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NON-LINEAR OPTICS AND SPECTROSCOPY OF ATOMIC AND LASER SYSTEMS WITH ELEMENTS OF A CHAOS

The whole class of modern problems of the nonlinear optics and spectroscopy of the atomic and laser systems is considered from the point of view of a chaos theory. An advanced techniques of using the non-linear analysis methods and chaos theory such as the wavelet analysis, multi-fractal formalism, mutual information approach, correlation integral analysis, false nearest neighbour algorithm, Lyapunov exponent's analysis, and surrogate data method are used in studying the cited problems.

1. Introduction

As it is well known in the modern quantum electronics, photoelectronics etc there are many physical systems (multielement semiconductors and gas lasers, different radiotechnical devices etc), which should be considered in the first approximation as set of autogenerators, coupled by different way (c.f.[1,2]). The typical schemes of different autogenerators (semiconductor quantum generators, coupled by means optical waveguide etc) are presented in refs. [1,2]. The key aspect of studying the dynamics of these systems is analysis of the temporal set for characteristic signals. In refs.[1-4] it has been numerically studied a regular and chaotic dynamics of the system of the Van-der-Poll autogenerators with account of finiteness of the signals propagation time between them and also with special kind of inter-oscillators interaction forces. Chaos theory establishes that apparently complex irregular behaviour could be the outcome of a simple deterministic system with a few dominant nonlinear interdependent variables. The past decade has witnessed a large number of studies employing the ideas gained from the science of chaos to characterize, model, and predict the dynamics of various systems phenomena (c.f.[1-25]). The outcomes of such studies are very encouraging, as they not only revealed that the dynamics of the apparently irregular phenomena could be understood from a chaotic deterministic point of view but also reported very good predictions using such an approach for different systems. Here we consider some problems of the nonlinear optics and spectroscopy of the atomic and laser systems from the point of view of a chaos theory, namely we consider the hydrogen atom and laser with absorbing cell. An advanced techniques of using the non-linear analysis methods and chaos theory such as the wavelet analysis, multi-fractal formalism, mutual information approach, correlation integral analysis, false nearest neighbour algorithm, Lyapunov exponent's analysis, and surrogate data method are applied to these systems.

2. Advanced technique of a chaos theory in optics and spectroscopy

In this section we briefly present an advanced technique of nonlinear analysis methods and chaos theory following to the Refs. [2-4,24,25]. Let us consider scalar measurements $s(n) = s(t_0 + n\mathrm{D}t) = s(n)$, where t_0 is the start time, Dt is the time step, and is n the number of the measurements. In a general case, $s(n)$ is any time series, particularly the amplitude level of any optical or spectroscopic characteristics. Since processes resulting in the chaotic behaviour are fundamentally multivariate, it is necessary to re-

construct phase space using as well as possible information contained in the $s(n)$. Such a reconstruction results in a certain set of d-dimensional vectors $\mathbf{y}(n)$ replacing the scalar measurements. Packard et al. [7] introduced the method of using time-delay coordinates to reconstruct the phase space of an observed dynamical system. The direct use of the lagged variables $s(n + t)$, where t is some integer to be determined, results in a coordinate system in which the structure of orbits in phase space can be captured. Then using a collection of time lags to create a vector in d dimensions,

$$\mathbf{y}(n) = [s(n), s(n + t), s(n + 2t), \ldots, s(n + (d\text{-}1)t)], \quad (1)$$

the required coordinates are provided. In a nonlinear system, the $s(n + jt)$ are some unknown nonlinear combination of the actual physical variables that comprise the source of the measurements. The dimension d is called the embedding dimension, d_E. Example of the Lorenz attractor given by Abarbanel et al. [5,6] is a good choice to illustrate the efficiency of the method.

According to Mañé [13] and Takens [12], any time lag will be acceptable is not terribly useful for extracting physics from data. If t is chosen too small, then the coordinates s(n + jt) and s(n + (j + 1)t) are so close to each other in numerical value that they cannot be distinguished from each other. Similarly, if t is too large, then s(n + jt) and s(n + (j + 1)t) are completely independent of each other in a statistical sense. Also, if t is too small or too large, then the correlation dimension of attractor can be under- or overestimated respectively [8,18]. It is therefore necessary to choose some intermediate (and more appropriate) position between above cases. First approach is to compute the linear autocorrelation function

$$C_L(\delta) = \frac{\frac{1}{N}\sum_{m=1}^{N}[s(m+\delta) - \bar{s}][s(m) - \bar{s}]}{\frac{1}{N}\sum_{m=1}^{N}[s(m) - \bar{s}]^2}, \quad (2)$$

where $\bar{s} = \frac{1}{N}\sum_{m=1}^{N} s(m)$, and to look for that time lag where C_L(d) first passes through zero (see [18]). This gives a good hint of choice for t at that $s(n + jt)$ and $s(n + (j + 1)t)$ are linearly independent. However, a linear independence of two variables does not mean that these variables are nonlinearly independent since a nonlinear relationship can differs from linear one. It is therefore preferably to utilize approach with a nonlinear concept of independence, e.g. the average mutual information. Briefly, the concept of mutual information can be described as follows. Let there are two systems, A and B, with measurements a_i and b_k. The amount one learns in bits about a measurement of a_i from a measurement of b_k is given by the arguments of information theory [9] as

$$I_{AB}(a_i, b_k) = \log_2\left(\frac{P_{AB}(a_i, b_k)}{P_A(a_i)P_B(b_k)}\right), \quad (3)$$

where the probability of observing a out of the set of all A is $P_A(a_i)$, and the probability of finding b in a measurement B is $P_B(b_i)$, and the joint probability of the measurement of a and b is $P_{AB}(a_i, b_k)$. The mutual information I of two measurements a_i and b_k is symmetric and non-negative, and equals to zero if only the systems are independent. The average mutual information between any value a_i from system A and b_k from B is the average over all possible measurements of $I_{AB}(a_i, b_k)$,

$$I_{AB}(\tau) = \sum_{a_i, b_k} P_{AB}(a_i, b_k) I_{AB}(a_i, b_k) \quad (4)$$

To place this definition to a context of observations from a certain physical system, let us think of the sets of measurements $s(n)$ as the A and of the measurements a time lag t later, $s(n + t)$, as B set. The average mutual information between observations at n and $n + t$ is then

$$I_{AB}(\tau) = \sum_{a_i, b_k} P_{AB}(a_i, b_k) I_{AB}(a_i, b_k). \quad (5)$$

Now we have to decide what property of I(t) we should select, in order to establish which among the various values of t we should use in making the data vectors $\mathbf{y}(n)$. In ref. [11] it has been suggested, as a prescription, that it is necessary to choose that t where the first minimum of I(t) occurs. On the other hand, the autocorrelation coefficient failed to achieve zero, i.e. the autocorrelation function analysis not provides us

with any value of t. Such an analysis can be certainly extended to values exceeding 1000, but it is known [15] that an attractor cannot be adequately reconstructed for very large values of t. The mutual information function usually [4] exhibits an initial rapid decay (up to a lag time of about 10) followed more slow decrease before attaining near-saturation at the first minimum.

One could remind that the autocorrelation function and average mutual information can be considered as analogues of the linear redundancy and general redundancy, respectively, which was applied in the test for nonlinearity. If a time series under consideration have an n-dimensional Gaussian distribution, these statistics are theoretically equivalent as it is shown by Paluš (see [15]). The general redundancies detect all dependences in the time series, while the linear redundancies are sensitive only to linear structures. Further, a possible nonlinear nature of process resulting in the vibrations amplitude level variations can be concluded.

The goal of the embedding dimension determination is to reconstruct a Euclidean space R^d large enough so that the set of points d_A can be unfolded without ambiguity. In accordance with the embedding theorem, the embedding dimension, d_E, must be greater, or at least equal, than a dimension of attractor, d_A, i.e. $d_E > d_A$. In other words, we can choose a fortiori large dimension d_E, e.g. 10 or 15, since the previous analysis provides us prospects that the dynamics of our system is probably chaotic. However, two problems arise with working in dimensions larger than really required by the data and time-delay embedding [5,6,18].

First, many of computations for extracting interesting properties from the data require searches and other operations in R^d whose computational cost rises exponentially with d. Second, but more significant from the physical point of view, in the presence of noise or other high dimensional contamination of the observations, the extra dimensions are not populated by dynamics, already captured by a smaller dimension, but entirely by the contaminating signal. In too large an embedding space one is unnecessarily spending time working around aspects of a bad representation of the observations which are solely filled with noise. It is therefore necessary to determine the dimension d_A.

There are several standard approaches to reconstruct the attractor dimension (see, e.g., [5,6,15]), but let us consider in this study two methods only. The correlation integral analysis is one of the widely used techniques to investigate the signatures of chaos in a time series. The analysis uses the correlation integral, $C(r)$, to distinguish between chaotic and stochastic systems. To compute the correlation integral, the algorithm of Grassberger and Procaccia [10] is the most commonly used approach. According to this algorithm, the correlation integral is

$$C(r) = \lim_{N\to\infty} \frac{2}{N(n-1)} \sum_{\substack{i,j \\ (1\le i<j\le N)}} H\left(r - |\ \mathbf{y}_i - \mathbf{y}_j\ |\right), \tag{6}$$

where H is the Heaviside step function with $H(u) = 1$ for $u > 0$ and $H(u) = 0$ for $u \le 0$, r is the radius of sphere centered on $\mathbf{y}_i$ or $\mathbf{y}_j$, and N is the number of data measurements. If the time series is characterized by an attractor, then the integral $C(r)$ is related to the radius r given by

$$d = \lim_{\substack{r\to 0 \\ N\to\infty}} \frac{\log C(r)}{\log r}, \tag{7}$$

where d is correlation exponent that can be determined as the slop of line in the coordinates log $C(r)$ versus log r by a least-squares fit of a straight line over a certain range of r, called the scaling region. If the correlation exponent attains saturation with an increase in the embedding dimension, then the system is generally considered to exhibit chaotic dynamics. The saturation value of the correlation exponent is defined as the correlation dimension (d_2) of the attractor. The nearest integer above the saturation value provides the minimum or optimum embedding dimension for reconstructing the phase-space or the number of variables necessary to model the dynamics of the system. On the other hand, if the correlation exponent increases without bound with increase in the embedding dimension, the system under investigation is generally considered stochastic.

There are certain important limitations in the use of the correlation integral analysis in the search for chaos. For instance, the selection of inappropriate values for the parameters involved

in the method may result in an underestimation (or overestimation) of the attractor dimension [8]. Consequently, finite and low correlation dimensions could be observed even for a stochastic process [18]. To verify the results obtained by the correlation integral analysis, we use surrogate data method.

The method of surrogate data [16] is an approach that makes use of the substitute data generated in accordance to the probabilistic structure underlying the original data. This means that the surrogate data possess some of the properties, such as the mean, the standard deviation, the cumulative distribution function, the power spectrum, etc., but are otherwise postulated as random, generated according to a specific null hypothesis. Here, the null hypothesis consists of a candidate linear process, and the goal is to reject the hypothesis that the original data have come from a linear stochastic process. One reasonable statistics suggested by Theiler et al. [16] is obtained as follows.

If we denote Q_{orig} as the statistic computed for the original time series and Q_{si} for the *i*th surrogate series generated under the null hypothesis and let m_s and s_s denote, respectively, the mean and standard deviation of the distribution of Q_s, then the measure of significance *S* is given by

$$S = \frac{|Q_{orig} - \mu_s|}{\sigma_s} \tag{8}$$

An *S* value of ~2 cannot be considered very significant, whereas an *S* value of ~10 is highly significant [16]. The details on the null hypothesis and surrogate data generation are described in ref. [18]. To detect nonlinearity in the amplitude level data, the one hundred realizations of surrogate data sets were generated according to a null hypothesis in accordance to the probabilistic structure underlying the original data. The correlation integrals and the correlation exponents, for embedding dimension values from 1 to 20, were computed for each of the surrogate data sets using the Grassberger-Procaccia algorithm as explained earlier.

Often, a significant difference in the estimates of the correlation exponents, between the original and surrogate data sets, can be observed. In the case of the original data, a saturation of the correlation exponent is observed after a certain embedding dimension value (i.e., 6), whereas the correlation exponents computed for the surrogate data sets continue increasing with the increasing embedding dimension. The high significance values of the statistic indicate that the null hypothesis (the data arise from a linear stochastic process) can be rejected and hence the original data might have come from a nonlinear process.

It is worth consider another method for determining d_E, namely, a method of false nearest neighbours. In practice, the percentage of false nearest neighbours is determined for each dimension *d*. A value at which the percentage is almost equal to zero can be considered as the embedding dimension. In ref. [4] under studying the chaotic dynamics of the quantum generat0rs was shown that the percentage of false neighbours drops to almost zero at 4 or 5, i.e. a four or five-dimensional phase-space is necessary to represent the dynamics (or unfold the attractor) of the amplitude level series. From the other hand, the mean percentage of false nearest neighbours computed for the surrogate data sets decreases steadily but at 20 is about 35%. Such a result seems to be in close agreement with that was obtained from the correlation integral analysis, providing further support to the observation made earlier regarding the presence of low-dimensional chaotic dynamics in the amplitude level variations.

The Lyapunov exponents are the dynamical invariants of the nonlinear system. In a general case, the orbits of chaotic attractors are unpredictable, but there is the limited predictability of chaotic physical system, which is defined by the global and local Lyapunov exponents. A negative exponent indicates a local average rate of contraction while a positive value indicates a local average rate of expansion. In the chaos theory, the spectrum of Lyapunov exponents is considered a measure of the effect of perturbing the initial conditions of a dynamical system. Note that both positive and negative Lyapunov exponents can coexist in a dissipative system, which is then chaotic. Since the Lyapunov exponents are defined as asymptotic average rates, they are independent of the initial conditions, and

therefore they do comprise an invariant measure of attractor. In fact, if one manages to derive the whole spectrum of Lyapunov exponents, other invariants of the system, i.e. Kolmogorov entropy and attractor's dimension can be found. The Kolmogorov entropy, *K*, measures the average rate at which information about the state is lost with time. An estimate of this measure is the sum of the positive Lyapunov exponents. The inverse of the Kolmogorov entropy is equal to the average predictability. The estimate of the dimension of the attractor is provided by the Kaplan and Yorke conjecture (see [15,18]). There are several approaches to computing the Lyapunov exponents (see, e.g., [5,6,18]). One of them [18] is in computing the whole spectrum and based on the Jacobin matrix of the system function [14]. To calculate the spectrum of Lyapunov exponents from the amplitude level data, one could determine the time delay t and embed the data in the four-dimensional space. In this point it is very important to determine the Kaplan-Yorke dimension and compare it with the correlation dimension, defined by the Grassberger-Procaccia algorithm. The estimations of the Kolmogorov entropy and average predictability can further show a limit, up to which the amplitude level data can be on average predicted. Surely, the important moment is a check of the statistical significance of results.

3. Atomic system in electromagnetic field and Laser with absorbing cell: Chaotic dynamics

One of actual problem of modern optics and spectroscopy of atomic systems is their behaviour in an external field. The classical task is studying a dynamics of a hydrogen atom in an external microwave field. This problem has been in details studied in Refs. [6-8] from the point of view of classical mechanics. Here we apply the above presented method to it. As the first example of chaotic atomic systems, in figures 1,2 we present the characteristic behaviour of the ionization probability for the hydrogen atom in the microwave electromagnetic field and correlation dimension results for chaotic dynamics (the relationship between the correlation exponent and embedding dimension values).

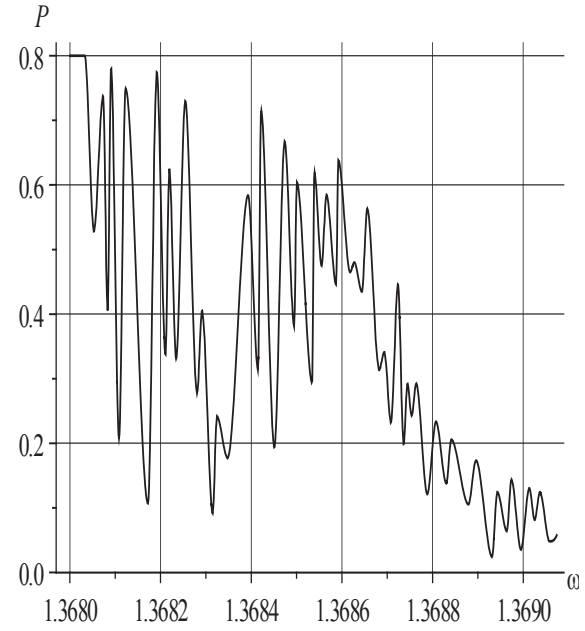

Fig.1. **Characteristic behaviour of the ionization probability for the hydrogen atom in the microwave electromagnetic field [25]**

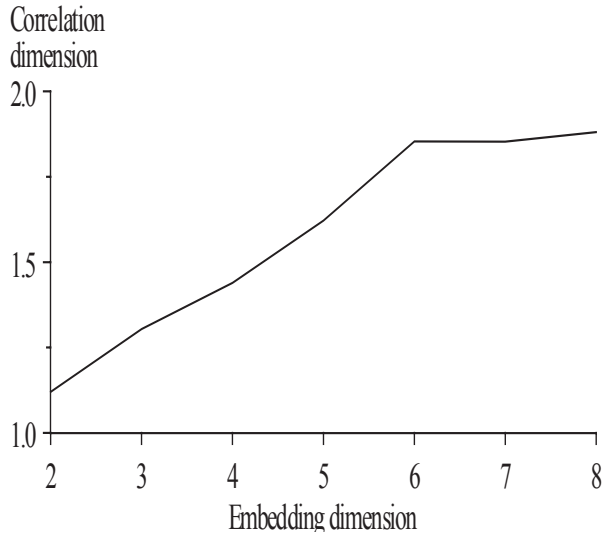

Fig.2. **The relationship between correlation exponent and embedding dimension values the hydrogen atom in the microwave electromagnetic field)**

As it can be seen, the correlation exponent value increases with embedding dimension up to a certain value, and then saturates beyond that value. The saturation of the correlation exponent beyond a certain embedding dimension is an indication of the existence of deterministic dynamics. The saturation value of the correlation exponent, i.e. correlation dimension of attractor, for the amplitude level series is about 1.8 and occurs

at the embedding dimension value of 6.The low, non-integer correlation dimension value indicates the existence of low-dimensional chaos in the dynamics of the hydrogen atom in the microwave electromagnetic field . the same picture has been found for the vibrations dynamics of the autogenerators [4]. The nearest integer above the correlation dimension value can be considered equal to the minimum dimension of the phase-space essential to embed the attractor. The value of the embedding dimension at which the saturation of the correlation dimension occurs is considered to provide the upper bound on the dimension of the phase-space sufficient to describe the motion of the attractor. Furthermore, the dimension of the embedding phase-space is equal to the number of variables present in the evolution of the system dynamics. The results of such studying can indicate that to model the dynamics of process resulting in the amplitude level variations the minimum number of variables essential is equal to 4 and the number of variables sufficient is equal to 6. Therefore, the amplitude level attractor should be embedded at least in a four-dimensional phase-space. The results can indicate also that the upper bound on the dimension of the phase-space sufficient to describe the motion of the attractor, and hence the number of variables sufficient to model the dynamics of process resulting in the level variations is equal to 6.

We performed a calculation of the energies and widths of the resonances in the hydrogen atom for the parameters of the external magnetic field corresponding to Kleppnera experiments and calculations using the models of the IWC and TMM [7,37,52]. The classical dynamics of the system depends on the scaled energy : $e=Eg^{-2/3}$ and is completely chaotic at $e>-0.12$. Detected resonances correspond in the experiments Kleppner etal. (look the review in [24]) are related to the already chaotic regime in the dynamics of the system. Examined several ranges of values of the magnetic field and, in particular, the value of B 6T. There are analyzed fully convergent series of resonances in the energy ranges: [(n-0.5)g, (n-0.3g] for n=1,2,3,4. Rydberg series of resonances are converging to the Landau ionization limit: $E_{ion}(n_r) = (n_r+1/2)\ g$. At each interval there is studied the distribution of levels and widths. For the energy interval between the first and second outside ionization (n_r=0, only one channel is open), the ratio of average width to average levels interval is equal [25]: G_{av}/DE_{av} =0.22±0.01 , that is agreed with results, obtained on the basis of the complex coordinates method (CCM): G_{av}/DE_{av} =0,23±0,01. Respectively, it can be written for the n opened channels: G_{av}/DE_{av} =0,23n (in the CCM), G_{av}/DE_{av} =0,22n (on the operator perturbation theory=OPT). To identify the statistical properties of the resonance it has been normalized the levels and widths interval with respect to the mean. The above determined value G_{av}/DE_{av} =0.22±0.01 is used used to scan the distribution of widths. Figure 3 shows the integral distribution of the energy levels: $N(s)=\int_0^s P(x)dx$,calculated within the random matrix theory (RMT), advanced OPT model (look [24,25]). Each system in Figure 3 corresponds to a fixed number of open channels intervals: a, b, c, d correspond to the number of open channels 1,2,3,4. The notations are used as follows: dotted line-prediction models RMT, the line points - CCM model and our model [25], solid line - model 2D-OPT. As can be seen between the results of calculations within all three models have a fairly good agreement.

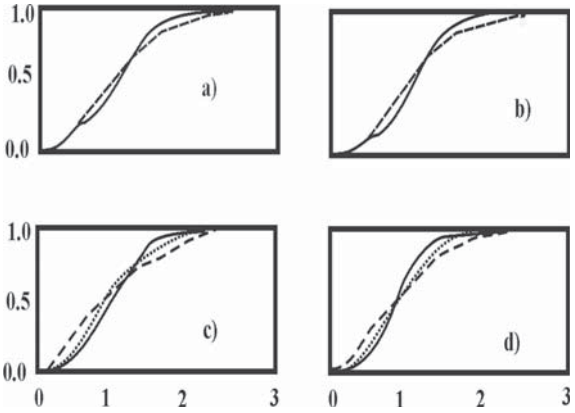

Fig. 3. **The integral (cumulative) distribution of the energy intervals for the resonances in the hydrogen atom in a magnetic field 6T.**

It is important to note that for n=1 all distributions are satisfying to the Porter-Thomas distribution:

$$P_n(\Gamma) = \{1/(2\Gamma_{av})^{n/2}\Gamma(n/2)\}\ \Gamma^{(n/2)-1}\exp[-\Gamma/2\Gamma_{av}] \quad (9)$$

where n-number of open channels, Γ(n/2) is the gamma function (not to be confused with a width Γ). The density of states in the middle of the each Landau channel is: to our data ~ 33 resonances at cm^{-1}, according to the CMM model ~ 40 cm^{-1}, that is in good agreement with the experimental value of 30 resonances cm^{-1} . The average width of the resonance is, to our data, 0.0055 cm^{-1}, which is also consistent with the experiment Kleppner et al.: 0.004-0.006 cm^{-1} , the OPT result: 0.005 cm^{-1} [24,25]. In the energy range [25,30 cm^{-1}], the average width of the resonance in our data is 0,034 cm^{-1}, which is in agreement with the experimental value of 0.03 cm^{-1} and in the evaluation of the CCM model 0.04 cm^{-1} and OPT model: 0,035 cm^{-1}. From a physical point of view, the presence in the spectrum of the hydrogen atom in a magnetic field, many resonances with anomalously small widths explained quite naturally. Their appearance is obviously not due to some hidden symmetry or the phenomenon of localization, and is due to interference effects and random fluctuations inherent in general to all chaotic systems. The numerical calculation of the Lyapunov exponents for the hydrogen atom in a magnetic field (6T) gives the following results: l_1=0.484; l_2 =0.195, that confirms the conclusion regarding the chaotic behaviour of the system.

Further we consider a chaotic dynamics of a laser system with absorbing cell. It is known that for a single-mode laser, described by the equations of the Lorentz needed to return to the region of chaotic generation combination of parameters is difficult to achieve. The results of study [23] indicate that the laser with a nonlinear absorption cell may be more convenient physical system for the experimental observation of dynamic chaos. We consider a theoretical model of a single-mode laser resonator in which the reinforcement is placed along with a nonlinear absorbing medium. Each of the environments consists of identical two-level atoms. The gain and absorption lines are uniformly broadened and their centers align and coincide with one of the frequencies of the cavity. Such a model can describe the real system of five differential equations [23]:

$$d_e/d\tau = -e + p_1 + p_2,$$
$$dp_1/d\tau = -\delta_1(p_1 + em_1),$$
$$dp_2/d\tau = -\delta_2(p_2 + em_2),$$
$$dp_2/d\tau = -\delta_2(p_2 + em_2),$$
$$dm_1/d\tau = -\rho_1(m_1 - m_{01} - ep_1),$$
$$dm_2/d\tau = -\rho_2(m_2 - m_{02} - \beta ep_2).$$

(10)

Here, the index 1 refers to intensify, and the index of 2 - to an absorbing medium; e, p_1p_1, p_2p_2, m_1m_1 и m_2m_2- the dimensionless variables, e - the amplitude of the laser of the field, p_kp_k- polarization in the environment, m_km_k- the difference between the populations of the working levels; p_k and d_k - respectively the longitudinal and transverse relaxation rate, related to the half-width of the resonator $\delta\omega_p/2\delta\omega_p/2$, k=1,2; $m_{0k}m_{0k}$- the difference between the populations of the working levels in the absence of generation $(m_{01} < 0,\ m_{02} > 0)$;$(m_{01} < 0,\ m_{02} > 0)$; β β-the ratio of the coefficients of saturation of the absorbing and amplifying media; $\tau = t\,\delta\omega_p/2$ $\tau = t\,\delta\omega_p/2$ is the dimensionless time. According to ref. [23], the system (10) is invariant under the substitution $e \rightarrow -e$, $p_kp_k \rightarrow -p_k\,(k = 1,2)$ $p_k(k = 1,2)$. Attractor of the system can be as invariant with respect to this change (let's call this attractor «symmetrical») and non-invariant («asymmetric»). In the latter case certainly, there are two attractor into each other after this change. In fig. 3 we list present the results of numerical simulation for the system (10) [23]. Strange attractors occur as a result of sequence of bifurcations of solutions (1), the first of which is Hopf bifurcation of stationary solutions with zero intensity of the laser field. This bifurcation occurs $\eta = \delta_2[1 + (\delta_2)(1 + \delta_1 + m_{02})/\delta_1(1 + \delta_1)]$ $\eta = \delta_2[1 + (\delta_2)(1 + \delta_1 + m_{02})/\delta_1(1 + \delta_1)]$, if $\eta < m_{02}\eta < m_{02}$. According to data [23]

and our analysis the Hopf bifurcation occurs at moderate values $\eta\eta$, if the relative width of the absorption line $\delta_2\delta_2$ is quite small, and the relative width of the gain lineδ_1 is quite large. The numerical calculation shows that in order to get the chaotic lasing it is necessary the following: to saturate the absorber should be saturated stronger than the amplifier ($\beta > 1$). At low β the limit cycles generated from the stationary solutions with the zeroth intensity is stable up to very large values of ŋ. . In table 1 we list the numerical parameters of the chaotic regime for the laser system with absorbing

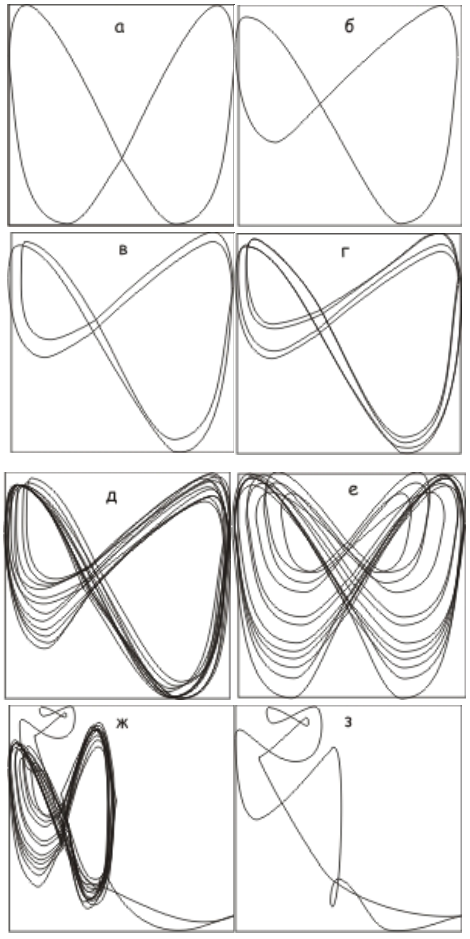

Figure 3. **Projections of the phase trajectories for different values of the parameter ŋ.**
ŋ: а - 1.7000, б - 1.8200, в - 1.8350, г - 1.8385, е - 1.8500, ж - 1.8800, з - 1.9000

cell: l_1-l_6 are the Lyapunov exponents in descending order, *K* - Kolmogorov entropy

Table 1. **Parameters of chaotic regimes in the laser system: l_1-l_6 are the Lyapunov exponents in descending order (our results)**

Regime	λ_1	λ_2	λ_3
Weak chaos	0.175	-0.0001	-0.0003
Strong chaos	0.542	0.203	-0.0001
	λ_4	λ_5	λ_6
Weak chaos	–0.244	–	–
Strong chaos	–0.0004	–0.067	–0.188

The main conclusion of this work is that application of the different chaos theory and nonlinear analysis methods and algorithms to studying chaotic elements in dynamics of the different atomic and laser systems is very useful. In particular, their using allows to study and confirm an existence of chaotic behaviour for the H atom in a microwave and magnetic field and laser with absorbing cell.

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NON-LINEAR OPTICS AND SPECTROSCOPY OF ATOMIC AND LASER SYSTEMS WITH ELEMENTS OF A CHAOS

Abstract

The whole class of modern problems of the nonlinear optics and spectroscopy of the atomic and laser systems is considered from the point of view of a chaos theory. An advanced techniques of using the non-linear analysis methods and chaos theory such as the wavelet analysis, multi-fractal formalism, mutual information approach, correlation integral analysis, false nearest neighbour algorithm, Lyapunov exponent's analysis, and surrogate data method are used in studying the cited problems.

Key words: optics and spectroscopy, atomic and laser systems, nonlinear analysis methods and chaos theory

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НЕЛИНЕЙНАЯ ОПТИКА И СПЕКТРОСКОПИЯ АТОМНЫХ И ЛАЗЕРНЫХ СИСТЕМ С ЭЛЕМЕНТАМИ ХАОСА

Резюме

Класс современных задач нелинейной оптики и спектроскопии атомных и лазерных систем рассматривается с точки зрения теории хаоса. Для решения искомых задач применены усовершенствованные нелинейные методы анализа и теории хаоса, в частности, вэйвлет-анализ, мультифрактальный формализм, метод взаимной информации, метод корреляционного интеграла, алгоритм ложных ближайших соседей, анализ показателей Ляпунова, метод сурогатных данных.

Ключевые слова: оптика и спектроскопия, атомные и лазерные системы, нелинейные методы, теория хаоса

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НЕЛІНІЙНА ОПТИКА І СПЕКТРОСКОПІЯ АТОМНИХ І ЛАЗЕРНИХ СИСТЕМ З ЕЛЕМЕНТАМИ ХАОСА

Резюме

Клас сучасних задач нелінійної оптики і спектроскопії атомних і лазерних систем розглядається з точки зору теорії хаосу. Для вирішення шуканих завдань застосовані вдосконалені нелінійні методи аналізу та теорії хаосу, зокрема, вейвлет-аналіз, мультіфрактальний формалізм, метод взаємної інформації, метод кореляційного інтеграла, алгоритм помилкових найближчих сусідів, аналіз показників Ляпунова, метод сурогатних даних.

Ключові слова: оптика і спектроскопія, атомні, лазерні системи, нелінійні методи, теорія хаосу