of the capacity causes difficulty in matching of the electrodes with the generators thus affecting the efficiency.

To provide the high level of homogeneity of electric field in the interaction zone, high energy efficiency and significantly lower capacity, compared to plain electrodes, without the degradation of productivity it is proposed the following electrode structure [2].

The structure consists of n arc-shaped electrodes placed on the side of a hollow cylinder of radius r and length l, where $l >> r$. It is assumed that the electrodes are perfectly conductive and are of negligibly small thickness, and the whole structure is surrounded with vacuum. The potential of each electrode is V_{01}, V_{02}, ... , V_{0n}, respectively.

The objective of this research is to provide a mathematical model for describing the proposed structure in order to obtain the expressions for electric field calculation within the structure volume. Thus it is necessary to determine the distribution of the electric field in the region inside the electrodes structure, which then specify the optimal number and sizes of the electrodes to provide the required size of the interaction area. Herewith the area of interaction is to be located within the field homogeneity region.

2. Analytical analysis techniques. Since $l >> r$ the problem of calculation of the potential and strength of the electric field within the cylinder is reduced to a plane problem in an infinite complex plane with a ring-shaped n-electrode boundary L of radius r. Ring-shaped boundary L is divided into separate segments of disconnected arcs $L_1, L_2, ..., L_n \in L$, which have no common points (Fig. 1). Arcs $L_1, L_2, ..., L_n$, located in the intervals a_1b_1, a_2b_2, a_3b_3, ..., a_nb_n, are equipotential because of being perfectly conductive. The potential of each arc, respectively, equals $V_{01}, V_{02},..., V_{0n}$. With respect to a circle of radius r full complex plane z is divided into two symmetrical sections: external S_+, for which $|z| \geq r$, and internal $S_–$, for which $|z| \leq r$, where z is an independent complex variable. The problem is to determine the complex potential $\Phi(z) = U(z) + jV(z)$, where $V(z)$ is a potential of the field, and the electric field $\bar{E}(z) = –j[\Phi'(z)]$ [3], where $\Phi'(z)$ is the derivative of the potential with respect to z. Electric field strength $\bar{E}(z)$ is a single-valued analytic function, and a sign […] stands for a complex conjugation. These functions are specified at every point within the complex plane z [4]. All over the complex plane function $\bar{E}(z)$ possess the following properties:

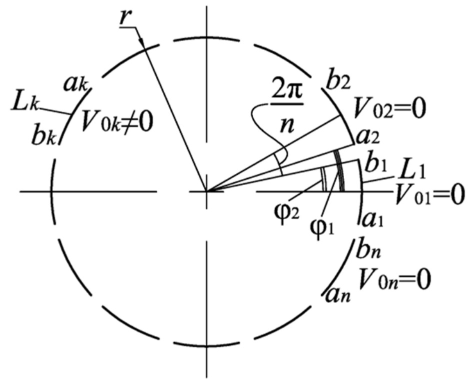


Fig. 1. Arrangement of electrodes of the structure

1. $\bar{E}(z)$ is limited in value everywhere except the arcs' ends (points a_k and b_k), at which $\bar{E}(z)\to\infty$.

2. Due to the symmetry of $\bar{E}(z)$ with respect to the boundary circle $\bar{E}^+(\gamma) = –\bar{E}^-(\gamma)$ on arcs a_kb_k, and $\bar{E}^+(\gamma) = \bar{E}^-(\gamma)$ on arcs b_ka_{k+1}, where $\bar{E}^+(\gamma)$ is the electric field at the boundary arcs on the outside and $\bar{E}^-(\gamma)$ is the electric field on the inside of the circle, γ denotes the complex coordinate on the boundary.

3. On arcs a_kb_k tangential component of the electric field $\bar{E}_\tau(\gamma)$ with respect to the boundary circle takes zero value, and on arcs b_ka_{k+1} the perpendicular component of the electric field is equal to zero both on the external and on the 4. At infinity $\bar{E}(z)$ has a zero of second order.

5. With respect to the boundary circle the electric field and the complex potential are related by the equations

$$\overline{E}(z)_{S_+} = \left(\overline{E}\left(\frac{1}{z^*}\right)\Big/ z^{*2}\right)^*_{S_-} \quad \text{and} \quad \Phi(z) = -\left(\Phi\left(\frac{1}{z^*}\right)\right)^*,$$

where S_+ and $S_–$ denote the external and internal sides of the boundary, respectively.

According to the properties referred to above, $\bar{E}(z)$ is a piecewise holomorphic function and L denotes a set of finite number of simple (smooth) arcs ($L_1, L_2, …, L_n \in L$), which have no common points, in addition the electric field $\bar{E}^+(\gamma)$ at L obeys the expression: $\bar{E}^+(\gamma) = G(\gamma)\bar{E}^-(\gamma)$.

Due to the property 2 the function $G(\gamma)$ is equal to –1 at arcs a_kb_k and is equal to 1 at b_ka_{k+1}, i.e. $G(\gamma)$ is piecewise constant function with the discontinuity of the first kind when going through the points a_k and b_k. So points a_k and b_k are nodal and singular, and $G(\gamma)$ is constant all over on the L except nodes.

The above properties of $\bar{E}(z)$ fit the field coupling problem of the theory of singular integral equations [2].

A general form of the expression of homogeneous problem of field coupling in the case when the function to be found (the electric field) is finite can be written at infinity as follows [2]:

$$\bar{F}(\xi)=X(\xi)P(\xi), \qquad \textbf{(1)}$$

where $F(\xi)$ is the function to be found, $X(\xi)$ is some canonical solution, $P(\xi)$ is an arbitrary polynomial of power k, ξ is independent complex variable.

With respect to the problem, expression (1) takes the form:

$$\bar{E}(z)=X(z^*)P(z^*). \qquad \textbf{(2)}$$

Following the above theory the power of the polynomial $P(z^*)$ and the so-called index of the problem of coupling χ is determined by the behavior of $E(z)$ at infinity.

Since $E(z)$ has at infinity a zero of the second order, $P(z^*)$ is of power m, so the power of $X(z^*)$ is equal to $\chi = (m + 2)$. According to definition [3]:

$$\chi = [\arg G(\gamma)]_L/2\pi,$$

where the sign $[\ldots]_L$ denotes the increment of expression enclosed in parentheses when passing the contour L once in a positive direction. Due to the fact that a_k and b_k are singular nodes $[\arg(G(\gamma))]_L = 2\pi n$, $\chi = n$, $m = n - 2$. With the known index of the problem of coupling the polynomial expression will take the form

$$P(z^*)=\sum_{k=2}^{n} C_{k-1}\left(z^*\right)^{n-k}, \qquad \textbf{(3)}$$

where C_k stands for arbitrary complex constant.

The class of the problem is determined depending on the behavior of $G(\gamma)$ and $F(z)$ in nodes. Accordingly, the nodes are singular when $F(z)$ at nodes is infinite, and non-singular, when the function in the nodes is limited. Conventionally, the class of the problem is denoted by h_i, where i is the number of non-singular nodes at the boundary line.

Thus, in general, the problem of class h_q has the canonical function:

$$X(\xi)=Q\frac{\sqrt{R_1(\xi)}}{\sqrt{R_2(\xi)}}, \qquad \textbf{(4)}$$

where Q is an arbitrary constant and

$$R_1(\xi)=\prod_{k=1}^{q}(\xi-c_k), \qquad R_2(\xi)=\prod_{k=q+1}^{2n}(\xi-c_k), \qquad \textbf{(5)}$$

where c_1, c_2, …, c_q are non-singular nodes; c_{q+1}, c_{q+2}, …, c_{2n} are singular nodes. The discussed problem has no non-singular nodes, therefore $q = 0$ and the problem is characterized as a problem of class h_0, and its canonical function is

$$X(z^*)=\frac{Q}{\sqrt{R(z^*)}}, \qquad \textbf{(6)}$$

where $R(z^*)=\prod_{k=1}^{n}(z^*\text{-}a_k)(z^*\text{-}b_k)$ and constant Q can be obtained according to the property 3 on the boundary circle from the expression $Q=\sqrt[4]{\prod_{k=1}^{n}a_k b_k}$ [4].

In what fallows the expressions of the electric field and potential, it is advisable, to be represented in the form of normalized variable $Z = z/r$, which is achieved by conformal mapping of the complex plane z onto the plane of a single boundary circle Z.

Using expressions (2), (3), (5), (6) and performing the procedure of conformal mapping we obtain the expression of the electric field

$$\bar{E}\left(Z^{*}\right)=\frac{\sqrt[4]{\prod_{k=1}^{n} a_k b_k}}{r}\frac{\sum_{k=2}^{n} C_{k-1}\left(Z^{*}\right)^{n-k}}{\sqrt{\prod_{k=1}^{n}\left(Z^{*}-a_k\right)\left(Z^{*}-b_k\right)}}, \quad \textbf{(7)}$$

and potential

$$V=-\mathrm{Re}\left[\frac{\sqrt[4]{\prod_{k=1}^{n} a_k b_k}}{R}\int_{z}\frac{\sum_{k=2}^{n} C_{k-1}\left(Z^{*}\right)^{n-k} dZ^{*}}{\sqrt{\prod_{k=1}^{n}\left(Z^{*}-a_k\right)\left(Z^{*}-b_k\right)}}\right]^{*}. \quad \textbf{(8)}$$

The two expressions above can be used as initial for calculating the electric field and potential at each point in the region Z for arbitrary distribution of the points a_k and b_k within the boundary circle L as well as for random distribution of potential on the boundary arcs L_k. According to the statement of the problem it is necessary to create uniform distribution of electric field within the electrode structure, which is only possible when $L_1 = L_2 = L_3 = \ldots = L_m$ and $b_1a_2 = b_2a_3 = \ldots = b_na_1$. Let $2\varphi_1$ denotes the angle by which all the arcs are bent down, respectively, the angular interval between the adjacent arcs is denoted by $2\varphi_2$, the angular distance between the midpoints of adjacent arcs is $2\pi/n$ (Fig. 1). Assuming the linearity of dielectric properties of treated medium the total electric field and potential within the given structure with an arbitrary distribution of the voltages among the arcs can be found as the sum of strengths and potentials obtained from the partial solution of the problem where only one arc L_k possesses potential V_{0k} and the others arcs are of zero potential. With regard to these conditions, the expression of the electric field (7) can be written as follows:

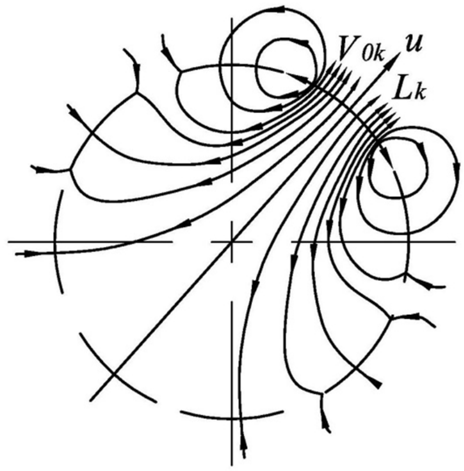


Fig. 2. Electric field lines pattern

$$\bar{E}\left(Z^{*}\right)=\frac{1}{r}\frac{\sum_{k=2}^{n} C_{k-1}\left(Z^{*}\right)^{n-k}}{\sqrt{Z^{*2n}-2Z^{*n}\cos n\varphi_1+1}}, \quad \textbf{(9)}$$

and the electric field lines pattern is depicted in Fig. 2. The electric field structure is symmetric with respect to line u, which passes through the center of arc L_k, which possesses potential V_{0k} (Fig. 2), therefore the symmetric coefficients must be equal to each other: $C_1 = C_{n-1}$, $C_2 = C_{n-2}$... Hence the number of unknown coefficients is reduced to $N = n/2$ for even n and to $N = (n + 1)/2$ for odd values of n.

After performing the boundary conditions symmetrization with respect to the u-axis, the equation (9) takes the form:

$$\bar{E}(Z)_k=\frac{(-1)^{k-1}}{r}Z^{*\frac{n}{2}-1}\times\frac{\sum_{i=1}^{N} C_i\left\{Z^{*\frac{n}{2}-i}\exp\left[j\left(\frac{n}{2}-i\right)\frac{2\pi}{n}(k-1)\right]+Z^{*i-\frac{n}{2}}\exp\left[-j\left(\frac{n}{2}-i\right)\frac{2\pi}{n}(k-1)\right]\right\}}{\sqrt{Z^{*2n}-2Z^{*n}\cos n\varphi_1+1}}. \quad \textbf{(10)}$$

Taking into account that the distance from all the points on the boundary circle to the origin equals r, in order to describe location of any point on the circle, it is sufficient to specify only the angular coordinate φ, which corresponds to a polar coordinate system.

Hence on boundary circle we have $Z = \exp(j\varphi)$. Substituting the value of Z in (10) and using Euler transformation, we obtain the expression for electric field strength on the boundary circle:

$$\bar{E}(Z)_k = \sqrt{2}\,\frac{(-1)^{k-1}}{r} \times \frac{\sum_{i=1}^{N} C_i \cos(n/2 - i)\left[\varphi - \frac{2\pi}{n}(k-1)\right]}{\sqrt{\cos n\varphi - \cos n\varphi_1}}\,\mathrm{e}^{-j2\varphi}. \quad \textbf{(11)}$$

In expression (11) the electric field is presented by means of components E_x and E_y. However, since the boundary is circular it is convenient to present electric field vector by sum of components E_r and E_φ as follows: $E_x + jE_y = (E_r + jE_\varphi)e^{j\varphi}$. Expression (11) is used to determine the unknown coefficients C_i by integrating $E(Z)_\varphi^k$ between the nodes b_k and a_{k+1}, whose difference of potential is known. As the number of the intervals is equal to N, the number of the linear independent equations, wherefrom the N unknown coefficients C_i are defined, is also equal to N. The N-order set when applying the potential V_{0k} to the k-th arc is written as

$$\sum_{i=1}^{N} C_i \sin\left(i\frac{\pi}{n}(2p-1)\right) P_{-i/n}(\cos n\varphi_2) = \frac{nV_{b_{k+p-1}a_{k+p}}}{2\pi}, \quad \textbf{(12)}$$

where $P_{-i/n}(\cos(n\varphi_2))$ is a Legendre function of order $(-i/n)$, p denotes the number of equation ($p\in[1, 2, ..., N]$), $V_{b_{k+p-1}a_{k+p}}$ is the difference of potential between the points b_{k+p-1} and a_{k+p}. In this set only the first equation, for which $p = 1$, has a right-hand side ($V_{0k} \neq 0$), in other equations for which $p > 1$, the right side is zero. A more detailed mathematical explanation of the given expressions has been provided in [5, 6].

The high degree of homogeneity of the electric field within the inside region z and rotation of field can be obtained if the potential distribution on the boundary arcs is provided according to the expression:

$$V_{0k} = V_{0m}\sin[\omega t + (k-1)2\pi/n], \quad \textbf{(13)}$$

where ω and V_{0m} represent the cyclic frequency and amplitude of the electrodes supply signal, accordingly, t is time. Thus the total vector of the electric field will be oriented between the electrodes with the maximum instantaneous voltage, phase of signal at which is equal to $\pi/2 + \pi M$, where M is arbitrary integer.

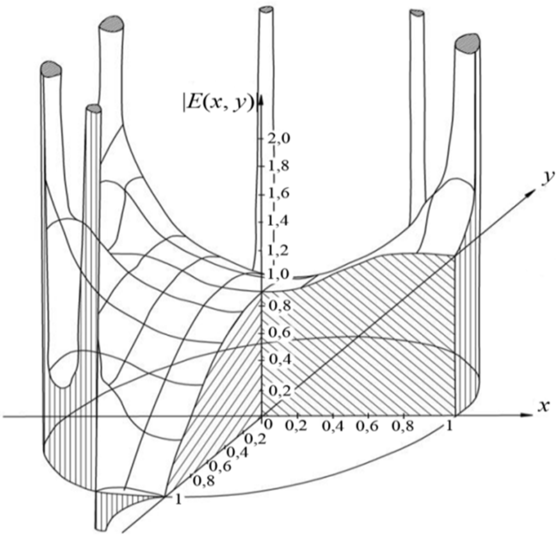
Since $C_i = V_{0k}F_i$, and $F_i = -(n/2\pi)\cdot(A/A_i)\cdot(1/P_{-i/n}(\cos n\varphi_2))$, where A is the determinant of the set (12) and A_i is the algebraic complement of its i-th element [6], so the expression (10), after (13) being substituted in it, and finding the total field generated by all the arcs by calculating the corresponding sum by k, takes the form:

$$\bar{E}(Z)_n = \frac{n}{2}\frac{V_{0m}}{r}F_1\left[\frac{\sin\omega t\left(1+Z^{*(n-2)}\right) + j\cos\omega t\left(1-Z^{*(n-2)}\right)}{\sqrt{Z^{*2n} - 2Z^{*n}\cos n\varphi_2 + 1}}\right]. \quad \textbf{(14)}$$

3. Results and discussions. Calculation of the electric field according to equation (14) shows that for small values of n the electric field in the inner region of complex plane z is considerably inhomogeneous, but there is a rise in homogeneity when n increases. To illustrate this statement in Fig. 3 and Fig. 4 the graphical results of calculations of $|E(z)|$ for $n = 4$ and $n = 8$ are presented.

Here in the figures it is assumed that $z = x + jy$, where x and y are the coordinates normalized to the radius r. It is apparent that for $n = 8$ the radius of homogeneity zone r_h is equal to 60 % of the boundary circle radius. At the same time for $n = 4$ the size of uniformity zone is virtually absent. The size of the uniformity zone also depends on the angle $2\varphi_2$. For small values of n the effect of this parameter on the homogeneity of the field is significant, but with the increase of n the effect on the zone size decreases. A major advantage of this electrode structure is the fact that the electric field in the zone of homogeneity has rotational character.

4. Conclusions. The problem of field coupling of the theory of singular integral equations is an effective tool for analyzing the distribution of the electric field inside the space bounded by multiply connected circular arc-shaped boundary, and allows us to obtain precise analytical expressions for the electric field distribution within the electrode structure. The calculations showed that the size of the zone of uniform field distribution inside the structure is determined by the phase and amplitude distribution on each electrode. When excited the regularly placed electrodes with a harmonic voltage with relative phase shift corresponded to the angular position of the electrode on the circle, with a number of elements of structure equal to 8 the size of uniformity zone is 60% of the diameter of the structure. Thus excited the electrode structure provides the uniform rotation of the field.



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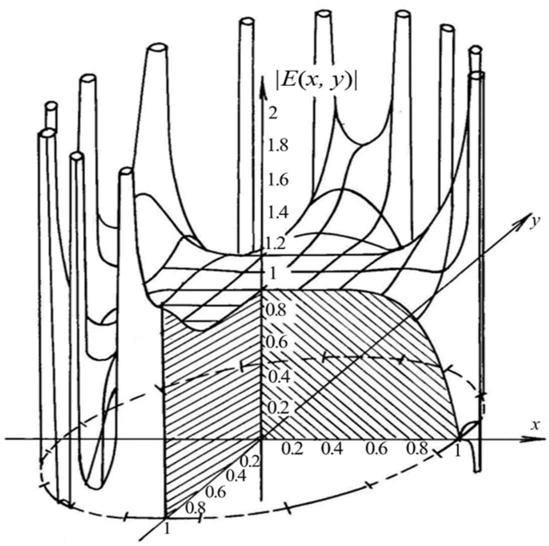

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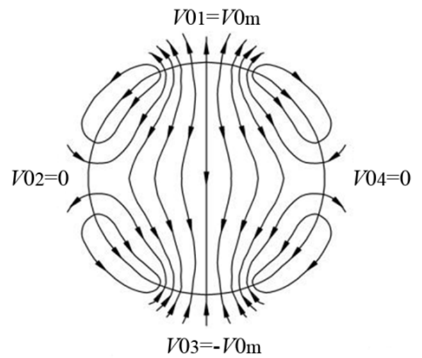
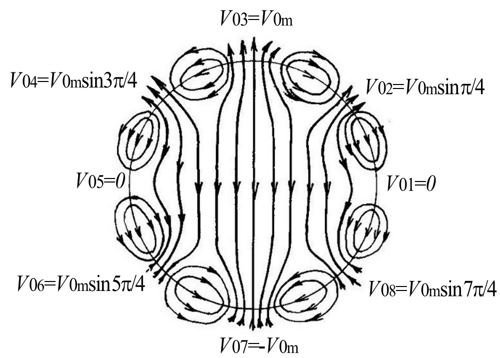


b)

Fig. 3. Dependence of |*E*(*x*, *y*)| (a) and the electric field lines pattern (b) for *n* = 4



b)

Fig. 4. Dependence of |*E*(*x*, *y*)| (a) and the electric field lines pattern (b) for *n* = 8

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