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A. N. Shakhman

Odessa National Polytechnical University, 1, Shevchenko pr., Ukraine
e-mail: quantsha@mail.ru

RELATIVISTIC THEORY OF SPECTRA OF PIONIC ATOMS WITH ACCOUNT OF STRONG PION-NUCLEAR INTERACTION EFFECTS

It is presented a consistent relativistic theory of spectra of the pionic atoms on the basis of the Klein-Gordon-Fock with a generalized radiation and strong pion-nuclear potentials. There are presented data of calculation of the energy and spectral parameters for pioninc neon, cesium, holmium, thulium, ytterbium, lutetium, thallium, lead, and others, including the calculation of energy shifts, the widths of the levels due to the strong interaction with accounting for the the radiation (vacuum polarization), nuclear (finite size of a nucleus) and other corrections.

1. Introduction

In previous papers [1-3] we have developed a new relativistic method of the Klein-Gordon-Fock equation with the simplified pion-nuclear potential to determine transition energies in spectroscopy of light, middle and heavy pionic atoms with accounting for the strong interaction effects.

Here we generalize this theory in order to describe pion-nuclear interaction more consistently using generalized radiation and strong pion-nuclear potentials. As illustration there are presented data of calculation of the energy and spectral parameters for pioninc neon, cesium, holmium, thulium, ytterbium, lutetium, thallium, lead, and others, including the calculation of energy shifts, the widths of the levels due to the strong interaction with accounting for the the radiation (vacuum polarization), nuclear (finite size of a nucleus) and other corrections.

Following [1-3], let us remind that spectroscopy of hadron atoms has been used as a tool for the study of particles and fundamental properties for a long time. Exotic atoms are also interesting objects as they enable to probe aspects of atomic and nuclear structure that are quantitatively different from what can be studied in electronic or "normal" atoms. At present time one of the most sensitive tests for the chiral symmetry breaking scenario in the modern hadron's physics is provided by studying the exotic hadron-atomic systems. Nowadays the transition energies in pionic atoms are measured with an unprecedented precision and from studying spectra of the hadronic atoms it is possible to investigate the strong interaction at low energies measuring the energy and natural width of the ground level with a precision of few meV [1-13]. The strong interaction is the reason for a shift in the energies of the low-lying levels from the purely electromagnetic values and the finite lifetime of the state corresponds to an increase in the observed level width. The most known theoretical models to treating the hadronic (pionic, kaonic, muonic, antiprotonic etc.) atomic systems are presented in refs. [1-5,7,8]. The most difficult aspects of the theoretical modelling are reduced to the correct description of pion-nuclear strong interaction [1-3] as the electromagnetic part of the problem is reasonably accounted for.

2. Total relativistic theory of spectra of pionic atoms

As the basis's of a new method has been published, here we present only the key topics of an approach [1-3]. All available theoretical models to treating the hadronic (kaonic, pionic) atoms are naturally based on the using the Klein-Gordon-Fock equation [2,5], which can be written as follows :

$$m^2c^2\Psi(x) = \{\frac{1}{c^2}[i\hbar\partial_t + eV_0(r)]^2 + \hbar^2\nabla^2\}\Psi(x) \quad (1)$$

where c is a speed of the light, *h* is the Planck constant, and Ψ_0(x) is the scalar wave function of the space-temporal coordinates. Usually one considers the central potential [V_0(r), 0] approximation with the stationary solution:

$$\Psi(x) = \exp(-iEt/\hbar)\,\varphi(x), \quad (2)$$

where $\varphi(x)$ is the solution of the stationary equation:

$$\{\frac{1}{c^2}[E + eV_0(r)]^2 + \hbar^2\nabla^2 - m^2c^2\}\varphi(x) = 0 \quad (3)$$

Here E is the total energy of the system (sum of the mass energy mc^2 and binding energy e_0). In principle, the central potential V_0 naturally includes the central Coulomb potential, the vacuum-polarization potential, the strong interaction potential.

The most direct approach to treating the strong interaction is provided by the well known optical potential model (c.g. [2]). Practically in all papers the central potential V_0 is the sum of the following potentials. The nuclear potential for the spherically symmetric density $\rho(r|R)$ is [6,13]:

$$V_{nucl}(r|R) = -((1/r)\int_0^r dr' r'^2\rho(r'|R) + \int_r^\infty dr' r'\rho(r'|R) \quad (4)$$

The most popular Fermi-model approximation the charge distribution in the nucleus $\rho(r)$ (c.f.[11]) is as follows:

$$\tilde{n}(r) = \tilde{n}_0 / \{1 + \exp[(r-c)/a)]\}, \quad (5)$$

where the parameter *a=0.*523 fm, the parameter *c* is chosen by such a way that it is true the following condition for average-squared radius:

$$<r^2>^{1/2} = (0.836\times A^{1/3} + 0.5700)\text{fm}.$$

The effective algorithm for its definition is used in refs. [12] and reduced to solution of the following system of the differential equations:

$$V'_{nucl}(r,R) = (1/r^2)\int_0^r dr' r'^2\rho(r',R) \equiv (1/r^2)y(r,R), \quad (6)$$

$$y'(r,R) = r^2\rho(r,R), \quad (7)$$

$$\tilde{n}'(r) = (\tilde{n}_0/a)\exp[(r-c)/a]\{1 + \exp[(r-c)/a)]\}^2 \quad (8)$$

with the corresponding boundary conditions. Another, probably, more consistent approach is in using the relativistic mean-field (RMF) model, which been designed as a renormalizable meson-field theory for nuclear matter and finite nuclei [13].To take into account the radiation corrections, namely, the effect of the vacuum polarization we have used the generalized Ueling-Serber potential with modification to take into account the high-order radiative corrections [5,12].

The most difficult aspect is an adequate account for the strong interaction. On order to describe the strong p^-N interaction we have used the optical potential model in which the generalized Ericson-Ericson potential is as follows:

$$V_{\pi^-N} = V_{opt}(r) = -\frac{4\pi}{2m}\left\{q(r)\nabla\frac{\alpha(r)}{1+4/3\pi\xi\alpha(r)}\nabla\right\} \quad (9)$$

$$q(r) = \left(1+\frac{m_\pi}{m_N}\right)\{b_0\rho(r) + b_1[\rho_n(r) - \rho_p(r)]\} + \left(1+\frac{m_\pi}{2m_N}\right)\{B_0\rho^2(r) + B_1\rho(r)\delta\rho(r)\}, \quad (10)$$

$$\alpha(r) = \left(1+\frac{m_\pi}{m_N}\right)^{-1}\{c_0\rho(r) + c_1[\rho_n(r) - \rho_p(r)]\} + \left(1+\frac{m_\pi}{2m_N}\right)^{-1}\{C_0\rho^2(r) + C_1\rho(r)\delta\rho(r)\}. \quad (11)$$

Here $\rho_{p,n}(r)$ – distribution of a density of the protons and neutrons, respectively, ξ – parameter ($\xi = 0$ corresponds to case of “no correlation”, $\xi = 1$, if anticorrelations between nucleons); respectively isoscalar and isovector parameters b_0, c_0, B_0, b_1, c_1, C_0 B_1, C_1 –are corresponding to the s-wave and p-wave (repulsive and attracting potential member) scattering length in the combined spin-isospin space with taking into account the absorption of pions (with different channels at p-p pair $B_{0(pp)}$ and *p-n* pair $B_{0(pn)}$), and isospin and spin dependence of an amplitude p^-N scattering

$$(b_0\rho(r) \rightarrow b_0\rho(r) + b_1\{\rho_p(r) - \rho_n(r)\},$$

the Lorentz-Lorentz effect in the p-wave interaction. For the pionic atom with remained electron shells the total wave-function is a product of the product Slater determinant of the electrons subsystem (Dirac equation) and the pionic wave function. In whole the energy of the hadronic atom is represented as the sum:

$$E \approx E_{KG} + E_{FS} + E_{VP} + E_N; \qquad (12)$$

Here E_{KG} -is the energy of a pion in a nucleus (Z, A) with the point-like charge (dominative contribution in (12)), E_{FS} is the contribution due to the nucleus finite size effect, E_{VP} is the radiation correction due to the vacuum-polarization effect, E_N is the energy shift due to the strong interaction V_N.

The strong pion-nucleus interaction contribution can be found from the solution of the Klein-Gordon equation with the corresponding pion-nucleon potential.

3. Results and conclusions

In table 1 we present theoretical and experimental data for shift and widths (keV) provided by the strong pion-nuclear interaction for a number of pionic atoms. The shortened designation of the parameter sets for the strong p^-N interaction potential: Tauscher -Tau1; Tauscher, -Tau2; Batty etal-Bat; Seki etal- Sek; de Laat-Konijin et al - Laat, this work -Sha. In our parameterization of the strong p^-N interaction potential the most reliably defined (B_0,c_0,c_1,C_0) parameters are remained unchanged, and the parameters whose values differ greatly in different sets, in particular, b_1 (b_1 = -0.094) plus still not included ones ImB_1, ImC_1 have been optimized by calculating dependencies strong shifts for p^--^{20}Ne, ^{24}Mg, ^{93}Nb, ^{133}Cs,^{175}Lu, ^{181}Ta, ^{197}Au, ^{208}Pb and further check that satisfies the smallest standard deviation of reliable experimental values.

Table 1

Theoretical and experimental data for shift and widths (keV) provided by the strong pion-nuclear interaction for a number of pionic atoms (see text)

ε_{4f},Γ_{4f}	Exp	H-like Func.	Tau1 $\xi = 0$	Tau2 $\xi = 1$
^{165}Ho: ε	0.29±0.01	0.21	0.25 0.27	0.24 0.26
^{169}Tm: ε	-	-	-	-
^{173}Yb: ε	-	-	-	-
^{175}Lu: ε	0.51±0.04	0.36	0.43	0.42
^{181}Ta: ε	0.56±0.04	0.47	0.57	0.54
^{197}Au: ε	1.25±0.07	-	1.21	1.14
^{208}Pb: ε	1.68±0.04	-	1.76	1.62
^{209}Bi: ε	1.78±0.06	-	1.94	1.80
^{165}Ho: Γ	0.21±0.02	0.08	0.13	0.12
^{169}Tm: Γ	-	-	-	-
^{173}Yb: Γ	-	-	-	-
^{175}Lu: Γ	0.27±0.07	0.14	0.23	0.22
^{181}Ta: Γ	0.31±0.05	0.16	0.31	0.30
^{197}Au: Γ	0.77±0.04	-	0.73	0.68
^{208}Pb: Γ	0.98±0.05	-	1.18	1.04
^{209}Bi: Γ	1.24±0.09	-	1.35	1.18

Table 1 (continuation)

Theoretical and experimental data for shift and widths (keV) provided by the strong pion-nuclear interaction for a number of pionic atoms (see text)

ε_{4f},Γ_{4f}	Bat $\xi = 1$	Sek $\xi = 1$	Laat $\xi = 1$	Sha $\xi = 1$
^{165}Ho: ε	0.24	0.21	0.26	0.29
^{169}Tm: ε	-	-	-	0.38
^{173}Yb: ε	-	-	-	0.44
^{175}Lu: ε	0.41	0.36	0.46	0.50
^{181}Ta: ε	0.53	0.47	0.60	0.55
^{197}Au: ε	1.12	0.98	1.25	1.24
^{208}Pb: ε	1.58	1.39	1.68	1.65
^{209}Bi: ε	1.78	1.57	1.83	1.77
^{165}Ho: Γ	0.13	0.11	0.13	0.20
^{169}Tm: Γ	-	-	-	0.23
^{173}Yb: Γ	-	-	-	0.26
^{175}Lu: Γ	0.24	0.20	0.24	0.28
^{181}Ta: Γ	0.31	0.27	0.31	0.30
^{197}Au: Γ	0.69	0.58	0.67	0.75
^{208}Pb: Γ	1.03	0.86	0.98	0.97
^{209}Bi: Γ	1.17	0.99	1.10	1.22

Analysis shows that the data from Table 1 of all alternative theories (except the column «H-like Func», containing data calculation within variation theory with relativistic H-like functions; here there is very unsatisfactorily agreement with experimental data) are obtained on the basis of the Klein-Gordon-Fock equation with nuclear potential V_{pN} by Erickson-Erickson with different parametrization [2]. More precise calculation data are based on the theory of Klein-Gordon-Fock equation with nuclear potential V_{pN} with parameters Tau1, Tau2, Laat, Sha (see table 1). Our theory shows that more optimal parameterization V_{pN} can significantly improve a quality of determining characteristics of the pionic atoms, which are provided by the strong pion-nuclear interaction. This conclusion is confirmed by the data of computing quadrupole shift of the 4f level in spectra of some p^-A, in particular, ^{165}Ho, ^{169}Tm, ^{173}Yb, ^{175}Lu, ^{209}Bi [2-7, 13], provided by the strong pion-nuclear interaction.

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Abstract.

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Key words: strong interaction, pionic atom, relativistic theory

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А. Н. Шахман

РЕЛЯТИВИСТСКАЯ ТЕОРИЯ СПЕКТРОВ ПИОННЫХ АТОМОВ С УЧЕТОМ ЭФФЕКТОВ СИЛЬНОГО ПИОН-ЯДЕРНОГО ВЗАИМОДЕЙСТВИЯ

Резюме.

Представлена последовательная релятивистская теория спектров пионных атомов на основе уравнения Клейна-Гордона-Фока с обобщенными радиационным и сильным пион-ядерным потенциалом. Выполнен расчет энергетических и спектральных параметров для пионных атомов неона, цезия, гольмия, тулия, иттербия, лютеция, таллия, свинца и других, включая расчет энергетических сдвигов, ширин уровней вследствие сильного взаимодействия с учетом радиационных (поляризация вакуума), ядерных (конечный размер ядра) и других поправок.

Ключевые слова: сильное взаимодействие, пионный атом, релятивистская теория

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А. М. Шахман

РЕЛЯТИВІСТСЬКА ТЕОРІЯ СПЕКТРІВ ПІОННИХ АТОМІВ З УРАХУВАННЯМ ЕФЕКТІВ СИЛЬНОЇ ПІОН-ЯДЕРНОЇ ВЗАЄМОДІЇ

Резюме.

Представлена послідовна релятивістська теорія спектрів півоній атомів на основі рівняння Клейна-Гордона-Фока з узагальненими радіаційним і сильним півонія-ядерним потенціалом. Виконано розрахунок енергетичних і спектральних параметрів для піоних атомів неону, цезію, гольмію, тулія, ітербію, лютецію, талію, свинцю та інших, включаючи розрахунок енергетичних зрушень, ширин рівнів внаслідок сильної взаємодії з урахуванням радіаційних (поляризація вакууму), ядерних (кінцевий розмір ядра) та інших поправок.

Ключові слова: сильна взаємодія, піонний атом, релятивістська теорія