Narrow-pass-band filters based on binary superlattices with strong impedance contrast

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The photonic band structure and transmission coefficient of a dielectric bilayer periodic array with strong impedance contrast are calculated and analyzed. It is found that the photonic band structure has broad pass bands and very narrow gaps. However, because of the strong impedance contrast and the use of lossless dielectric layers, the transmission spectrum exhibits sharp peaks inside the pass bands. The remarkable properties of the superlattice can be used for the fabrication of narrow-pass-band filters.

1. Introduction

Optical periodic nanostructures are of great importance for modern photonics because of the possibility to control the light propagation through them in a wide frequency range. Such nanostructures, being commonly composed of dielectric and metal components, have been called photonic metamaterials. They have promising applications in nanophotonics due to their exotic optical properties as, for example, hyperbolic photonic dispersion and negative refractive index [1–3].

In the case of one-dimensional periodic dielectric-metal nanostructures (i.e. superlattices), it is well established (see, e.g., [4–6]) that if the metallic layers are sufficiently thin then the photonic band structure exhibits narrow pass bands due to the very strong contrast between the impedances of the dielectric and the metal. The slightly dispersive (almost flat) photonic bands are associated with Fabry–Perot resonances occurring inside the relatively thick dielectric layer. However, the light transmission through a dielectric-metal multilayer stack diminishes with its length because of the energy losses in the metal component, which are particularly considerable in the THz and infrared frequency range.

Energy losses in optic superlattices with narrow photonic pass bands might be reduced by substituting the metal layers for dielectric ones of extremely high refractive index [7,8]. Indeed, the impedance of the ordinary dielectric layer, which has low enough refractive index, and that of the layer with high refractive index turn out to be essentially different and, therefore, sharp transmission resonances should be observed.

In this work the optical properties of one-dimensional periodic nanostructures composed of alternating dielectric layers with rather different impedances are investigated. In particular, the photonic band structure and transmission optical spectrum for binary dielectric superlattices are analyzed and discussed. We show that such periodic nanostructures with strong impedance contrast can operate as narrow-pass-band filters of electromagnetic radiation.

2. Problem formulation: Basic relations

We consider a plane electromagnetic wave propagating along the growth direction of a periodic multilayer stack, see Fig. 1. The wave has frequency ω, its electric $\mathbf{E}(x,t)$ and magnetic $\mathbf{H}(x,t)$ components read

$$\mathbf{E}(x,t) = \{0, E(x), 0\}\exp(-i\omega t), \qquad (1a)$$

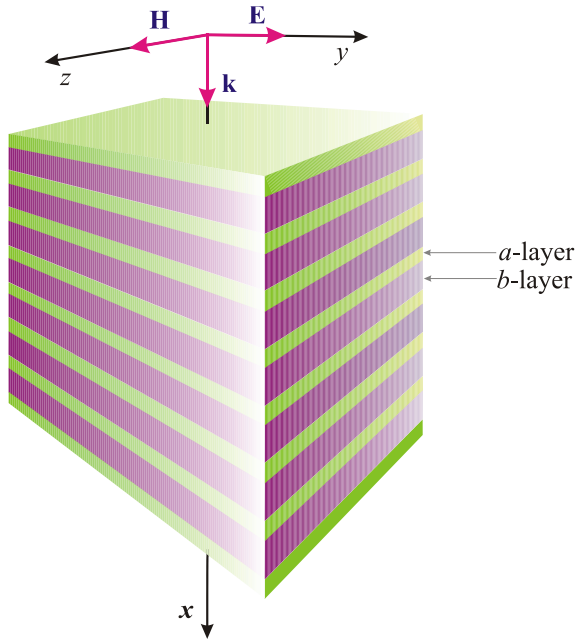$$\mathbf{H}(x,t) = \{0, 0, H(x)\}\exp(-i\omega t). \qquad (1b)$$

Fig. 1. (Color online) A sketch of the problem.

A unit (a,b) cell of the stack consists of two dielectric a- and b-slabs with the constant thickness d_a and d_b, respectively. The size of the unit cell is $d = d_a + d_b$. The array contains N identical unit cells with the whole length Nd.

Every kind of dielectric a- and b-layers is respectively specified by the dielectric constant (permittivity) $\varepsilon_{a,b}$, magnetic permeability $\mu_{a,b}$, corresponding refractive index $n_{a,b}$, impedance $Z_{a,b}$, wave number $k_{a,b}$, and wave phase shift $\varphi_{a,b}$,

$$n_a = \sqrt{\varepsilon_a \mu_a}, \quad Z_a = \frac{\mu_a}{n_a}, \quad k_a = \frac{\omega}{c} n_a, \quad \varphi_a = k_a d_a; \quad (2a)$$

$$n_b = \sqrt{\varepsilon_b \mu_b}, \quad Z_b = \frac{\mu_b}{n_b}, \quad k_b = \frac{\omega}{c} n_b, \quad \varphi_b = k_b d_b. \quad (2b)$$

For definiteness, we assume that all optical parameters (2) are of positive values.

With the use of the transfer matrix method (see, e.g., the book [9]), the Bloch wave number κ and the transmission coefficient (or, the same, the transmittance) T_N of such a system can be readily obtained to obey two following equations:

$$\cos(\kappa d) = \cos \varphi_a \cos \varphi_b - \alpha_+ \sin \varphi_a \sin \varphi_b, \quad (3)$$

$$T_N = \frac{1}{1 + \alpha_-^2 \sin^2 \varphi_b \sin^2(N\kappa d)/\sin^2(\kappa d)}. \quad (4)$$

In these expressions

$$\alpha_\pm = \frac{1}{2}\left(\frac{Z_a}{Z_b} \pm \frac{Z_b}{Z_a}\right), \qquad \alpha_+^2 - \alpha_-^2 = 1, \quad (5)$$

where the parameter α_- is so-called mismatching factor. It is worth noting that the dispersion relation (3) defines the

spectrum of the Bloch wave number, $\kappa = \kappa(\omega)$. Specifically, the Bloch wave number κ turns out to be real inside the spectral pass bands (where $|\cos(\kappa d)| \le 1$) and of complex value within the reflection bands (gaps with $|\cos(\kappa d)| > 1$). In addition, one can reveals that expression (4) for the transmittance adequately describes the wave transmission both within spectral pass bands and gaps.

Equations (3) and (4) are quite general and applicable to the wide class of bilayer stack-structures. In the present study we shall analyze such systems whose unit-cell optical parameters are assumed to satisfy three constitutive conditions. First, the strong contrast between the impedances must be met: the impedance Z_b of the b-layer must be much smaller than the impedance Z_a of the a-layer,

$$Z_b \ll Z_a \quad \rightarrow \quad \alpha_\pm \approx \frac{Z_a}{2Z_b} \gg 1. \quad (6)$$

Second, the optic paths, $n_a d_a$ and $n_b d_b$, of the wave traveling through the corresponding basic a- and b-slabs, are substantially different. However, as opposed to the condition (6), the optic path of the b-layer is assumed to be much greater than that of the a-layer, i.e.

$$n_a d_a \ll n_b d_b \quad \rightarrow \quad \varphi_a \ll \varphi_b = \frac{n_b d_b}{n_a d_a} \varphi_a. \quad (7)$$

Apart from the two parameters introduced above, great Z_a/Z_b, Eq. (6), and small $n_a d_a / n_b d_b$, Eq. (7), there is one more, a third parameter that is the product of the two previous ones. The third parameter has to be small: the first great ratio is compensated by the second small one,

$$\frac{Z_a}{Z_b} \frac{n_a d_a}{n_b d_b} \ll 1 \quad \rightarrow \quad \frac{\mu_a d_a}{\mu_b d_b} \ll 1. \quad (8)$$

When the permeabilities $\mu_a \sim \mu_b \sim 1$, the third requirement (8) means that the thickness d_a of the a-layer with smaller refractive index n_a is much smaller than the thickness d_b of the b-layer with the higher refractive index n_b. Therefore, three basic conditions (6)–(8) can be presented in the form of a double inequality,

$$\frac{n_a}{n_b} \ll 1 \ll \frac{d_b}{d_a}. \quad (9)$$

Note that this inequality is valid not only when $n_a \sim 1$ and $n_b \gg 1$, but also when $0 < n_a \ll 1$ and $n_b \sim 1$.

3. Spectrum

The band structure of the spectral dependence $\kappa = \kappa(\omega)$ governed by the dispersion relation (3) starts with so-called lowest path band that is not of great interest. As for the higher bands, one can draw some important qualitative conclusions about them without actual solving the problem completely. Indeed, under the strong-contrast condition (6), the factor α_+ appearing in front of the product $\sin \varphi_a \sin \varphi_b$

in Eq. (3) has a great value. It is obvious that inside the pass bands, this great value should be compensated by very low values of $\sin \varphi_a$ or $\sin \varphi_b$. Hence, the transmission bands can be situated only in the vicinity of the a- and b-resonances arising, respectively, in a- and b-layers, i.e. in the vicinity of the points

$$\varphi_a = j_a \pi \quad \text{or} \quad \varphi_b = j_b \pi, \tag{10}$$

where j_a and j_b are positive integer numbers. Correspondingly, we shall call these bands a-band or b-band.

As a consequence of the condition (7), within the region between the top $\varphi_b = \pi$ of the lowest propagation band and the first a-resonance $\varphi_a = \pi$, i.e., when

$$\pi \lesssim \varphi_b \lesssim \frac{n_b d_b}{n_a d_a} \pi, \tag{11}$$

there are a lot of transmission b-bands associated with the b-resonances. At the same time, within the interval (11), the phase shift φ_a changes from very small values, in the lower b-bands, to the values of the order of π, in the higher b-bands. Due to peculiar properties of the lower b-bands, which are demonstrated below, here we restrict ourself to the study of the lower b-bands only where the values of the phase shift φ_a are small,

$$\varphi_a = \frac{n_a d_a}{n_b d_b} \varphi_b \sim \frac{n_a d_a}{n_b d_b} j_b \pi \ll 1. \tag{12}$$

There exists common belief that the pass bands of a bi-layer periodic structure with strong contrast of the impedances always should be narrow. Remarkably, the b-bands of the superlattice under consideration are expected to be broad, in spite of the strong contrast (6). Indeed, it easy to show that under conditions (6), (7) and within the frequency range (11), (12), the dispersion relation (3) can be represented in the following approximate form

$$\cos(\kappa d) = \left[1 - \frac{1}{2} \left(\frac{n_a d_a}{n_b d_b} \right)^2 \varphi_b^2 \right] \cos \varphi_b -$$

$$- \frac{Z_a}{2 Z_b} \frac{n_a d_a}{n_b d_b} \varphi_b \sin \varphi_b. \tag{13}$$

Surprisingly, the asymptotics (13) does not contain any great parameter, as opposed to initial Eq. (3). Moreover, the factor in front of $\varphi_b \sin \varphi_b$, turns out to be small due to condition (8). This is the reason why the b-bands are expected to be broad, whereas the gaps between them are expected to be narrow.

Within the zero approximation with respect to the second and third small parameters of the problem, see Eqs. (7) and (8), for the lower b-resonances (12), the simplified dispersion relation (13) degenerates into

$$\cos(\kappa d) = \cos \varphi_b. \tag{14}$$

In such a case the stack-structure is effectively equivalent to a homogeneous medium with linear spectrum and average refractive index $\bar{n}$. In the representation of extended Brillouin zones one gets

$$\kappa d = \varphi_b \quad \rightarrow \quad \frac{\omega \bar{n}}{c} = \kappa, \quad \bar{n} = \frac{n_b d_b}{d}. \tag{15}$$

Note that in this case there are no gaps in the spectrum. Due to the inequality (7), the average refractive index does not depend on the optic path $n_a d_a$ of a-layers.

In the first Brillouin zone representation, in which $|\kappa d| \le \pi$, the spectrum has the form of a vertical "saw",

$$\frac{\omega \bar{n}}{c} = (-1)^{j_b} \kappa + \frac{2\pi}{d} \left(\frac{j_b}{2} + \left\{ \frac{j_b}{2} \right\} \right), \tag{16}$$

$$j_b = 0, 1, 2, \ldots, \quad 0 \le \kappa d \le \pi,$$

depicted in Fig. 2 by the dash-dot line. Here the braces $\{x\}$ refer to the fractional part of the inner number x. As j_b is an integer, the fractional part $\{j_b/2\}$ takes solely two values: it is zero for even j_b and a half for odd j_b. Index j_b enumerates the transmission b-bands: the larger j_b, the higher spectral b-band. The edges of the b-bands are defined by

$$\varphi_b = 0, 2\pi, 4\pi, \ldots \quad \text{at} \quad \kappa d = 0, \tag{17a}$$

$$\varphi_b = \pi, 3\pi, 5\pi, \ldots \quad \text{at} \quad \kappa d = \pi. \tag{17b}$$

As seen, the edges (17) of the unperturbed b-bands (16) are situated at the b-resonances (10). In the zero approximation, the width $\Delta \varphi_b^{(\text{band})}$ of the b-bands is equal to π.

The zero-approximation band structure is degenerated because the top and the bottom of the neighboring unperturbed b-bands coincide. Accounting for the small terms
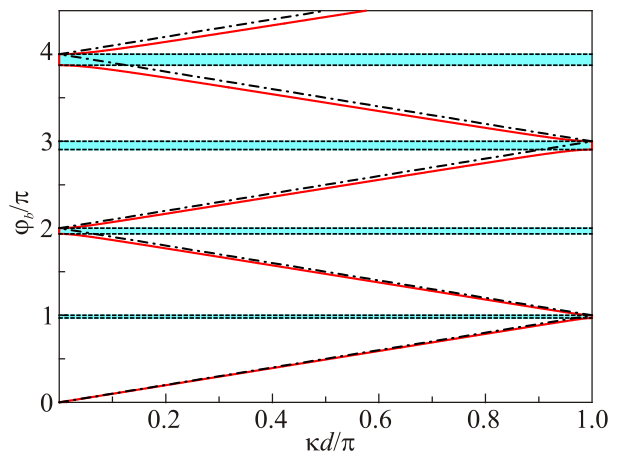

Fig. 2. (Color online) Solid line — band structure (3) for $d_b/d_a = 30$, $n_b/n_a = 25$. Dash-dot line — unperturbed band structure displayed by vertical "saw" (16). Horizontal dashed straight lines indicate the edges of the spectral broad bands and narrow gaps.

in the dispersion relation (13) that have been omitted in Eq. (14), results in the non-zero width of the gaps as a consequence of the splitting of the *b*-band edges in the *b*-resonances.

For such a band structure, one might expect a practically perfect transmission for the whole spectrum except the narrow gaps. Remarkably, the analysis following below, completely disproves this supposition.

In the middle of the *b*-bands, the spectrum is linear. It can be approximately described by the vertical "saw" (16), see Fig. 2. Therefore, the analysis of the spectrum in the vicinity of the *b*-resonances, Eq. (17), is of actual interest only.

It can be shown that the width of the gaps is given by

$$\Delta\varphi_b^{(\text{gap})} = j_b \pi \frac{Z_a}{Z_b} \frac{n_a d_a}{n_b d_b} \ll 1. \tag{18}$$

The *b*-resonances are situated very close to the top edges of the gaps: the relative distance between the *b*-resonance and the top edge of the gap, with respect to the width of the gap is $(Z_b/Z_a)^2 \ll 1$.

The width of the bands, $\Delta\varphi_b^{(\text{band})}$, is found to be slightly smaller than π,

$$\Delta\varphi_b^{(\text{band})} = \left[1 - (j_b + 1)\frac{Z_a}{Z_b} \frac{n_a d_a}{n_b d_b}\right]\pi. \tag{19}$$

The bands are asymmetric because the corresponding *b*-resonances are placed within the bands, however, near their bottom edges: the ratio of the distance between the *b*-resonance and the bottom edge of the band to the width of the band is $j_b Z_a n_a d_a / Z_b n_b d_b \ll 1$.

To summarize, the stack-structure represents a *quasi-homogeneous* medium. Specifically, the wide *b*-layers with high refractive index n_b are separated from each other by the narrow *a*-layers with low refractive index n_a, i.e. $d_a \ll d_b$ and $n_a \ll n_b$, see Eq. (9). Therefore, within the zero approximation (neglecting *a*-layers), the spectrum of this structure is described by the linear dependence (15), or, the same, by the saw-like expression (16). The presence of the narrow *a*-layers gives rise to the emergence of the narrow gaps and slight changes of the spectrum at the edges of the broad bands whose width $\Delta\varphi_b^{(\text{band})}$ is approximately equal to π. The exact band structure described by the exact dispersion relation (3) is illustratively compared with the unperturbed band structure (16) in Fig. 2.

4. Transmittance

With thorough analysis of the dispersion relation (3), we have shown that the band structure of the bilayer periodic array under consideration consists of broad bands separated by narrow gaps. At first glance, it seems that the system with such a band structure is perfectly transparent within the whole spectrum except the narrow gaps. Nevertheless, the analytical and numerical study of the expres-

sion (4) for the transmittance, reveals that the behavior of the transmittance drastically differs from that based on the form of the band structure only. In fact, the transmittance has form of a "comb": inside each pass band, the transmittance, getting very sharp peaks, turns out to be very small within large intervals between them. We show below that this unexpected effect is a direct consequence of the strong contrast condition (6). Within every *b*-band, the number of the peaks equals to the number N of the unit (a,b) cells that constitute the stack-structure. The leftmost peak is originated from the corresponding *b*-resonance (10). To the right from the *b*-peak, there are $(N-1)$ equidistant peaks contributed by the Fabry–Perot resonances,

$$\kappa d = m\pi/N, \qquad m = 1, 2, 3, \ldots, N-1, \tag{20}$$

arising due to multiple wave-reflections from the boundaries of the whole superlattice.

The predicted behavior of the transmittance can be understood as follows. By substitution of the unperturbed spectrum (15) into the general Eq. (4) for the transmittance T_N, one can readily express the latter in the terms of the phase shift φ_b,

$$T_N = \frac{1}{1 + \alpha_-^2 \sin^2(N\varphi_b)}. \tag{21}$$

This equation shows that the transmission is perfect, i.e., $T_N = 1$, when the phase shift φ_b within the j_bth *b*-band, takes values

$$\varphi_b^{(\text{res})} = \frac{m\pi}{N}, \quad m = 1, 2, 3 \ldots$$

$$\frac{m}{N} = j_b, \ j_b + \frac{1}{N}, \ j_b + \frac{2}{N}, \ldots, j_b + \frac{N-1}{N}. \tag{22}$$

Here the ratio $m/N = j_b$ defines the *b*-resonances, whereas the rest of the ratios, from $(j_b + 1/N)$ to $[j_b + (N-1)/N]$, refer to the Fabry–Perot resonances within the j_bth *b*-band.

Between the peaks, the transmittance takes small values of the order of

$$T_N \sim \alpha_-^{-2} \sim \left(\frac{Z_b}{Z_a}\right)^2 \ll 1. \tag{23}$$

The small values of the transmittance between the peaks are caused by the mismatching factor α_- which is great in the strong contrast regime, see Eq. (6). The value of T_N given by Eq. (23) is inverse to the squared mismatching factor. Therefore, the transmittance T_N decreases with increase of the mismatching factor rapidly enough.

Since the transmittance significantly differs from the small values of the order of $\alpha_-^{-2} \ll 1$ only in the narrow vicinities of the very sharp peaks situated at the resonances (22), we can determine the form of these peaks. By replacing the sine in Eq. (21) with its approximate expression in the vicinity of the resonances (22), we get
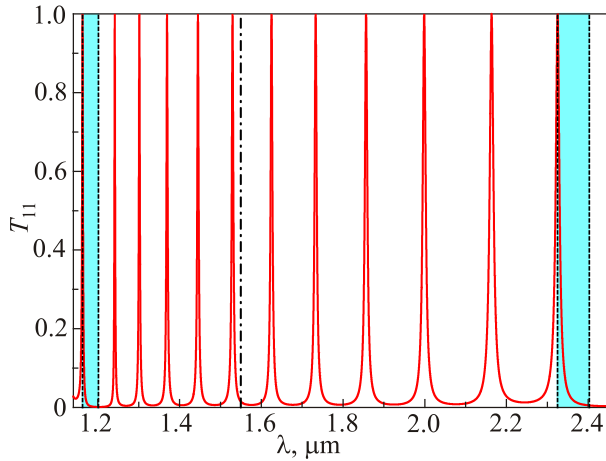
Fig. 3. (Color online) Transmittance (4) of the superlattice with number of unit cells $N = 11$, within the second b-band (the band with $j_b = 1$ that follows the lowest band with $j_b = 0$) indicated in Fig. 2. Vertical dash-dot line shows wavelength 1.55 μm. Vertical dashed lines display the edges of the spectral gaps and bands.

$$T_N = \frac{\Delta^2}{\Delta^2 + \left(\varphi_b - \varphi_b^{(res)}\right)^2}. \qquad (24)$$

The peaks (24) have form of Lorentzian lines centered on $\varphi_b^{(res)}$ with the half width at half maximum

$$\Delta = \frac{1}{N\alpha_-} \ll 1. \qquad (25)$$

From Eq. (25) it follows that the sharpness of the peaks of the transmittance spectrum results from the inequality $N\alpha_- \gg 1$ which is always met in the strong contrast regime (6).

Figure 3 displays the resonant structure of the transmittance within the second b-band ($j_b = 1$) for the superlattice with $N = 11$ unit (a,b) cells, refractive indices $n_a = 1$, $n_b = 25$, and the relation between the layer thicknesses $d_b/d_a = 30$. The thickness d_b of the b-layer is chosen in such a way that the frequency corresponding to wavelength 1.55 μm (1550 nm), the standard wavelength for optical transfer-lines, is situated in the middle of the second band, where $\varphi_b/\pi = 1.5$, see Fig 2. This gives $d_b = 46.5$ nm and $d_a = 1.55$ nm.

5. Conclusions

We have investigated the propagation of the electromagnetic radiation through the dielectric bilayer periodic array with strong contrast of impedances. Specifically, we have obtained and studied both the band structure and the transmission coefficient using analytical and numerical calculations.

We have found that, in spite of such a strong impedance contrast, lower b-bands are broad, whereas the gaps between them are narrow. This is because the strong contrast condition alone does not define the structure of the bands uniquely. Besides the strong contrast condition (6), there are two additional conditions (7) and (8) which reveal the structure of the pass bands. As for the transmission coefficient, we have clearly demonstrated that the array is not transparent within the almost whole spectrum. On the contrary, the transparency occurs in the quite narrow vicinity of the b- and Fabry–Perot resonances only. In the intervals between these resonances, the transmission coefficient is very close to zero.

The remarkable properties of the superlattices described in the present study can be used for the fabrication of narrow-pass-band filters. The main condition is using dielectrics with strong contrast of their impedances. For this purpose, as materials for the a- and b-layers, one can respectively employ dielectrics with low and very high refractive index, $n_a \sim 1$ and $n_b \gg 1$. Another possibility is to use a low refractive index material with $n_b \sim 1$ for the b-layers and a material with unnaturally low refractive index, $0 < n_a \ll 1$, for the a-layers.

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