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THERMOELASTIC DAMPING IN FGM NANO-ELECTROMECHANICAL SYSTEM IN AXIAL VIBRATION BASED ON ERINGEN NONLOCAL THEORY

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Abstract. The thermo-elastic damping is a dominant source of internal damping in micro-electromechanical systems (MEMS) and nano-electromechanical systems (NEMS). The internal damping cannot neither be controlled nor minimized unless either mechanical or geometrical properties are changed. Therefore, a novel FGMNEM system with a controllable thermo-elastic damping of axial vibration based on Eringen nonlocal theory is considered. The effects of different parameter like the gradient index, nonlocal parameter, length of nanobeam and ambient temperature on the thermo-elastic damping quality factor are presented. It is shown that the thermo-elastic damping can be controlled by changing different parameter.

Key words: FGM nanobeam, Eringen nonlocal theory, axial vibrations, thermoelastic damping.

1. Introduction.

Nowadays MEMS and NEMS industries are playing important roles in scientific and engineering communities. There are a lot of advantages that make MEMS and NEMS commercialization attractive. Ink jet printer heads, micropumps, airbag accelerometers and micro sensors are a few examples of devices which these systems have successfully replaced more conventional systems [1].

There are different kind of loss mechanism in MEMS and NEMS which classified into the two categories: extrinsic and intrinsic losses. [2]. internal damping cannot neither be controlled norminimized unless either mechanical or geometrical properties are changed [3]TED is a dominant source of intrinsic damping in MEMS. [4] and NEMS. [5] working under vacuum condition. [6]. It has been identified as one of important loss mechanisms in NEMS and MEMS [7]. From the other hand development of low-power, high performance MEM and NEM systems are of great importance [8] so TED is a very active research approach [7]. Lifshitz and Roukes [9] derived an analytical expression for the QF of TED in-micro-beams and studied the effect of different geometrical parameters. Saeedi vahdat et al. [4] study the effects of axial and residual stresses on thermoelastic damping in capacitive micro-beam. Lepage et al. [10] studied thermoelastic effects in a vibrating beam accelerometer that modelled using finite elements. Nayfeh and Younis [11] presented ananalytical expression for the QF of micro-plates of general shapes and boundary conditions due to TED. Rezazadeh et al. [6] studied thermoelastic damping in a micro-beam resonator using modified couple stress theory and also thermoelastic damping in capacitive micro-beam resonators using hyperbolic heat conduction model [12].

As can be seen there are a lot of work has been done on the TED of transverse vibration but investigations about longitudinal vibrations are few in comparison with it. These vibrations are quite different [13]for example, the natural frequencies in transversal vibration are much lower than that of longitudinal vibration [13]and probably can achieve high QF. [14]. TED of the longitudinal vibration of micro beams is studied by maroofi et al. [14] and they showed increasing of the ambient temperature and length of micro beam increases the QF.

In the present work, by considering the coupled equations of motion and heat conductivity, TED of nano FGM beam's axial vibration based on nonlocal theory has been presented and has been proposed a novel FGMNEM system, with a controllable thermo-elastic damping of axial vibration.

The Material properties of the FGM beam vary continuously along the beam thickness according to the power law distribution. The coupled equations is solved by Galerkin method for a clamped-clamped boundary condition and finally, the effects of different parameters like gradient index, nonlocal parameter, length of nano beam and ambient temperature has been presented on the QF.

2. Formulation.

Consider functionally graded clamped nanobeam of length L, width b, and thickness h. Material properties of the beam vary continuously along the beam thickness are functions of z according to the power law distribution [15]:

$$p(z) = (p_l - p_u)\left(\frac{2z+h}{2h}\right)^m + p_u,$$

where subscripts u and l refer to material properties of the upper and lower surfaces respectively. m is a non-negative number that dictates the material variation profile through the thickness of the beam. The general strain field results from both mechanical and thermal effects [16, 17]

$$e_{ij} = e_{ij}{}^M + e_{ij}{}^T, \tag{1}$$

where $e_{ij}{}^M$ and $e_{ij}{}^T$ are mechanical and thermal strain as bellow:

$$e_{ij}^M = \frac{1+\nu}{E(z)}\sigma_{ij} - \frac{\nu}{E(z)}\sigma_{kk}\delta_{ij};\ e_{ij}^T = \alpha(z)(T-T_0)\delta_{ij}, \tag{2}$$

where ν is passion's ratio, E is young modulus, α is coefficient of the linear thermal expansion and T_0 is equal to the ambient temperature so the stress-strain relation is as follow [16]:

$$\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij} - (3\lambda + 2\mu)\alpha(z)(T-T_0)\delta_{ij}. \tag{3}$$

For both plane strain and plane stress condition we have [18]:

$$\sigma_{xx} = E^{ef}[e_{xx}{}^M + e_{xx}{}^T] = E^{ef}e_{xx} - \zeta(T-T_0). \tag{4}$$

In plane stress condition $\zeta = E^{ef}(z)\alpha(z)$ and in plane strain condition $\zeta = E^{ef}(z)\alpha(z)/(1-\upsilon)$ and where $b \ge 5h,\ E^{ef} = E/(1-\nu^2)$ otherwise $E^{ef} = E.$

2.1 Nonlocal theory. According to Eringen [19 – 21], the stress field at a point x in an elastic continuum not only depends on the strain field at the point but also on strains at all other points of the body. Eringen attributed this fact to the atomic theory of lattice dynamics and experimental observations on phonon dispersion. Thus, the nonlocal stress tensor components σ_{ij} at point x are expressed as:

$$\sigma_{ij}(x) = \int_V k\ (|x'-x|, \tau)\ t_{ij}(x'\ )dx', \tag{5}$$

where tij(x) are the components of the classical macroscopic stress tensor at point x and the kernel function $k\ (|x'-x|, \tau)$ represents the nonlocal modulus, $|x'-x|$ being the distance (in Euclidean norm) and τ is a material constant that depends on internal and external characteristic lengths (such as the lattice spacing and wavelength, respectively). It is possible (see Eringen [20]) to represent the integral constitutive relations in an equivalent differential form as:

$$\left(1-\mu\frac{\partial^2}{\partial x^2}\right)\sigma_{xx}=E^{ef}e_{xx}\quad(\mu=(e_0a)^2),\tag{6}$$

where a is an internal characteristic length and e_0 is a constant. Now based on the Eringen nonlocal theory Eq. (4) changes to following form.

$$\left(1-\mu\frac{\partial^2}{\partial x^2}\right)\sigma_{xx}=E^{ef}[e_{xx}{}^M+e_{xx}{}^T]=E^{ef}e_{xx}-E^{ef}\alpha(z)(T-T_0).\tag{7}$$

The equation for the axial motion of the beam in the absence of external force can be obtained as. [16]:

$$\frac{\partial N}{\partial x}=I_1\frac{\partial^2 u(x,t)}{\partial t^2},\tag{8}$$

where $u(x,t)$ is the axial displacement, $\rho(z)$ is the mass per unit length, N is the axial force per unit length and I is as bellow,

$$N=\int_A\sigma_{xx}dA;\ I_1=b\int_{-h/2}^{h/2}\rho(z)dz,\tag{9}$$

where A is the cross-sectional area of the beam. Integrating Eq. (6) with respect to area gives the following relation:

$$\left(1-\mu\frac{\partial^2}{\partial x^2}\right)N=B_1e_{xx}-B_2(T-T_0),\tag{10}$$

where $B_1=b\int_{-h/2}^{h/2}E^{ef}(z)\,dz$ and $B_2=b\int_{-h/2}^{h/2}E^{ef}(z)\,\alpha(z)\,dz$.

Using Eqs. (7) – (9), the equation of axial vibration can be found in terms of displacement

$$I_1\frac{\partial^2u(x,t)}{\partial t^2}-\mu\,I_1\frac{\partial^4u(x,t)}{\partial t^2\partial x^2}=B_1\frac{\partial^2u(x,t)}{\partial x^2}-B_2\frac{\partial\theta}{\partial x}\quad(\theta=T-T_0),\tag{11}$$

where $\mu=0$ Eq. (10) reduces to the classic form.

In the other hand the heat conduction equation without any thermal source is. [16]:

$$k\theta_{,ii}=\rho(z)c(z)\dot{\theta}+\frac{E(z)}{1-2\upsilon}\alpha(z)T_0\dot{e}_{ii},\tag{12}$$

where k and c are thermal conductively and specific heat constant at a constant volume. Eq. (11) in the form of displacement obtained as:

$$k(z)\left(\frac{\partial^2\theta}{\partial x^2}\right)=\rho(z)c(z)\frac{\partial\theta}{\partial t}+\frac{E(z)}{1-2\upsilon}\alpha(z)T_0\frac{\partial^2u}{\partial x\partial t}.\tag{13}$$

By using the following non-dimensional parameters:

$$\hat{u}=\frac{u}{l};\quad\hat{x}=\frac{x}{l};\quad\hat{\theta}=\frac{\theta}{T_0};\quad\hat{t}=\frac{t}{t_0};\quad t_0^2=\frac{\rho(z)L^2(1+\nu)(1-\nu)}{E(z)}.$$

The dimensional-less form of coupled Eqs. (10), (12) obtains as bellow:

$$D_1\frac{\partial^2\hat{u}}{\partial\hat{x}^2}-D_2\frac{\partial\hat{\theta}}{\partial\hat{x}}=D_3\frac{\partial^2\hat{u}}{\partial\hat{t}^2}-D_4\frac{\partial^4\hat{u}}{\partial\hat{t}^2\partial\hat{x}^2};\tag{14a}$$

$$C_1 \frac{\partial^2 \hat{\theta}}{\partial \hat{x}^2} = C_2 \frac{\partial \hat{\theta}}{\partial \hat{t}} + C_3 \frac{\partial^2 \hat{u}}{\partial \hat{x} \partial \hat{t}}, \tag{14b}$$

where D and C coefficients are as:

$$D_1 = \frac{B_1}{L}; \; D_2 = \frac{B_2 T_0}{L}; \; D_3 = I_1 \frac{L}{t_0{}^2}; \; D_4 = \frac{\mu I_1}{t_0{}^2 L};$$

$$C_1 = \frac{k(z)T_0}{L^2}; \; C_2 = \frac{\rho(z)c(z)T_0}{\sqrt{t_0}}; \; C_3 = \frac{E^{ef}\alpha(z)T_0}{\sqrt{t_0}(1-2\upsilon)}.$$

3. Numerical solution.

To solve the coupled equation Galerkin method has been used. Based on the Galerkin method:

$$\hat{u}(\hat{x},\hat{t}) = \sum_{n=1}^{\infty} \varphi_n(\hat{x}) a_n(\hat{t}) \approx \sum_{n=1}^{N} \varphi_n(\hat{x}) a_n(\hat{t}); \; \hat{\theta}(\hat{x},\hat{t}) = \sum_{m=1}^{\infty} \eta_m(\hat{x}) b_m(\hat{t}) \approx \sum_{m=1}^{M} \eta_m(\hat{x}) b_m(\hat{t}), \tag{15}$$

where $\varphi_n(\hat{x})$, $\eta_m(\hat{x})$ are the suitable shape functions and $a_n(\hat{t})$, $b_m(\hat{t})$ are time dependent coefficients. The shape functions satisfy the both end clamped boundary conditions of our problem so the form of they is as bellow:

$$\varphi_n(\hat{x}) = \sin(n\pi\hat{x}); \; \eta_m(\hat{x}) = \sin(m\pi\hat{x}). \tag{16}$$

Substituting Eq. (15) into Eq. (14) and multiplying Eq. (14a) into $\varphi_i(\hat{x})$ and Eq. (15) into $\eta_j(\hat{x})$ and integrating outcome from 0 to 1 and also considering first mode shape of displacement and second one of thermo leads to:

$$(-\bar{D}_3 + \bar{D}_4)\ddot{a}_1(\hat{t}) + \bar{D}_1 a_1(\hat{t}) - \bar{D}_2 b_2(\hat{t}) = 0; \; \bar{C}_1 b_2(\hat{t}) - \bar{C}_2 \dot{b}_2(\hat{t}) - \bar{C}_3 \dot{a}_1(\hat{t}) = 0, \tag{17}$$

By considering $a_1(\hat{t}) = ae^{\omega\hat{t}}$ and $b_2(\hat{t}) = be^{\omega\hat{t}}$ Eq. (17) changes to the following form:

$$((-\bar{D}_3 + \bar{D}_4)s^2 + \bar{D}_1)a - \bar{D}_2 b = 0; \; (\bar{C}_1 - \bar{C}_2 s)b - \bar{C}_3 s a = 0. \tag{18}$$

So the natural frequencies of the system obtain by solving Eq. (18). The $\bar{D}$ and $\bar{C}$ coefficients are as bellow:

$$\bar{D}_1 = D_1 \int_0^1 \varphi_n''(\hat{x})\varphi_i(\hat{x})\, d\hat{x}; \; \bar{D}_2 = D_2 \int_0^1 \eta_m'(\hat{x})\varphi_i(\hat{x})\, d\hat{x};$$

$$\bar{D}_3 = D_3 \int_0^1 \varphi_n(\hat{x})\varphi_i(\hat{x})\, d\hat{x}; \; \bar{D}_4 = D_4 \int_0^1 \varphi_n''(\hat{x})\varphi_i(\hat{x})\, d\hat{x};$$

$$\bar{C}_1 = C_1 \int_0^1 \eta_m''(\hat{x})\eta_j(\hat{x}) d\hat{x}; \; \bar{C}_2 = C_2 \int_0^1 \eta_m(\hat{x})\eta_j(\hat{x})\, d\hat{x}; \; \bar{C}_3 = C_3 \int_0^1 \varphi_n'(\hat{x})\eta_j(\hat{x})\, d\hat{x}.$$

4. Results.

In this section the numerical results has been presented. According to the power-law distribution of a material's property, as an example, variation of the elastic modulus and thermal conductivity through the thickness with different material-variation profile parameter m is shown in Fig. 1. As can be noticed, the variation of a material property can be enhanced via manipulating the control parameter *m*. The FGM nano beam has been composed of Nickel and silicon nitride. Its property varying through the thickness based on the power low. Its bottom and upper surface is pure nickel and pure silicon nitride respectively. Materials properties have been presented in Table 1.

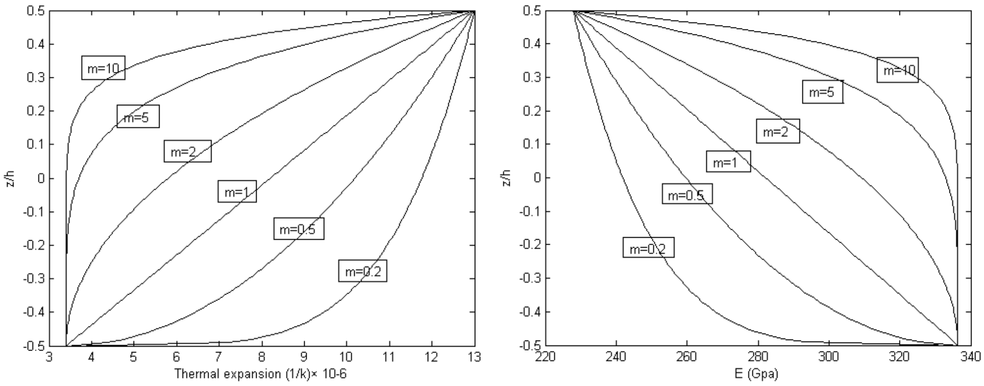

Fig. 1. Variation of the elastic modulus and thermal conductivity through the thickness with different material-variation profile parameter *m*.

Geometrical and material properties.

Table 1

Material	Nickel [14]	Silicon nitride [22]
Density (kg/m^3)	8900	2300
Thermal conductivity (w/mk)	92	30
Young modulus(Gpa)	210	310
Thermal expansion (1/k)× 10^{-6}	13	3.4
Specific heat at constant volume (j/kgk)	438	680
Poisson's ratio	0,31	0,27

According to the complex frequency approach, quality factor of thermo-elastic damping (Q_{TED}) can be achieved as [6, 23]:

$$Q_{TED} = \frac{1}{2\zeta} \cong \frac{1}{2}\left|\frac{\mathrm{Re}(\omega)}{\mathrm{Im}(\omega)}\right|. \tag{19}$$

To validate our results a comparison has been made with the results of Maroofi et al. [14] in Table. 2 for silicon micro beam where $L = 200\,\mu\text{m}, E = 169\,\text{Gpa}, k = 150\ \text{w/mk}$, $c = 695\ \text{j/kgk}, \rho = 2300\ \text{kg/m}^3, \vartheta = 0,28$ and $\alpha = 2,6\ \times 10^{-6}$. As it can be seen there is a good agreement between them.

A comparison between quality factor ($Q_{TED}\times 10^6$) in longitudinal vibration for silicon beam.

Table 2

Temperature (T_0)	Maroofi et al. [47]	present
300 K	2,861	3,107
400 k	2,146	2,331

In Fig. 2 the effects of gradient index $(m = 0 - 40)$ for three value of nonlocal parameter $(\mu = 0,\ 24\ \text{nm}^2)$ have been showed. As it can be seen increasing of gradient index increases the quality factor in the nonlocal and classic theory. For gradient index between 0 – 10 the variation of the Q_{TED} is large but as the gradient index increases the diagram will be linear. Note that $\mu = 0$ presents classic theory and where $m = 0$ the beam is pure nickel and by increasing gradient index the silicon bromide's present increases. Fig. 3 shows the effects of nonlocal parameter on the quality factor. As there is no value has been introduced for the FGM beam so here the nonlocal parameter has been chosen between 0 – 4 nm^2. As it is visible increasing of μ causes increase in the Q_{TED}. Comparison of Fig. 3, *a* and Fig. 3, *b* shows that variation of μ for $L = 20\ \mathrm{nm}$ has lower effects than $L = 10\,\mathrm{nm}$ on the Q_{TED}. This is means that increasing of the length decreases the effects of nonlocal parameter on the Q_{TED}.

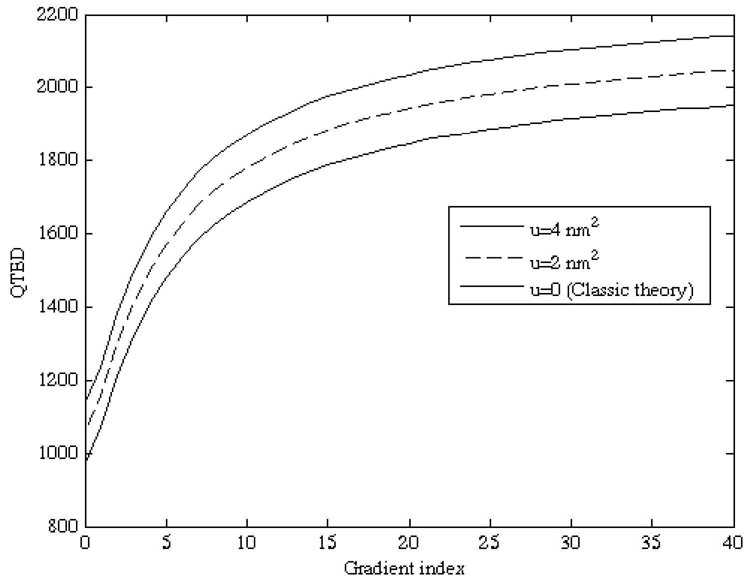

Fig. 2. Effects of gradient index on the thermo-elastic damping quality factor where $L = 10\,\mathrm{nm}, T_0 = 300\,\mathrm{K}$.

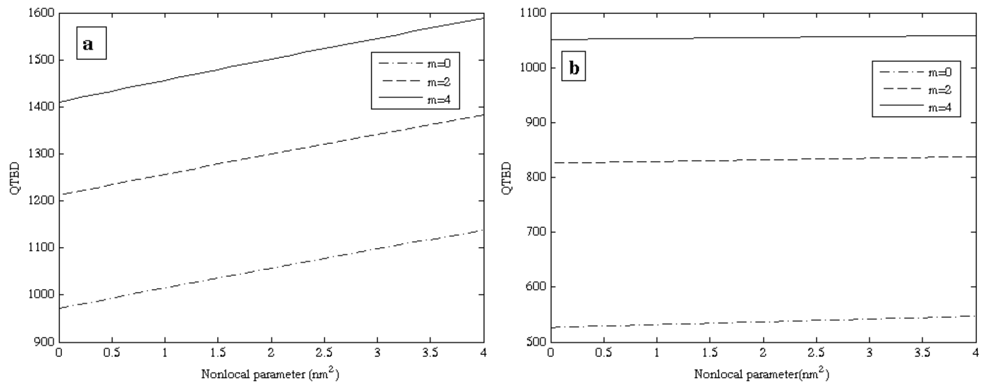

Fig. 3. Effects of nonlocal parameter on the thermo-elastic damping quality factor a: $L = 10\ \mathrm{nm}, T_0 = 300\ \mathrm{kb}: L = 10\ \mathrm{nm}, T_0 = 300\,\mathrm{K}$.

Effects of length variation have been showed in Fig. 4, where length varies from 10 – 100 nm and gradient index is $m = 0, 2, 4, 10$. As expected decreasing of length causing decrease in Q_{TED}. As it can be seen increasing of the gradient index decreases this variation. In other hand it can be found that length variation's effects on the Q_{TED} decreases as the silicon-nitride's present of the FGM beam increases.

In Fig. 5 ambient temperature versus Q_{TED} is showed for nickel nano beam based on classic and Eringen nonlocal theory $(m = 0, \mu = 0 \text{ and } m = 0, \mu = 2)$ and the FGM nano beam based on classic and Eringen nonlocal theory $(m = 2, \mu = 0 \text{ and } m = 2, \mu = 2)$. It is visible by increasing of ambient temperature the Q_{TED} decreases. Also it can be seen that by increasing of T_0 the effects of gradient index and nonlocal parameter on the Q_{TED} decreases. In table 3 Q_{TED} in the $T_0 = 200$ and 500 K and also the difference of this two quality factor ($Q_{\text{TED1}} - Q_{\text{TED2}}$) has been showed. This table shows that by increasing of gradient index and nonlocal parameter the effects of ambient temperature's variation on the Q_{TED} increases.

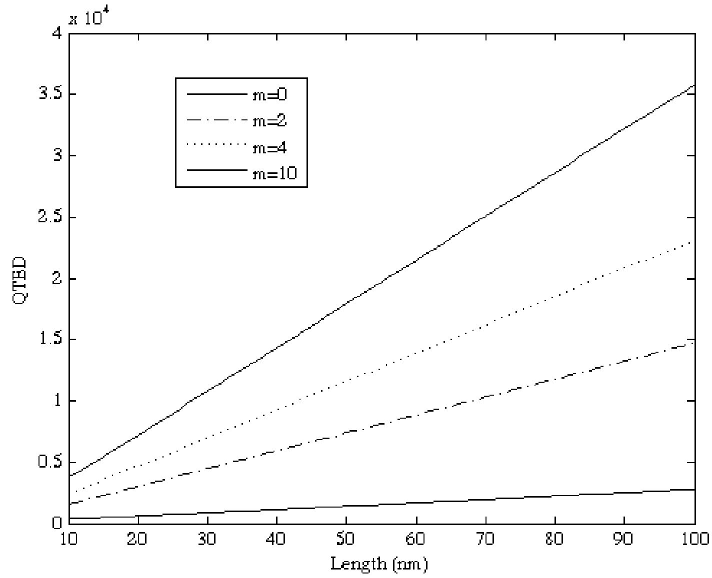

Fig. 4. Length variation versus thermo-elastic damping quality factor (μ=0).

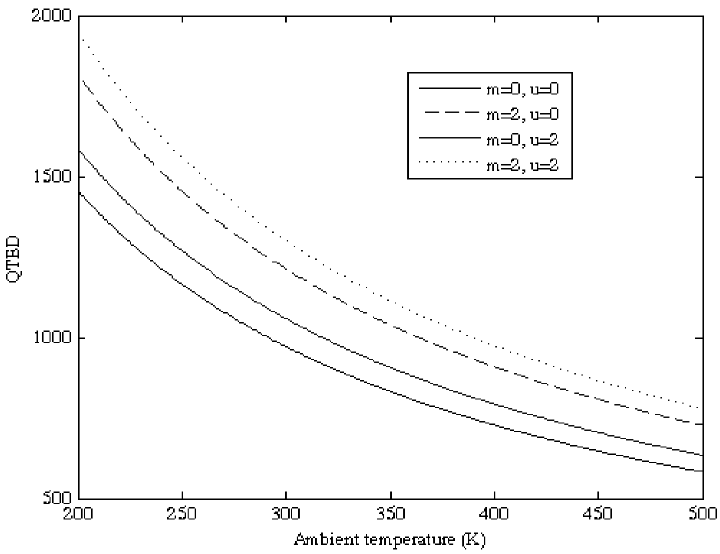

Fig. 5. Ambient temperature versus thermo-elastic damping quality factor $(L = 10\,\text{nm})$.

Q_{TED} of $T_0 = 200$ & 500 for different values of gradient index and nonlocal parameter.

Table 3

m	$\mu(\text{nm}^2)$	Q_{TED1} ($T_0 = 200\,\text{K}$)	Q_{TED2} ($T_0 = 500\,\text{K}$)	$Q_{TED1} - Q_{TED2}$
0	0	1456	582,5	873,5
	2	1586	634,5	951,5
	4	1706	682,5	1023,5
2	0	1818	727,1	1090,9
	2	1950	779,8	1170,2
	4	2074	829,4	1244,6
4	0	2116	846,2	1269,8
	2	2253	901	1352
	4	2383	953,1	1429,9

5. Conclusion.

Thermoelastic damping of axial vibration of the FGM nano beam has been presented based on the Eringen nonlocal theory. It has been composed of Nickel and silicon nitride. Its property varying through the thickness based on the power low. Its bottom and upper surface is pure nickel and pure silicon nitride respectively. The effects of gradient index, nonlocal parameter, Length of the beam and ambient temperature has been presented on the thermo-elastic damping quality factor. In the other word a FGMNEM system, with a controllable thermo-elastic damping of axial vibration based on Eringen nonlocal theory has been presented and has been showed that increasing of gradient index (increasing of silicon bromide's present), nonlocal parameter and length of the nano beam, increases the quality factor and increasing of ambient temperature decreases it. Also is showed that by increasing of ambient temperature decreases the effects of gradient index and nonlocal parameter and increasing of the gradient index decreases the effects of the nano beam's length variation, on the Q_{TED}. The achieved results can be used as a design implement for the designers to control TED.

РЕЗЮМЕ. Термопружне демпфування є головним джерелом внутрішнього демпфування в мікроелектромеханічних і наноелектромеханічних системах. Внутрішнє демпфування не може бути ні кероване, ні мінімізоване, поки механічні і геометричні властивості матеріалу не є змінними. Тому розглянуто осьові коливання балки з нового матеріалу – функціонально градієнтного матеріалу з врахуванням наноелектромеханічних явищ – в рамках нелокальної теорії Ерінгена. Розглянуто вплив різних параметрів – показника градієнтості, параметрів нелокальності, довжини нанобалки, температури навколишнього середовища – на коефіцієнт термопружного демпфування. Показано, що термопружне демпфування може бути кероване зміною цих параметрів.

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