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Ключові слова: *тривимірні скінченні елементи, вал, втулка, поля переміщень та температур, зазор, натяг.*

УДК 539.3

IMPROVING THE MATHEMATICAL MODELS APPLIED FOR THE SOLUTION OF SOLID ASSEMBLY CONSTRUCTIONS THERMOELASTICITY PROBLEM

Викладено методику побудови уточненої скінченно-елементної математичної моделі збірних конструкцій типу «вал» – «втулка», що мають значне розповсюдження в енергомашинобудуванні. З використанням розроблених тривимірних скінченних елементів розв'язано контактну термопружну задачу для даного типу з'єднань. Отримано поле розподілу переміщень на торцевих контактуючих поверхнях валу та втулки, а також поле розподілу температур в з'єднанні.

Introduction

The working process of attached solid constructions like shaft and sleeve subassemblies that are used in modern turbines is steadily influenced by various mechanical and thermal effects of high intense. This fact causes a connection between changes of the matched solid bodies mechanical contact and a heat flow through their surfaces. Especially important this correlation is for details of gas turbine engines due to their extremely hard working process.

It should be noticed that the main conditions of contact conjugation between the details in such types of subassemblies always change sharply for every type of mechanism's working state [1]. Firstly, the shaft and sleeve subassemblies are fitted with a gap or negative allowances before the start of working process. This means that each pair of contacting surfaces has its own definite conjugation conditions. But during the working process the conjugation conditions can rapidly change. Therefore we can observe the changes of heat flow parameters on the shaft and sleeve contacting surfaces [2]. So a mathematical model used for such subassemblies thermoelasticity problem solution needs to take into consideration all these changes on the details contacting surfaces that also cause the variations of temperature and displacement fields on the aforementioned surfaces.

There are two main approaches that are used for the solution of contact problems for deformable solids by a finite elements method (FEM). The main idea of the first approach is to use the contact layer of definite thermal conductivity, that is located among the solid bodies contacting surfaces. The finite elements model of the contact layer is based on finite elements similar to the elements of the solid bodies. But the thermo conductivity features of the layer elements are different from the features of solid bodies' elements [1]. This approach is rather useful, but its application to the thermoelasticity problems solution of real assemblies and subassemblies is inconvenient, because it's extremely hard to calculate the layer's deformation caused by the thermal gradient on the contacting surfaces. The second main approach is to use the definite function, that distinctly determines the dependences between the heat flux and displacements of finite elements nodes located on the contacting surfaces [3–6]. The foregoing problems solution could be obtained only in the case of several conditions fulfillment. The first one is the condition of details' junction unpenetration; the second one is the condition of normal and tangential forces equality for each pair of contacting finite elements nodes [4–7].

The main aim of this article is to develop more correct mathematical model based on three dimensional finite elements, that could be used for shaft and sleeve subassemblies thermoelasticity problems solution.

Formulation of the problem

The investigated subassembly is located in the right rectangular Cartesian coordinate system *xyz* with the beginning at the shaft's butt centre O. *Z* axis is normal to the shaft axis of rotation; *x* matches the shaft axis of rotation. The whole coordinate system is rotating with constant angular velocity together with shaft and sleeve subassembly (Fig. 1).

The considered mechanical deformable system energy state could be described by Lagrange variation principle. Thus

$$\begin{aligned} \delta L &= 0 \\ L &= \Pi - W \end{aligned}, \tag{1}$$

where L – Lagrange function; Π – potential energy of system's resistance to deformation; W – the work of external forces.

After FEM approximation the main equation of the mechanical system balance (1) is transformed to

$$\mathbf{K\delta} = \boldsymbol{F}\ , \tag{2}$$

where **K** – global stiffness matrix of finite elements model; δ – vector of finite elements nodes generalized displacement; ***F*** – vector of external forces.

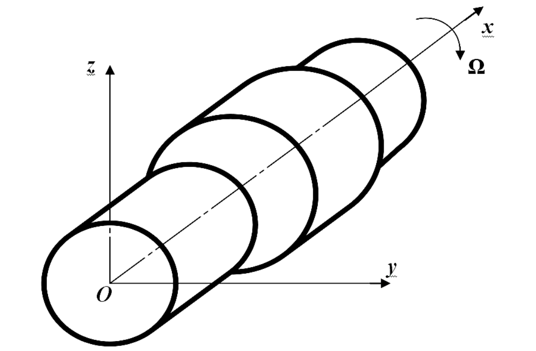

Fig. 1. The shaft and sleeve subassembly in the Cartesian coordinate system

The temperature state of the solid body caused by the stationary heat transfer could be described by next equation [5]

$$\lambda\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+Q=0\,, \tag{3}$$

where T – body temperature, K; λ – thermal conductivity coefficient W/m·K; x, y, z – Cartesian coordinates of the solid body; Q – internal source of heat.

For solution of the equation (3) next boundary conditions should be applied

$$\lambda\left(\frac{\partial T}{\partial x}l_x+\frac{\partial T}{\partial y}l_y+\frac{\partial T}{\partial z}l_z\right)+\lambda(T-T_o)+q=0\,, \tag{4}$$

where T_o – ambient temperature, K; l_x, l_y, l_z – direction cosines of the normal vector to the surface; q – heat flow density, W/m.

Dependencies (3) and (4) form functional (5). Its minimization gives us solution of the temperature problem

$$\phi=\frac{1}{2}\int_V\left[\lambda\left(\left(\frac{\partial T}{\partial x}\right)^2+\left(\frac{\partial T}{\partial y}\right)^2+\left(\frac{\partial T}{\partial z}\right)^2-2QT\right)\right]dV+\int_S\left[qT+\frac{1}{2}h(T-T_o)^2\right]dS\,. \tag{5}$$

After the FEM approximation of (5) we could receive the mutual dependences of the aforementioned assembly heat balance

$$\mathbf{K_T T}=\mathbf{Q}\,, \tag{6}$$

where $\mathbf{K_T}$ – global matrix of the finite elements model thermal conductivity; **T** – vector of temperatures located in the nodes of finite elements; **Q** – vector of external heat load.

Therefore, for the solution of shaft and sleeve subassemblies thermoelasticity problem we need to solve the set of matrix equations, that is formed by the usage of dependencies (2) and (6).

Solution of the problem

For more correct solution of the aforementioned problem we must design the special three-dimensional finite element of hexahedron type (Fig. 2). It has eight nodes with five degrees of freedom in each node. Such type of finite elements allows to provide FEM sampling of solid bodies which form is fairly familiar to the details, forming the researched structure. That is why the usage of mathematical model formed on base of such finite elements gives an opportunity to solve the thermoelasticity problem of shaft and sleeve subassemblies more correctly [8].

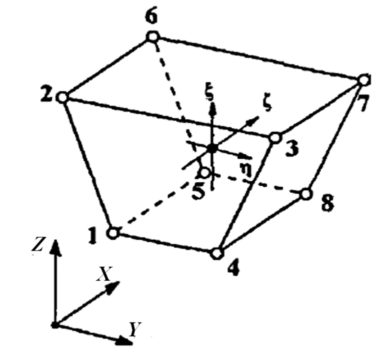

Fig. 2. Three-dimentional finite element of hexahedron type

Transfer from the global Cartesian coordinate system *xyz* of the mechanical system to the finite element's local coordinate system ζηξ should be described by the dependencies

$$x = \{N_i\}^T \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_8 \end{Bmatrix}; \quad y = \{N_i\}^T \begin{Bmatrix} y_1 \\ y_2 \\ \vdots \\ y_8 \end{Bmatrix}; \quad z = \{N_i\}^T \begin{Bmatrix} z_1 \\ z_2 \\ \vdots \\ z_8 \end{Bmatrix}, \; (i = 1, 2, ,\ldots, 8), \tag{7}$$

where (x, y, z) – global Cartesian coordinates of the element; $(x_1,x_2,\ldots,x_8;\ y_1,y_2,\ldots,y_8;\ z_1,z_2,\ldots,z_8)$ – Cartesian coordinates of the element's nodes; N_i – finite element's shape functions.

Shape functions for the developed finite element are presented by the dependences (8). Functional inequalities should be noticed: $(-1 \le \xi \le 1; -1 \le \zeta \le 1; -1 \le \eta \le 1)$. Then

$$\begin{gathered} N_1 = \frac{1}{8}(1-\eta)(1-\xi)(1-\zeta); \quad N_2 = \frac{1}{8}(1-\eta)(1+\xi)(1-\zeta); \\ N_3 = \frac{1}{8}(1+\eta)(1+\xi)(1-\zeta); \quad N_4 = \frac{1}{8}(1+\eta)(1-\xi)(1-\zeta); \\ N_5 = \frac{1}{8}(1-\eta)(1-\xi)(1+\zeta); \quad N_6 = \frac{1}{8}(1-\eta)(1+\xi)(1+\zeta); \\ N_7 = \frac{1}{8}(1+\eta)(1+\xi)(1+\zeta); \quad N_8 = \frac{1}{8}(1+\eta)(1-\xi)(1+\zeta).; \end{gathered} \tag{8}$$

The displacement of finite elements nodes towards the *xyz* directions could be obtained by solution of the dependencies (7) and (8). Thus

$$\boldsymbol{\delta}^e = \begin{bmatrix} N_1 & 0 & 0 & \cdots & N_8 & 0 & 0 \\ 0 & N_1 & 0 & \cdots & 0 & N_8 & 0 \\ 0 & 0 & N_1 & \cdots & 0 & 0 & N_8 \end{bmatrix} \begin{Bmatrix} \delta_1^1 \\ \delta_1^2 \\ \delta_1^3 \\ \vdots \\ \delta_8^1 \\ \delta_8^2 \\ \delta_8^3 \end{Bmatrix}, \; (e = 1, 2, \ldots, n), \tag{9}$$

where $\boldsymbol{\delta}^{\mathbf{e}}$ – vector of the e-st finite element generalized displacement; δ_1^1, δ_1^2, δ_1^3 – generalized displacements of the finite element node 1 towards the *xyz* directions; *n* – quantity of the finite elements taken into consideration.

Functions of temperature (T^e) for the *e*-st finite element of the developed three-dimentional finite elements model could be obtained by means of the dependencies

$$T^e = \sum_{i=1}^{n} N_i T_i \,, \ (i = 1, 2, ,\ldots, 8), \qquad (10)$$

where T_i – temperatures in the nodes of the finite element.

It should also be noticed that for the solution of linear algebraic equations systems (7–10) the numerical method of Kholetcski is used. The procedure of matrix reordering is also used to make global matrixes more compact. For the storage of global matrixes in the computer random access memory the Sherman's compact scheme is suitable.

Main results and their analysis

For the mathematical model adequacy and calculation algorithm efficiency verification the fields of shaft and sleeve subassembly temperatures and displacements are calculated. All calculations are realized with the usage of ANSYS program complex. In the researched subassembly the shaft's diameter is d = 120h7 mm; shaft's material is heat-resistant steel 20Х3НМФА. The internal diameter of the sleeve is D = 120M8; sleeve's material – structural steel 30Х. Shaft rotation frequency is 1000 revolutions per minute. Thermal conductivity coefficient λ = 500 W/m·K.

The sector of shaft and sleeve subassembly finite elements model is shown on the Figure 3.

The front surfaces of both details forming the subassembly are axially fixed. Wherein there is a gap between the shaft and sleeve front surfaces. Its value is near 0,01 mm. Between the shaft and sleeve radial surfaces there is a negative allowance, which value is 0,01 mm too. On the front surfaces of both details the boundary conditions of the first kind used for the thermal problem are given. In the initial state both parts of subassembly have the temperature of 293 K. Then the shaft front surface is heated up to 373 K.

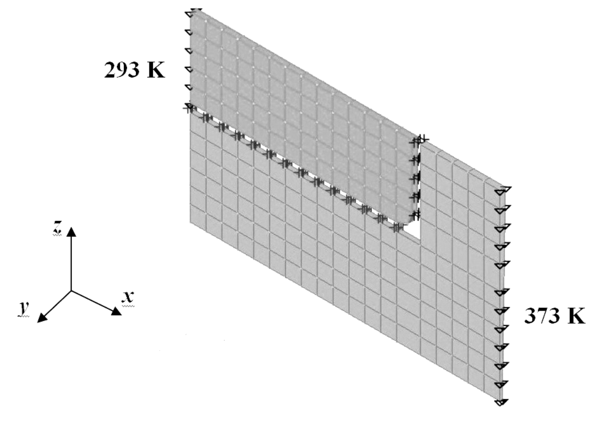

Fig. 3. Sector of shaft and sleeve subassembly of finite elements model

On the Figures 4 and 5 the fields of displacement and temperature in the specified shaft and sleeve subassembly are shown.

According to the fig. 4 we have to take into consideration that on the shaft and sleeve front surfaces the conjugation conditions have been changed from the gap to the negative allowance. Such changes could be explained by the influence of the heat flow,

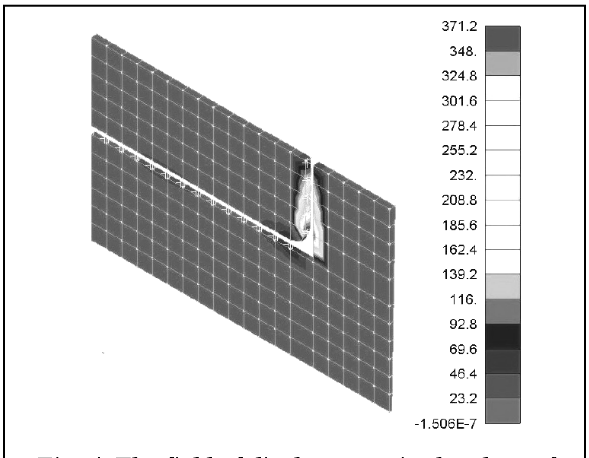

Fig. 4. The field of displacement in the place of shaft and sleeve front surfaces contact

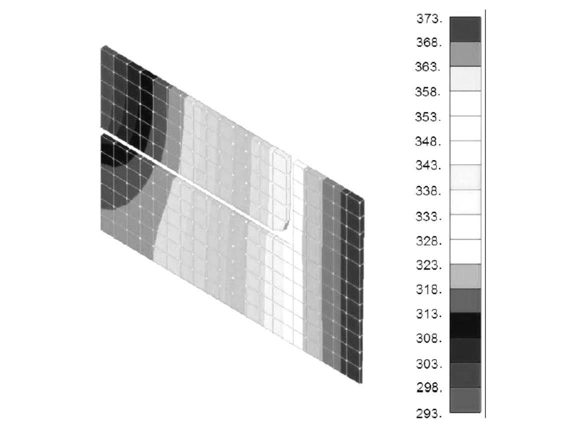

Fig. 5. The field of temperatures in the shaft and sleeve subassembly t

that causes heat extension of the contacting surfaces. These processes are described by the Fick's law [5].

The field of temperatures in the shaft and sleeve subassembly is shown on the Fig. 5. It could be observed that it is practically homogeneous and does not have sharp gradients. This fact could be explained by the change of the conjugation conditions from gap to negative allowance, caused by the influence of contact surfaces thermal deformation. So the absence of the air gap between the front surfaces of shaft and sleeve caused the absence of sharp temperature gradients in the places of their fitting.

Conclusions

The new more correct mathematical model that could be used for the shaft and sleeve thermoelasticity problems solution has been successfully created. This model is based on the three-dimentional finite elements of hexagon type usage. The fields of displacement and temperatures on the details contacting surfaces have also been obtained. It has also been established that the conjugation conditions in such type of subassemblies are changed from the gap to negative allowance type due to the heat extension of material. On the base of this mathematical model the thermal stress-strained state of such structures, widely used in marine engine building, could also be investigated.

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Ключові слова: *стрингерна пластина, круговий отвір, сили зчеплення в зв'язках, контакт берегів тріщин, контактні напруження.*

УДК 539.375

ЧАСТИЧНОЕ ЗАКРЫТИЕ ПРЯМОЛИНЕЙНЫХ ТРЕЩИН СО СВЯЗЯМИ В СТРИНГЕРНОЙ ПЛАСТИНЕ С ОТВЕРСТИЕМ

Досліджується підкріплена стрингерами пружна ізотропна пластина, що має круговий отвір, з якого виходять прямолінійні тріщини зі зв'язками між берегами. Розглянуто випадок часткового закриття тріщин. Для визначення параметрів, що характеризують закриття тріщин, отримано сингулярне інтегральне рівняння, яке за допомогою процедури алгебраізації зведене до скінченної нелінійної алгебраїчної системи. Розв'язуючи алгебраїчну систему методом послідовних наближень, знайдені сили зчеплення в зв'язках, контактні напруження й розмір контактних зон тріщин.

Введение

Тонкие пластины, имеющие отверстия, являются широко распространенным элементом конструкций. Поскольку отверстие создает повышенную концентрацию напряжений в пластине, пред-