UDC 517.9

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NON-LINEAR ANALYSIS OF CHAOTIC SELF-OSCILLATIONS IN BACKWARD-WAVE TUBE

The analysis techniques including multi-fractal approach, methods of correlation integral, false nearest neighbour, Lyapunov exponent's, surrogate data, is applied analysis of numerical parameters of chaotic self-oscillations in the backward-wave tube. There are presented the numerical data on the Lyapunov exponents' for two self-oscillations regimes in the backward-wave tube: i). the weak chaos (normalized length: L=4.24); ii) developed chaos (L=6.1).

As it is well known in the modern electronics, photoelectronics etc there are many physical systems (the backward-wave tubes, multielement semiconductors and gas lasers, different radiotechnical devices etc), which can manifest the elements of chaos and hyperchaos in their dynamics (c.f.[1-4]). The key aspect of studying the dynamics of these systems is analysis of the dynamical characteristics. Chaos theory establishes that apparently complex irregular behaviour could be the outcome of a simple deterministic system with a few dominant nonlinear interdependent variables. The past decade has witnessed a large number of studies employing the ideas gained from the science of chaos to characterize, model, and predict the dynamics of various systems phenomena (c.f.[1-20]). The outcomes of such studies are very encouraging, as they not only revealed that the dynamics of the apparently irregular phenomena could be understood from a chaotic deterministic point of view but also reported very good predictions using such an approach for different systems.

The backward-wave tube is an electronic device for generating electromagnetic vibrations of the superhigh frequencies range. In ref.[2] there have been presented the temporal dependences of the output signal amplitude, phase portraits, statistical quantifiers for a weak chaos arising via period-doubling cascade of self-modulation and for developed chaos at large values of the dimensionless length parameter. The authors of [2] have solved the equations of nonstationary nonlinear theory for the O type backward-wave tubes without account of the spatial charge, relativistic effects, energy losses etc. It has been shown that the finite-dimension strange attractor is responsible for chaotic regimes in the backward-wave tube. In our work in order to study the chaotic self-oscillations regimes in the backward-wave tube we have used earlier developed and adapted techniques of the non-linear analysis, such as the multi-fractal formalism, methods of correlation integral, false nearest neighbour, Lyapunov exponent's, surrogate data (code "Geomath"). As the key ideas of our technique for nonlinear analysis of chaotic systems have been in details presented in refs. [3,4,17-21], here we are limited only by brief representation.. Since processes resulting in the chaotic behaviour are fundamentally multivariate, it is necessary to reconstruct phase space using as well as possible information contained in the dynamical parameter $s(n)$, where n the number of the measurements. Such a reconstruction results in a certain set of d-dimensional vectors $\mathbf{y}(n)$ replacing the scalar measurements. Packard et al. [7] introduced the method of using time-delay coordinates to reconstruct the phase space of an observed dynamical system. The direct use of the lagged variables $s(n + t)$, where t is some integer to be determined, results in a coordinate system in which the structure of orbits in phase space can be captured. Then using a collection of time lags to create a vector in d dimensions,

$$\mathbf{y}(n) = [s(n), s(n+t), s(n+2t), \ldots, \; s(n+(d\text{-}1)t)], \qquad (1)$$

the required coordinates are provided.

In a nonlinear system, the $s(n + jt)$ are some unknown nonlinear combination of the actual physical variables that comprise the source of the measurements. The dimension d is called the embedding dimension, d_E. According to Mañé [13] and Takens [12], any time lag will be acceptable is not terribly useful for extracting physics from data. If t is chosen too small, then the coordinates s(n + jt) and s(n + (j + 1)t) are so close to each other in numerical value that they cannot be distinguished from each other. Similarly, if t is too large, then s(n + jt) and s(n + (j + 1)t) are completely independent of each other in a statistical sense. Also, if t is too small or too large, then the correlation dimension of attractor can be under- or overestimated respectively [8,18]. The autocorrelation function and average mutual information can be applied here. The first approach is to compute the linear autocorrelation function:

$$C_L(\delta) = \frac{\frac{1}{N}\sum_{m=1}^{N}[s(m+\delta)-\bar{s}][s(m)-\bar{s}]}{\frac{1}{N}\sum_{m=1}^{N}[s(m)-\bar{s}]^2},$$

$$\bar{s} = \frac{1}{N}\sum_{m=1}^{N} s(m) \qquad (2)$$

and to look for that time lag where C_L(d) first passes through zero (see [18]). This gives a good hint of choice for t at that $s(n + jt)$ and $s(n + (j + 1)t)$ are linearly independent. a time series under consideration have an n-dimensional Gaussian distribution, these statistics are theoretically equivalent as it is shown by Paluš (see [15]). The general redundancies detect all dependences in the time series, while the linear redundancies are sensitive only to linear structures. Further, a possible nonlinear nature of process resulting in the vibrations amplitude level variations can be concluded.

The goal of the embedding dimension determination is to reconstruct a Euclidean space R^d large enough so that the set of points d_A can be unfolded without ambiguity. In accordance with the embedding theorem, the embedding dimension, d_E, must be greater, or at least equal, than a dimension of attractor, d_A, i.e. $d_E > d_A$.

In other words, we can choose a fortiori large dimension d_E, e.g. 10 or 15, since the previous analysis provides us prospects that the dynamics of our system is probably chaotic. However, two problems arise with working in dimensions larger than really required by the data and time-delay embedding [5,6,18].

First, many of computations for extracting interesting properties from the data require searches and other operations in R^d whose computational cost rises exponentially with d. Second, but more significant from the physical point of view, in the presence of noise or other high dimensional contamination of the observations, the extra dimensions are not populated by dynamics, already captured by a smaller dimension, but entirely by the contaminating signal.

In the large an embedding space there is no necessity spending time working around aspects of a bad representation of the observations which are solely filled with noise. It is therefore necessary to determine the dimension d_A.

There are several standard approaches to reconstruct the attractor dimension (see, e.g., [3-6,15]). The correlation integral analysis is one of the widely used techniques to investigate the signatures of chaos in a time series. The analysis uses the correlation integral, $C(r)$, to distinguish between chaotic and stochastic systems.

To compute the correlation integral, the algorithm of Grassberger and Procaccia [10] is the most commonly used approach. If the time series is characterized by an attractor, then the integral $C(r)$ is related to the radius r given by

$$d = \lim_{\substack{r\to 0\\ N\to\infty}} \frac{\log C(r)}{\log r}, \qquad (3)$$

where d is correlation exponent that can be determined as the slop of line in the coordinates log $C(r)$ versus log r by a least-squares fit of a straight line over a certain range of r, called the scaling region. If the correlation exponent attains saturation with an increase in the embedding dimension, the system is generally considered to exhibit chaotic dynamics. The saturation value of correlation exponent is defined as the correlation dimension (d_2) of attractor.

Lyapunov exponents are the dynamical invariants of the nonlinear system. In a general case, the orbits of chaotic attractors are unpredictable, but there is the limited predictability of chaotic physical system, which is defined by the global and local Lyapunov exponents. A negative exponent indicates a local average rate of contraction while a positive value indicates a local average rate of expansion. In the chaos theory, the spectrum of Lyapunov exponents is considered a measure of the effect of perturbing the initial conditions of a dynamical system. Since the Lyapunov exponents are defined as asymptotic average rates, they are independent of the initial conditions, and therefore they do comprise an invariant measure of attractor. In fact, if one manages to derive the whole spectrum of Lyapunov exponents, other invariants of the system, i.e. Kolmogorov entropy and attractor's dimension can be found. The Kolmogorov entropy, K, measures the average rate at which information about the state is lost with time. An estimate of this measure is the sum of the positive Lyapunov exponents. The inverse of the Kolmogorov entropy is equal to the average predictability. There are several approaches to computing the Lyapunov exponents (see, e.g., [5,6,18]). One of them [18] is in computing the whole spectrum and based on the Jacobin matrix of the system function [14].

In table 1 we present the data on the Lyapunov exponents' for two self-oscillations regimes in the backward-wave tube: i). the weak chaos (normalized length: L=4.24); ii) developed chaos (L=6.1). The correlations dimensions are respectively as 2.9 and 6.2. Our analysis is in very good agreement with the similar data [2] and confirms a conclusion about realization of the chaotic features in dynamics of the backward-wave tube.

Table 1. **numerical parameters of the chaotic self-oscillations in the backward-wave tube: l_1-l_6 are the Lyapunov exponents in descending order, K is the Kolmogorov entropy**

Regime	λ_1	λ_2	λ_3	K
Weak chaos L=4.24	0.261	-0.0001	–0.0004	0.261
Hyper chaos L=6.1	0.514	0.228	0.0000	0.742
Regime	λ_4	λ_5	λ_6	K
Weak chaos L=4.24	–0.528	–	–	0.261
Hyper chaos L=6.1	-0.0002	–0.084	–0.396	0.742

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The article is received in editorial 25.05.2013

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Abstract

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Key words: backward-wave tube, chaos, non-linear methods

УДК 517.9

Г. П. Препелица, В. В. Буяджи, В. Б. Терновский

НЕЛИНЕЙНЫЙ АНАЛИЗ ХАОТИЧЕСКИХ АВТОКОЛЕБАТЕЛЬНЫХ РЕЖИМОВ В ЛАМПЕ ОБРАТНОЙ ВОЛНЫ

Резюме

Техника анализа, включающая мультифрактальный подход, методы корреляционных интегралов, ложных ближайших соседей, экспонент Ляпунова, суррогатных данных, использована для изучения числовых параметров хаотических автоколебательных режимов лампы обратной волны. Представлены данные о численных показателях Ляпунова для двух автоколебательных режимов лампы обратной волны: i).слабого хаоса (нормированная длина: L = 4.24); ii) развитого хаоса (L = 6.1).

Ключевые слова: лампа обратной волны, хаос, нелинейные методы

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Г. П. Препелица, В. В. Буяджи, В. Б. Терновський

НЕЛІНІЙНИЙ АНАЛІЗ ХАОТИЧНИХ АВТОКОЛИВАЛЬНИХ РЕЖИМІВ У ЛАМПІ ЗВЕРНЕНОЇ ХВИЛІ

Резюме.

Техніка нелінійного аналізу, яка включає мультіфрактальний підхід, методи кореляційних інтегралів, хибних найближчих сусідів, експонент Ляпунова, сурогатних даних, використана для аналізу чисельних параметрів хаотичних автоколивальних режимів лампи зверненої хвилі. Наведені дані по чисельних показниках Ляпунова для двох автоколивальних режимів лампи зворотної хвилі: i). слабкого хаосу (нормована довжина: L = 4.24); ii) розвиненого хаосу (L = 6.1).

Ключові слова: лампа зверненої хвилі, хаос, нелінійні методи