

Kherson Department, Odessa National Polytechnical University, 1, Shevchenko pr., Ukraine
e-mail: quantsha@mail.ru

STRONG π NUCLEAT INTERACTION EFFECTS IN SPECTROSCOPY OF HADRONIC ATOMS

New New relativistic method on the basis of the Klein-Gordon-Fock equation with a generalized pion-nuclear potential is used to determine the strong interaction effects in spectroscopy of pionic atoms. The numerical data on 2p-1s, 3d-2p, 4f-3d, 5g-4f transition energies on the basis of the Klein-Gordon-Fock theory with the pion-nucleus potential, the Klein-Gordon-Fock theory with account of the radiative and the finite nuclear size corrections are presented for in some middle and heavy pionic atoms. It is performed comparison between theoretical data and results of measurements in the Berkley, CERN and Virginia laboratories.

1. Introduction

In this paper we goes on our studying on spectroscopy of pionic atoms and quantitative estimate of the strong pion-nucleon interactions contributions to the transitions energies in the X-ray spectra of pionic atomic systems. Let us note that the spectroscopy of hadron atoms has been used as a tool for the study of particles and fundamental properties for a long time. Exotic atoms are also interesting objects as they enable to probe aspects of atomic and nuclear structure that are quantitatively different from what can be studied in electronic or “normal” atoms. As it is known, one of the fundamental questions in the modern hadron’s physics is that the hadron masses being much higher than the mass of their quark content. The current mass of the up (u) and down (d) quarks is two orders of magnitude smaller than a typical hadron’s mass of about 1 GeV. This extraordinary phenomenon is proposed to originate from spontaneous breaking of chiral symmetry of massless quarks in strong interaction physics [1-5]. At present time one of the most sensitive tests for the chiral symmetry breaking scenario in the modern hadron’s physics is provided by studying the exotic hadron-atomic systems. Nowadays the transition energies in pionic (kaonic, muonic etc.) atoms are measured with an unprecedented precision and from studying spectra of the hadronic atoms it is possible to investigate the strong inter-

action at low energies measuring the energy and natural width of the ground level with a precision of few meV [1-10].

The mechanism of creation of the hadronic atoms is well known now (e.g. [1,2]). Really, such an atom is formed when a negative pion enters a medium, loses its kinetic energy through ionization and excitation of the atoms and molecules and eventually is captured, replacing the electron, in an excited atomic orbit. The further de-excitation scenario includes the different cascade processes such as the Auger transitions, Coulomb de-excitation, scattering etc. When a pion reaches a low-n state with the little angular momentum, strong interaction with the nucleus causes its absorption. The strong interaction is the reason for a shift in the energies of the low-lying levels from the purely electromagnetic values and the finite lifetime of the state corresponds to an increase in the observed level width. The possible energy shifts caused by the pion-induced fluorescence X-rays were checked in the measurement of the pion beams at PSI in Switzerland. For a long time the similar experimental investigations have been carried out in the laboratories of Berkley, Virginia (USA), CERN (Switzerland).

The most known theoretical models to treating the hadronic (pionic, kaonic, muonic, antiprotonic etc.) atomic systems are presented in refs. [1-5,7,8]. The most difficult aspects of the theo-

retical modeling are reduced to the correct description of pion-nuclear strong interaction [1-3] as the electromagnetic part of the problem is reasonably accounted for. Besides, quite new aspect is linked with the possible, obviously, very tiny electroweak and hyperfine interactions.

2. Relativistic approach to pionic atoms spectra

All available theoretical models to treating the hadronic (kaonic, pionic) atoms are naturally based on the using the Klein-Gordon-Fock equation [2,5], which can be written as follows :

$$m^2 c^2 \Psi(x) = \left\{ \frac{1}{c^2} [i\hbar \partial_t + eV_0(r)]^2 + \hbar^2 \nabla^2 \right\} \Psi(x) \quad (1)$$

where c is a speed of the light, \hbar is the Planck constant, and $\Psi_0(x)$ is the scalar wave function of the space-temporal coordinates. Usually one considers the central potential $[V_0(r), 0]$ approximation with the stationary solution:

$$\Psi(x) = \exp(-iEt/\hbar) \varphi(x), \quad (2)$$

where $\varphi(x)$ is the solution of the stationary equation:

$$\left\{ \frac{1}{c^2} [E + eV_0(r)]^2 + \hbar^2 \nabla^2 - m^2 c^2 \right\} \varphi(x) = 0 \quad (3)$$

Here E is the total energy of the system (sum of the mass energy mc^2 and binding energy e_0). In principle, the central potential V_0 naturally includes the central Coulomb potential, the vacuum-polarization potential, the strong interaction potential.

The most direct approach to treating the strong interaction is provided by the well known optical potential model (c.g. [2]). Practically in all papers the central potential V_0 is the sum of the following potentials. The nuclear potential for the spherically symmetric density $\rho(r|R)$ is [6,13]:

$$V_{nucl}(r|R) = -\left(\frac{1}{r} \right) \int_0^r dr' r'^2 \rho(r'|R) + \int_r^\infty dr' r' \rho(r'|R) \quad (4)$$

The most popular Fermi-model approximation the charge distribution in the nucleus $\rho(r)$

(c.f.[12]) is as follows:

$$\tilde{n}(r) = \tilde{n}_0 \left[1 + \exp[(r-c)/a] \right], \quad (5)$$

where the parameter $a=0.523$ fm, the parameter c is chosen by such a way that it is true the following condition for average-squared radius:

$$\langle r^2 \rangle^{1/2} = (0.836 \times A^{1/3} + 0.5700) \text{fm}.$$

The effective algorithm for its definition is used in refs. [12] and reduced to solution of the following system of the differential equations:

$$V'_{nucl}(r, R) = \left(\frac{1}{r^2} \right) \int_0^r dr' r'^2 \rho(r', R) \equiv \left(\frac{1}{r^2} \right) y(r, R) \quad (6)$$

$$y'(r, R) = r^2 \rho(r, R), \quad (7)$$

$$\tilde{n}'(r) = (\tilde{n}_0 / a) \exp[(r-c)/a] \left[1 + \exp[(r-c)/a] \right]^2 \quad (8)$$

with the corresponding boundary conditions. Another, probably, more consistent approach is in using the relativistic mean-field (RMF) model, which been designed as a renormalizable meson-field theory for nuclear matter and finite nuclei [13]. To take into account the radiation corrections, namely, the effect of the vacuum polarization we have used the generalized Ueling-Serber potential with modification to take into account the high-order radiative corrections [5,12]. The most difficult aspect is an adequate account for the strong interaction. In the pion-nucleon state interaction one should use the following pulse approximation expression for scattering amplitude of a pion on the "i" nucleon [2,3]:

$$f_i(r) = \left\{ b'_0 + b'_1(t\tau) + [c'_0 + c'_1(t\tau)] k k' \right\} \delta(r-r_i); \quad (9)$$

where t and τ are the isospines of pion and nucleon. The nucleon spin proportional terms of the kind $\sigma[kk']$ are omitted. The constants in (9) can be expressed through usual s-wave (α_{2T}) and p-wave ($\alpha_{2T,2J}$) scattering length (T and J -isospin and spin of the system πN). The corresponding parameters in the Compton wave length $\tilde{\lambda}_\pi$ terms are as follows:

$$b'_0 = (\alpha_1 + 2\alpha_3)/3 = -0.0017 \lambda_\pi.$$

$$b'_0 = (\alpha_3 - \alpha_1)/3 = -0.086 \lambda_\pi.$$

$$c'_0 = (4\alpha_{33} + 2\alpha_{13} + 2\alpha_{31} + \alpha_{11})/3 = -0.208(\lambda_\pi)^3.$$

$$c'_1 = (2\alpha_{33} - 2\alpha_{13} + \alpha_{31} - \alpha_{11})/3 = -0.184(\lambda_\pi)^3. \quad (10)$$

The scattering amplitude for pion on a nucleus can be further received as a coherent sum of the πN -scattering lengths. πN -scattering lengths. πN -scattering lengths. In approximation of the only s-wave interaction the corresponding potential can be written in the Dezer form:

$$V_N(r) = -2\pi\hbar^2\mu_\pi^{-1} [ZA^{-1}a_p + (A-Z)A^{-1}a_n] \rho(r). \quad (11)$$

The s-wave lengths of the $\pi^{-1}p$ -scattering $a_p = (2\alpha_1 + \alpha_3)/3$ и $\pi^{-1}n$ - scattering $a_n = \alpha_3$; scattering are introduced to Eq. (11). Because of the equality between $a_n = b'_0 + b'_1$ and $a_p = b'_0 - b'_1$ (with an opposite sign) the theoretical shift of

the s-level with $T = 0$ ($A = 2Z$) from Eq. (12) is much less than the observed shift. So, the more correct approximation must take into account the effects of the higher orders.

In whole the energy of the hadronic atom is represented as the sum:

$$E \approx E_{KG} + E_{FS} + E_{VP} + E_N; \quad (12)$$

Here E_{KG} -is the energy of a pion in a nucleus (Z, A) with the point-like charge (dominative contribution in (12)), E_{FS} is the contribution due to the nucleus finite size effect, E_{VP} is the radiation correction due to the vacuum-polarization effect, E_N is the energy shift due to the strong interaction V_N . The last contribution can be defined from the experimental energy values as:

$$E_N = E - (E_{KG} + E_{FS} + E_{VP}) \quad (13)$$

From the other side the strong pion-nucleus interaction contribution can be found from the solution of the Klein-Gordon equation with the corresponding meson-nucleon potential. In this case,

this contribution E_N is the function of the potentials (8)-(11) parameters.

3. Results and conclusions

In table 1 we present the data on the transition energies in some pionic atoms (from. Refs. [1-4]): the measured values from the Berkley, CERN and Virginia laboratories, the theoretical values for the $2p-1s$, $3d-2p$, $4f-3d$, $5g-4f$ pionic transitions ($E_{\#1}^N$ - values from the Klein-Gordon-Fock equation with the pion-nucleus potential [2]; E_{KGF} - values from the Klein-Gordon-Fock equation with account of radiative corrections (our data); E_{KS} - RMF finite nuclear size contribution (our data), $E_{\#2}^N$ - values from the Klein-Gordon-Fock equation with the generalized pion-nuclear potential [5] (our data).

Table 1. Transition energies (keV) in the spectra of some pionic atoms (see text)

Atom	E_{EXP} Berkley	E_{EXP} CERN	E_{KGF}	E_{FS}	$E_{\#1}^N$	$E_{\#2}^N$
Transition 4f-3d						
Ca ⁴⁰	72,352 ± 0,009		72,357	0,00	72,361	72,362
I ¹²⁷	519,1 ± 1,1	520,8 ± 0,8	516,50	-0,22	520,25	520,68
Cs ¹³³	560,5 ± 1,1	562,0 ± 1,5	556,80	-0,33	561,47	562,12
La ¹³⁹	603,6 ± 0,9	604,9 ± 2,0	597,88	-0,45	604,56	604,84
Transition 5g-4f						
Ta ¹⁸¹	453,1 ± 0,4	453,90 ± 0,20	450,67	-0,01	453,56	453,56
Au ¹⁹⁷	532,5 ± 0,5	533,16 ± 0,20	531,99	-0,025	532,87	532,87
Hg ²⁰¹	-	547,14 ± 0,25	545,730	-0,036	546,85	547,22
Bi ²⁰⁹	589,8 ± 0,9	590,06 ± 0,30	588,24	-0,05	589,75	590,16
Th ²³²	698,0 ± 0,6	698,4 ± 0,4	693,88	-0,17	697,20	698,25
U ²³⁸	731,4 ± 1,1	732,0 ± 0,4	725,78	-0,23	729,80	730,52

Note: E_{EXP} (Virginia): Ta¹⁸¹(5g-4f)=453.4±0.3; U²³⁸(5g-4f)=730.88±0.75;

It is easily to understand that when there is the close agreement between theoretical and experimental shifts, the corresponding energy levels are not significantly sensitive to strong pion-nuclear interaction, i.e the electromagnetic contribution is dominative. In the opposite situation the strong-interaction effect is very significant. The analysis of the presented data indicate on the necessity of the further more exact experimental investigations and further improvement of the pion-nuclear potential modelling. One can see that the contributions provided by the finite size effect should be accounted in a precise theory. Really, under availability of the "exact" values of the transitions energies one can perform the comparison of the theoretically and experimentally defined transition energies in the X-ray spectra in order to make a redefinition of the pion-nucleon model potential parameters using Eqs. (9)-(11). Taking into account the increasing accuracy of the X-ray pionic atom spectroscopy experiments, one can conclude that the such a way will make more clear the true values for parameters of the pion-nuclear potentials and correct the disadvantage of widely used parameterization of the potentials (9)-(11).

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A. N. Shakhman

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Abstract

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Key words: strong interaction, hadronic atom, relativistic theory

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A. N. Шахман

ЭФФЕКТЫ СИЛЬНОГО ВЗАИМОДЕЙСТВИЯ В СПЕКТРОСКОПИИ АДРОННЫХ АТОМОВ

Резюме

Новый релятивистский метод на основе уравнения Клейна-Гордона-Фока с обобщенными пион-ядерным потенциалом применен для изучения эффектов сильного взаимодействия в спектроскопии пионных атомов. Представлены данные по энергиям 2p-1s, 3d-2p, 4f-3d,5g-4f переходов для некоторых средних и тяжелых пионных атомов, полученные на основе уравнения Клейна-Гордона-Фока с пион-ядерным потенциалом с учетом радиационных и конечно-размерных ядерных поправок. Проведено сравнение теоретических данных с данными измерений в лабораториях Беркли, ЦЕРН и Вирджиния.

Ключевые слова: сильное взаимодействие, адронный атом, релятивистская теория

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A. M. Шахман

ЕФЕКТИ СИЛЬНОЇ ВЗАЄМОДІЇ В СПЕКТРОСКОПІЇ АДРОННИХ АТОМІВ

Резюме

Новий релятивістський метод на основі рівняння Клейна-Гордона-Фока із узагальненим піон-ядерним потенціалом застосовано до опису ефектів сильної взаємодії в спектроскопії піонних атомів. Отримані чисельні дані про 2p-1s, 3d-2p, 4f-3d,5g-4f енергії переходів для деяких середніх і важких піонних атомів, отримані на основі рівняння Клейна-Гордона-Фока з піон-ядерним потенціалом та урахуванням радіаційних та скінченно-розмірних ядерних поправок. Проведено порівняння теоретичних даних та результатів вимірювань в лабораторіях Берклі, ЦЕРН і Вірджинія.

Ключові слова: сильна взаємодія, адронний атом, релятивістська теорія