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CALCULATING THE RADIATIVE VACUUM POLARIZATION CONTRIBUTION TO THE ENERGY SHIFT OF 2P-2S TRANSITION IN μ-HYDROGEN

Calculating the radiative contribution due to the vacuum polarization effect to energy shift for 2p-2s transition in muonic hydrogen has been carried out on the basis of gauge-invariant relativistic many-body perturbation theory.

Due to the significant progress in the modern experimental technologies now a great interest attracts studying spectra of heavy and super heavy elements atoms, exotic atomic systems, including hadronic and leptonic atoms [1-6]. Especial problem is connected with précised calculating the radiative corrections to the transition energies of the muonic atoms, namely, muonic hydrogen [1-5]. Naturally, it is provided by necessity of further developing the modern as atomic and as nuclear theories. From the other side, detailed information about spectra of the exotic atomic systems (kaonic, pionic, muonic atoms) can be very useful under construction of the new X-ray standards. One could remind a great importance of the muonic chemistry, muonic spectroscopy. Very attractive perspective of the thermonuclear fission through the mechanism of the muonic catalysis is still interesting and widely studied.

The standard Dirac approach is traditionally used as starting basis in calculations of the heavy ions [5]. The problem of accounting the radiative corrections, in particular, self-energy part of the Lamb shift and vacuum polarization contribution is mostly treated with using the perturbation theory (PT) on parameters $1/Z$, αZ (α is fine structure constant). It permits evaluations of the relative contributions of different expansion energy terms: non-relativistic, relativistic ones, as functions of Z. For high Z (Z is a nuclear charge) serious problems are connected with correct definition of the QED corrections: the Lamb shift, Lamb shift self-energy part, vacuum polarization, the nuclear finite size correction etc [6-13]. Further improvement of this method is linked with using gauge invariant procedures of generating relativistic orbital basises and more correct treating nuclear and QED effects [5-8,13-18].

In refs. [19-22] it has been proposed a new scheme to calculating spectra of heavy systems with account of nuclear and radiative effects, based on the relativistic many-body PT [13-18] (see also [3]) and advanced effective procedures for accounting the radiative corrections. Here we present the results of calculating the contribution due to the vacuum polarization effect to energy shift for 2p-2s transition in muonic hydrogen. The obtained result has to be more précised in comparison with our result [19].

The calculation of the radiative vacuum polarization shift in the muonic hydrogen can be performed using the relativistic Dirac approximation as a zeroth one. Further, the expectation value of the radiative vacuum polarization operator gives the corresponding correction. One can write the one-particle relativistic Dirac equation with a potential:

$$V(r)=V_n(r)+U(r). \tag{1}$$

This potential includes the electrical V_n and polarization $U(r)$ potentials of a nucleus. Further one could define the charge distribution in a nucleus, for example, by the Gaussian function (look details in Ref. [9]):

$$\rho(r|R)=\left(4\gamma^{3/2}/\sqrt{\pi}\right)\exp\left(-\gamma r^2\right). \quad (2)$$

Here $\gamma=4/\pi R^2$; R is an effective nucleus radius. As usually, the Coulomb potential for spherically symmetric density $\rho(r|R)$ can be written as follows:

$$V_{nucl}(r|R)=-\left((1/r)\int_0^r dr' r'^2 \rho\left(r'\middle|R\right)+\int_r^\infty dr' r' \rho\left(r'\middle|R\right)\right). \quad (3)$$

The details of the determination of this potential can be, for example, found in ref. [9]. The vacuum polarization part is usually accounted in the first PT order by using the Uehling potential [1,8,16,17]:

$$U(r)=-\frac{2\alpha}{3\pi r}\int_1^\infty dt\exp\left(-2rt/\alpha Z\right)\left(1+1/2t^2\right)\frac{\sqrt{t^2-1}}{t^2}\equiv-\frac{2\alpha}{3\pi r}C(g),\ g=\frac{r}{\alpha Z}. \quad (4)$$

The corresponding expectation value of this operator gives the corresponding vacuum polarization correction. In the scheme [17] this potential is approximated by quite precise analytical function (see details in refs. [3,15,16]). The most advanced version of the such potential (C®$\tilde{\tilde{C}}$) is presented as follows:

$$\tilde{\tilde{C}}(g)=\tilde{C}_1(g)\tilde{\tilde{C}}_2(g)/\left(\tilde{C}_1(g)+\tilde{\tilde{C}}_2(g)\right),$$

$$\tilde{\tilde{C}}_2(g)=\tilde{C}_2(g)f(g),$$

$$\tilde{C}_2(g)=-1.8801\exp(-g)/g^{3/2}$$

$$\tilde{C}_1(g)=\mathrm{h}(g/2)+1.410545-1.037837g,$$

$$f(g)=\left((1.1024/g-1.3361)/g+0.8027\right)$$

The using this formula permits one to decrease the calculation errors for this term down to ~0.1%. Error of usual calculation scheme is~10%. We carried out the calculation of the vacuum polarization contribution to the energy shift for 2p-2s transition in muonic hydrogen. It should be noted that the energy levels of muonic atoms are very sensitive to effects of QED, nuclear structure and recoil since the muon is about 206 times heavier than the electron. As usually the fundamental constants from the CODATA 1998 are used in the numerical calculations. The most QED effect for muonic atoms is the virtual production and annihilation of a single e^+e^- pair (the Uehling contribution). In table 1 we present our data for the generalized Uehling potential expectation values and results of other calculations [5,17]). The mixed muon-electron vacuum polarization correction is 0.00007 meV. Correction to the Lamb shift due to the anomalous magnetic moment has the value (accepted in literature value): -0.00002mev. The relativistic recoil correction is connected with the well known fact.

The centre of mass motion can be separated exactly from the relative motion only in the non-relativistic limit. Relativistic corrections have been calculated and its contribution to 2p-2s energy [1]: -0.00419 meV. To obtain total relativistic and recoil corrections, one must add the difference between the Uehling potential expectation calculated with relativistic and non-relativistic wave functions, giving total correction 0.0169meV. Two-photon recoil for μ- hydrogen is -0.04497 meV. Higher-order radiative recoil corrections give an additional contribution of -0.0096meV. The final result for the μ- hydrogen Lamb shift is 202.054 meV.

In conclusion we note that usually the radiative corrections are estimated in the light systems by means of the αZ expansion method and surely this approach is quite consistent in a case of the little values on nuclear charge. Obviously, the expansion method can not be used for studying the Lamb shift (polarization of vacuum effect) in more complicated and heavier muonic systems (than, say, hydrogen or the muonic hydrogen). At the same time our method is applicable to light atoms and more heavy ones as in free state as in an external field [21,22].

Table 1. The Uehling potential expectation value (R_p –proton radius in fm)

Point Nucleus		R_p=0,875 [5]		R_p=0,875 [19]		R_p=0,875 (our th.)	
$2p_{1/2}$-$2s_{1/2}$	$2p_{3/2}$-$2s_{1/2}$	$2p_{1/2}$-$2s_{1/2}$	$2p_{3/2}$-$2s_{1/2}$	$2p_{1/2}$-$2s_{1/2}$	$2p_{3/2}$-$2s_{1/2}$	$2p_{1/2}$-$2s_{1/2}$	$2p_{3/2}$-$2s_{1/2}$
205.0282	205.0332	205.0199	205.0250	205.0207	205.0256	205.0202	205.0254

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Abstract. Calculating the radiative contribution due to the vacuum polarization effect to energy shift for 2p-2s transition in muonic hydrogen has been carried out on the basis of gauge-invariant relativistic many-body perturbation theory.

Key words: muonic hydrogen, energy shift, relativistic theory

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РАСЧЕТ РАДИАЦИОННОГО ВКЛАДА ЗА СЧЕТ ЭФФЕКТА ПОЛЯРИЗАЦИИ ВАКУУМА В СДВИГ ЭНЕРГИИ 2P-2S ПЕРЕХОДА В m- ВОДОРОДЕ

Резюме. Проведен расчет радиационного вклада за счет эффекта поляризации вакуума в сдвиг энергии 2p-2s перехода в мюонном водороде методом калибровочно-инвариантной релятивисткой теории возмущений.

Ключевые слова: μ — водород, энергетический сдвиг, релятивистская теория

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РОЗРАХУНОК РАДІАЦІЙНОГО ВНЕСКУ ЗА РАХУНОК ЕФЕКТУ ПОЛЯРИЗАЦІЇ ВАКУУМУ У ЗСУВ ЕНЕРГІЇ 2P-2S ПЕРЕХОДУ У m- ВОДНІ

Резюме. Виконано розрахунок радіаційного внеску за рахунок ефекту поляризацій вакууму у зсув енергії 2p-2s переходу у мюонному водні методом калібровочно-інваріантної релятивістської теорії збурень

Ключові слова: μ — водень, енергетичний зсув, релятивістська теорія