UDK 539.184

Yu. G. Chernyakova, L. A. Vitavetskaya, P. G. Bashkaryov, I. N. Serga, A. G. Berestenko

Odessa National Politechnical University, 1, Shevchenko av., Odessa, Ukraine
Odessa State Environmental University, 15, Lvovskaya str., Odessa, Ukraine
e-mail: nucvita@mail.ru

THE RADIATIVE VACUUM POLARIZATION CONTRIBUTION TO THE ENERGY SHIFT OF SOME LEVELS OF THE PIONIC HYDROGEN

Calculating the radiative contribution due to the vacuum polarization effect to energy value for some levels in pionic hydrogen atom including in particular the main Uehling-Serber and the high-order Källen-Sabry and Wichmann - Kroll corrections has been carried out using the modified Uehling-Serber potential. The values for some characteristic energy corrections to 1s, 2p, 3p, 4p states of the pion hydrogen (in particular, radiation contributions and contributions due to the finite size of the proton and pion are presented and compared with alternative data by Schlesser-Indelicato et al.

1. Introduction

It is well known that the development of a new theoretical approach to the description of spectral parameters pionic atoms in the excited states with precise accounting relativistic, radiation, nuclear, electron screening effects (look [1-18]) on the basis of Klein-Gordon-Fock (Dirac) equation and a development of a consistent relativistic theory of hyperfine structure of spectra represents one of actual fundamental problem of moden optics and spectroscopy of hadronic atomic systems. Especial problem is connected with précised calculating the radiative corrections to the transition energies of the low-Z exotic (pionic, kaonic, muonic) atoms, namely, hydrogen and deuterium. Naturally, it is provided by necessity of further developing the modern as atomic and as nuclear spectra theories. From the other side, one could mention that the detailed data about spectra of the exotic atomic systems (kaonic, pionic, muonic atoms) can be very useful under construction of the new X-ray standards. It is worth to remind about known achievements and a great importance of the theoretical muonic, hadronic chemistry and hadronic spectroscopy as well as the significant progress in the modern experimental technologies applying to hadronic atoms [1-15].

The standard Dirac approach is traditionally used as starting basis in calculations of the heavy ions [2]. The problem of accounting the radiative corrections, in particular, self-energy part of the Lamb shift and vacuum polarization contribution is mostly treated with using the expansions on the natural physical parameters *1/Z, aZ* (*a* is fine structure constant) [5,10]. It permits evaluations of the relative contributions of different expansion energy terms: non-relativistic, relativistic ones, as functions of Z. For high Z (Z is a nuclear charge) it should be necessary to account for the high-order QED corrections and the nuclear finite size correction etc [1-3,10-12,16]. Further improvement of this method in a case of the heavy ions is linked with using gauge invariant procedures of generating relativistic orbital basises and more correct treating nuclear and QED effects [1-3]. In a case of the low-Z exotic atomic systems such as an exotic hydrogen (deuterium) a great interest attracts estimation of the radiative, in particular, vacuum polarization, correction. In refs. [17-19] it has been proposed a precise scheme to calculating spectra of heavy systems with account of nuclear and radiative effects, based on the relativistic many-body perturbation theory (see also [3]) and advanced effective procedures for accounting the radiative corrections.

In this paper we present the results of calculating the QED contribution and first of all due to the vacuum polarization effect to energy shift for some levels energies of pionic hydrogen. The obtained results are compared with the calculation data by [2]. As theoretical model we have used relativistic models, presented in Ref. [20,21] (look [23-25]).

The master equation for describing thepionic atom dynamics is the Klein-Gordon-Fock equation, which is in atomic units as follows:

$$\left\{\alpha^2\left[E-V_c(r)\right]^2+\vec{\nabla}^2-\mu^2c^2\right\}\psi(r)=0, \quad (1)$$

де m-наведеная маса p$^-$, E- енергія піон, V_c–загальний потенціал, який, зокрема, включає кулонівський потенціал взаємодії p$^-$ з ядром (з урахуванням скінченого розміру), узагальнений радіаційний потенціал, потенціал електронних оболонок.

where m- is a reduced mass, E- a pion energy V_c– total electromagnetic interaction which includes especially Coulomb interaction potential of p$^-$ with a nucleus (with accounting for the finite size effect), radiation potential (including thr vacuum polarization potential) and possibly the potential of electron shells (for multi-electron pionic atom).

The total electromagnetic interaction potential:

$$V_c(r)=V_n(r)+U_R(r). \quad (1)$$

includes the electrical V_n and radiation (including polarization) U_R(r) potential of a nucleus with accounting the finite size correction. The expectation value of the radiative vacuum polarization operator gives the corresponding correction. In ref. [21] it is presented a consistent approach to determining radiation QED corrections (main among them, of course, is the correction the polarization of the vacuum; effect which is typical for a distance of Compton wavelength $\lambda_C^e=\hbar/m_ec=$ 386.16Fm; while the Bohr radius of a pion orbit $r_B^\pi=194n^2/Z$ Fm) to the energy states of pionic atom, which is based on using the Uehling-Serber potential with effective accounting for the Breit-Rosenthal-Crawford-Schawlow):

$$U_{PV}^{FS}(r)=-\frac{2\alpha^2}{3\pi}\int d^3r'\int_m^\infty dt\exp\left(-2t|r-r'|/\alpha Z\right)\times$$
$$\times\left(1+\frac{1}{2t^2}\right)\frac{\sqrt{t^2-1}}{t^2}\frac{\rho(r')}{|r-r'|}, \quad (2)$$

and additional terms which take into account a contribution of the corrections of $[\alpha(Z\alpha)]^n$ order, in particular, the Källen-Sabry and Wichmann - Kroll corrections ($\alpha^2(\alpha Z)+\alpha(Z\alpha)^n$...).

A vacuum polarization potential without the Breit-Rosenthal-Crawford-Schawlow effect is usually represented as follows (in fact in the first PT order):

$$U(r)=-\frac{2\alpha}{3\pi r}\int_1^\infty dt\exp(-2rt/\alpha Z)\left(1+1/2t^2\right)\frac{\sqrt{t^2-1}}{t^2}\equiv \quad (3)$$
$$=-\frac{2\alpha}{3\pi r}C(g),$$

$$g=\frac{r}{\alpha Z}. \quad (4)$$

The corresponding expectation value of this operator gives the corresponding vacuum polarization correction. In the scheme [20,21] this potential is approximated by quite precise analytical function (see details in refs. [12-16]). The most advanced version of the such potential (C® $\tilde{\tilde{C}}$) is presented as follows:

$$\tilde{\tilde{C}}(g)=\tilde{C}_1(g)\tilde{\tilde{C}}_2(g)/\left(\tilde{C}_1(g)+\tilde{\tilde{C}}_2(g)\right), \quad (5)$$
$$\tilde{\tilde{C}}_2(g)=\tilde{C}_2(g)f(g),$$
$$\tilde{C}_2(g)=-1.8801\exp(-g)/g^{3/2}$$
$$\tilde{C}_1(g)=\text{h}(g/2)+1.410545-1.037837g,$$
$$f(g)=((1.1024/g-1.3361)/g+0.8027)$$

The using this formula permits one to decrease the calculation errors for this term down to ~0.1%. Error of usual calculation scheme is~10%.

Earlier we carried out the calculation of the vacuum polarization contribution to the energy shift of a number of the levels and transitions in kaonic and pionic nitrogen. One should keep in mind that the energy.

levels of exotic (pionic, kaomic etc) atoms are very sensitive to effects of QED, nuclear structure and recoil since the pion (kaon) is heavier than the electron. As usually the fundamental constants from the CODATA 1998 are used in the numerical calculations. We have evaluated the values for the QED contributions and other specific corrections to the energy 1s, 2p, 3p, 4p states of the pion hydrogen. In table 1 we present the values (in meV) for some characteristic energy corrections to 1s, 2p, 3p, 4p states of the pion hydrogen (in particular, radiation contribtions and contributions due to the finite size of the proton and pion) estimated by the theory and Schlesser-Indelicato et al. [5] and our theory. The following abbreviations are used: correction on polarization of the vacuum of Uehling Serber (PV-US), the correction to the Breit interaction (BI), correction to a size radius (SR) of a proton and a pion, the higher-order corrections for a vacuum polarization Kallen-Sabry (PV-KS) and Wichman-Kroll (PV-WC). Analyzing the results, it should be noted that, in general between the theoretical results (actually the contributions of electromagnetic energy in the state or transition) obtained under various theories, in particular, our theory and the theory Schlesser-Indelicato et al. [5] there is a fairly good agreement that is easily explained in principle (as in the case of the conventional hydrogen atom) in a sense negligible role of radiation and nuclear finite-size corrections, Obviously it is of a great interest application of the presented scheme in computing QED corrections to levels energies in heavy pionic atoms.

Table 1

The values (in meV) some specific corrections to the energy 1s, 2p, 3p, 4p states of the pion hydrogen (QED contributions and contributions by the proton and pion SR) according to the theory Schlesser-Indelicato et al. [5] and our theory (see. Text)

QED	*F*	1s [5]	1s (our)
PV-US		-3240.802	-3240.799
BI	1/2 3/2	-178.461	-178.458
SR	p π^-	61.711 39.33	61.711 39.33
PV-KS		-24.365	-24.363
PV-WK		-4.110	-4.113
QED	*F*	2p [5]	2p (our)
PV-US		-35.795	-35.793
BI	1/2 3/2	-11.655 -4.048	-11.652 -4.046
SR	p π^-	0 0	0 0
PV-KS		-0.346	-0.343
PV-WK		-0.008	-0.010
QED	*F*	3p [5]	3p (our.)
PV-US		-11.407	-11.405
BI	1/2 3/2	-4.221 -1.967	-4.219 -1.965
SR	p π^-	0 0	0 0
PV-KS		-0.108	-0.105
PV-WK		-0.002	-0.003
QED	*F*	4p [5]	4p (our)
PV-US		-4.921	-4.918
BI	1/2 3/2	-1.943 -0.992	-1.940 -0.989
SR	p π^-	0 0	0 0
PV-KS		-0.046	-0.044
PV-WK		-0.001	-0.002

References

1. Lyubovitskij V., Rusetsky A., πp atom in ChPT: Strong energy-level shift// Phys. Lett.B. – 2000. – Vol.494. – P.9-13..
2. Mohr P.J., Energy Levels of H-like atoms predicted by Quantum Electrodyna-mics,10<Z<40//Atom.Dat.Nucl.Dat. Tabl. – 1983 – Vol.24,N2. – P.453-470.
3. Glushkov A.V., Relativistic quantum theory. Quantum mechanics of atomic systems, Odessa: Astroprint, 2008. – 700P.
4. Hayano R.S., Hori M., Horvath D., Widman E., Antiprotonic helium and CPT invariance//Rep. Prog. Phys. – 2007. – Vol.70. – P.1995-2065.
5. Schlesser S., Le Bigot E.-O., Indelicato P., Pachucki K., Quantum-electrodynamics corrections in pionic hydrogen// Phys.Rev.C. – 2011. – Vol.84. – P.015211 (8p.).
6. Gotta D., Amaro F., Anagnostopoulos D., Pionic Hydrogen//Precision Physics of

Simple Atoms and Molecules, Ser. Lecture Notes in Physics (Springer, Berlin / Heidelberg).-2008.-Vol.745.-P.165-186.
7. Mitroy J., Quantum defect theory for the study of hadronic atoms/ Mitroy J., Ivallov I.A.// J. Phys. G: Nucl. Part. Phys. – 2001. – Vol.**27. – P.**1421–1433
8. Deslattes R., Kessler E., Indelicato P., de Billy L., Lindroth E. , Anton J., Exotic atoms//Rev. Mod. Phys. – 2003. –Vol.75. – P.35-70.
9. Borie E., Lamb shift in muonic hydrogen// Phys.Rev. A. – 2005. – Vol.71. – P.032508-1-8.
10. Mohr P.J. Quantum Electrodynamics Calculations in few-Electron Systems// Phys. Scripta. – 1993. – Vol.46, N1. – P.44-52.
11. Klaft I.,Borneis S., Engel T., Fricke B., Grieser R., Huber G., Kuhl T., Marx D., Neumann R., Schroder S., Seelig P., Volker L. Precision laser spectroscopy of ground state hyperfine splitting of H-like $^{209}Bi^{82+}$// Phys.Rev.Lett. – 1994. – Vol.73. – P.2425-2427.
12. Glushkov A.V., Relativistic quantum theory. Quantum mechanics of atomic systems, Odessa: Astroprint, 2008. – 700P.
13. Khetselius O.Yu., Relativistic perturbation theory calculation of the hyperfine structure parameters for some heavy-element isotopes// Int. Journ. of Quantum Chemistry. – 2009. – Vol.109. – N14. – P. 3330-3335.
14. Backenstoss G., Pionic atoms//Ann.Rev. Nucl.Sci. – 1970. – Vol.20. – P.467-510.
15. Menshikov L I and Evseev M K, Some questions of physics of exotic atoms// Phys. Uspekhi.2001 – Vol. 171. – P.150-184.
16. Ivanova E.P., Ivanov L.N., Aglitsky E.V., Modern trends in spectroscopy of multicharged ions// Phys.Rep. – 1991. – Vol. 166. – P.315-390.
17. Glushkov A.V., Ambrosov S., Loboda A., Chernyakova Y.G., Khetselius O., Svinarenko A., QED theory of the superheavy elements ions: energy levels, radiative corrections, and hyperfine structure for different nuclear models// Nuclear Phys.A – 2004. – Vol. 734. – P.21-28.
18. Glushkov A.V., Khetselius O.Yu., Gurnitskaya E.P., Loboda A.V., Florko T.A., Sukharev D.E., Lovett L., Gauge-invariant QED perturbation theory approach to calculating nuclear electric quadrupole moments, hfs structure constants for heavy atoms and ions// Frontiers in Quantum Systems in Chem. and Phys. (Springer). – 2008. – Vol.18. – P.505-522.
19. Kuznetsova A.A., Vitavetskaya L.A., Chernyakova Yu.G., Korchevsky D.A., Calculating radiative vacuum polarization contribution to the energy shift of 3p-1s transition in pionic deuterium// Photo-electronics. – 2013. – Vol.22. – P.108-111
20. Serga I.N., Dubrovskaya Yu.V., Kvasikova A.S., Shakhman A.N., Sukharev D.E., Spectroscopy of hadronic atoms: Energy shifts// Journal of Physics: C Series (IOP, London, UK). – 2012. – Vol.397. – P.012013 (5p.).
21. Serga I.N., Relativistic theory of spectra of pionic atoms with account of the radiative and nuclear corrections// Photoelectronics. – 2013. – Vol.22. – P.71-75.
22. Kvasikova A.S., Ignatenko A.V., Florko T.A., Sukharev D.E., Chernyakova Yu.G., Photoeffect and spectroscopy of the hydrogen atom in the crossed dc electric and magnetic field// Photoelectronics. – 2011. – Vol.20. – P.71-75.
23. Kuznetsova A.A., Kvasikova A.S., Shakhman A.N., Vitavetskaya L.A., Calculating the radiative vacuum polarization contribution to the energy shift of 2p-2s transition in m-hydrogen// Photoelectronics. – 2012. – N21. – P.66-67.
24. Khetselius O.Yu., Turin A.V., Sukharev D.E., Florko T.A., Estimating of X-ray spectra for kaonic atoms as tool for sensing the nuclear structure// Sensor Electr. and Microsyst. Techn. - 2009. - N1. – P.30-35..
25. Serga I.N., Electromagnetic and strong interactions effects in X-ray spectroscopy of pionic atoms // Photoelectronics. – 2011. – Vol.20. – P.109-112.

This article has been received within 2015

UDK 539.184

Yu. G. Chernyakova, L. A. Vitavetskaya, P. G. Bashkaryov, I. N. Serga, A. G. Berestenko

THE RADIATIVE VACUUM POLARIZATION CONTRIBUTION TO THE ENERGY SHIFT OF SOME LEVELS OF THE PIONIC HYDROGEN

Abstract.
Calculating the radiative contribution due to the vacuum polarization effect to energy value for some levels in pionic hydrogen atom including in particular the main Uehling-Serber and the high-order Källen-Sabry and Wichmann - Kroll corrections has been carried out using the modified Uehling-Serber potential. The values for some characteristic energy corrections to 1s, 2p, 3p, 4p states of the pion hydrogen (in particular, radiation contributions and contributions due to the finite size of the proton and pion are presented and compared with alternative data by Schlesser-Indelicato et al.
Key words: pionic hydrogen, radiative corrections

УДК 539.184

Ю. Г. Чернякова, Л. А. Витавецкая, П. Г. Башкарьов, И. Н. Серга, А. Г. Берестенко

РАДИАЦИОННЫЕ ВКЛАДЫ ЗА СЧЕТ ЭФФЕКТА ПОЛЯРИЗАЦИИ ВАКУУМА В СДВИГ ЭНЕРГИИ РЯДА УРОВНЕЙ ПИОННОГО ВОДОРОДА

Резюме.
Проведен расчет радиационного вклада за счет эффекта поляризации вакуума в величину энергии ряда уровней в пионном водороде в том числе, в частности, основной вклад Юлинга-Сербера и вклады высоких порядков Каллена-Сабри и Вичманна-Кролла с использованием модифицированного потенциала Юлинга-Сербера. Приведены значения некоторых характерных энергетических поправок к энергии 1s, 2p, 3p, 4p состояний пионного водорода (в частности, радиационные поправки, поправки за счет конечного размера протона и пиона представлены и др.) и проведено их сравнение по сравнению с альтернативными данным Schlesser-Indelicato и др.
Ключевые слова: пионный водород, радиационные поправки

УДК 539.184

Ю. Г. Чернякова, Л. А. Вітавецька, П. Г. Башкарьов, І. М. Сєрга, А. Г. Берестенко

РОЗРАХУНОК РАДІАЦІЙНОГО ВНЕСКУ ЗА РАХУНОК ЕФЕКТУ ПОЛЯРИЗАЦІЇ ВАКУУМУ У ЗСУВ ЕНЕРГІЇ ДЕКОТРИХ РІВНІ ПІОННОГО ВОДНЮ

Резюме.
Виконано розрахунок радіаційного внеску за рахунок ефекту поляризації вакууму у величину енергії декотрих рівнів у піонному водні у тому числі, зокрема, основний внесок Юлінг-Сербер і вклади високих порядків Каллена-Сабрі і Вічманна –Кролла, з використанням модифікованого потенціалу Юлінга-Сербера. Наведені значення деяких характерних енергетичних поправок до енергії 1s, 2p, 3p, 4p станів піонного водню (зокрема, радіаційні поправки, поправки за рахунок кінцевого розміру протона і півонії представлені і ін.) І проведено їх порівняння в порівнянні з альтернативними даними Schlesser-Indelicato та ін.
Ключові слова: піонний водень, радіаційні поправки