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G. P. Prepelitsa, S. V. Brusentseva, A. V. Duborez, O. Yu. Khetselius, P. G. Bashkaryov

Odessa State Environmental University, 15, Lvovskaya str., Odessa, Ukraine
Odessa National Polytechnical University, 1, Shevchenko av., Odessa, Ukraine
e-mail: quantpre@mail.ru

NEW NONLINEAR ANALYSIS, CHAOS THEORY AND INFORMATION TECHNOLOGY APPROACH TO STUDYING DYNAMICS OF CHAIN OF QUANTUM AUTOGENERATORS

A chaos-geometric approach [7-11] that consistently includes a number of new or improved known methods of analysis (the correlation integral, the fractal analysis, algorithms of the average mutual information, and false nearest neighbors, the Lyapunov exponents analysis, the Kolmogorov entropy, the method of surrogate data, a set of the spectral methods, a neural network algorithms, etc. .) is used to solve the problem of quantitative modeling and analysis of chaotic dynamics of a chain of two quantum autogenerators. There are theoretically studied a chaos scenario generation and obtained quantitative data on the dynamic and topological invariants of the system in the chaotic regime.

1 Introduction

In many papers (see, for example, [1-18]) it has been noted that a chaos is alternative of randomness and occurs as in very simple deterministic systems as quite complex ones. Although chaos theory places fundamental limitations for long-rage prediction (see e.g. [1-9]), it can be used for short-range prediction since ex facte random data can contain simple deterministic relationships with only a few degrees of freedom. Chaos theory establishes that apparently complex irregular behaviour could be the outcome of a simple deterministic system with a few dominant nonlinear interdependent variables. The past decade has witnessed a large number of studies employing the ideas gained from the science of chaos to characterize, model, and predict the dynamics of various systems phenomena (see e.g. [1-13]). The outcomes of such studies are very encouraging, as they not only revealed that the dynamics of the apparently irregular phenomena could be understood from a chaotic deterministic point of view but also reported very good predictions using such an approach for different systems.

In a modern quantum electronics and laser physics etc there are many systems and devices (such as multi-element semiconductors and gas lasers etc), dynamics of which can exhibit chaotic behaviour. These systems can be considered in the first approximation as a grid of autogenerators (quantum generators), coupled by different way [2,14,15].

In this paper a chaos-geometric approach [7-11] that consistently includes a number of new or improved known methods of analysis (the correlation integral, the fractal analysis, algorithms of the average mutual information, and false nearest neighbors, the Lyapunov exponents analysis, the Kolmogorov entropy, the method of surrogate data, a set of the spectral methods, a neural network algorithms, etc.; see details in Refs. [1-34]) is used to solve the problem of quantitative modeling and analysis of chaotic dynamics of a chain of two quantum autogenerators. There are theoretically studied a chaos scenario generation and obtained quantitative data on the dynamic and topological invariants of the system in the chaotic regime

2. Methods of studying dynamics of the laser systems

As used non-linear analysis, chaos theory and information technology methods to studying non-linear dynamics of the laser systems have been earlier in details presented [1-20] here we are limited only by the key ideas. As usually, we formally consider scalar measurements $s(n) = s(t_0 + n\Delta t) = s(n)$, where t_0 is the start time, D*t* is the time step, and is *n* the number of the measurements. Packard et al. [18] introduced the method of using time-delay coordinates to reconstruct the phase space of an observed dynamical system. The direct use of the lagged variables $s(n + t)$, where t is some integer to be determined, results in a coordinate system in which the structure of orbits in phase space can be captured. First approach to compute t is based on the linear autocorrelation function. The second method is an approach with a nonlinear concept of independence, e.g. the average mutual information. Briefly, the concept of mutual information can be described as follows [5,7,13]. One could remind that the autocorrelation function and average mutual information can be considered as analogues of the linear redundancy and general redundancy, respectively, which was applied in the test for nonlinearity. If a time series under consideration have an *n*-dimensional Gaussian distribution, these statistics are theoretically equivalent as it is shown in Ref. [22].

The goal of the embedding dimension determination is to reconstruct a Euclidean space R^d large enough so that the set of points d_A can be unfolded without ambiguity. There are several standard approaches to reconstruct the attractor dimension (see, e.g., [1,7,23]), but let us consider in this study two methods only. The correlation integral analysis is one of the widely used techniques to investigate the signatures of chaos in a time series. The analysis uses the correlation integral, $C(r)$, to distinguish between chaotic and stochastic systems. To compute the correlation integral, the algorithm of Grassberger and Procaccia [23] is the most commonly used approach. According to this algorithm, the correlation integral is

$$C(r) = \lim_{N\to\infty} \frac{2}{N(n-1)} \sum_{\substack{i,j \\ (1\le i<j\le N)}} H\left(r - |\mathbf{y}_i - \mathbf{y}_j|\right) \tag{1}$$

where H is the Heaviside step function with $H(u) = 1$ for $u > 0$ and $H(u) = 0$ for u £ 0, r is the radius of sphere centered on $\mathbf{y}_i$ or $\mathbf{y}_j$, and N is the number of data measurements. If the time series is characterized by an attractor, then the integral $C(r)$ is related to the radius r given by

$$d = \lim_{\substack{r\to 0 \\ N\to\infty}} \frac{\log C(r)}{\log r}, \tag{2}$$

where d is correlation exponent that can be determined as the slop of line in the coordinates log $C(r)$ versus log r by a least-squares fit of a straight line over a certain range of r, called the scaling region.

There are certain important limitations in the use of the correlation integral analysis in the search for chaos. To verify the results obtained by the correlation integral analysis, we use surrogate data method. The method of surrogate data [1,7,19] is an approach that makes use of the substitute data generated in accordance to the probabilistic structure underlying the original data. Advanced version is presented in [7-9].

The next step is computing the Lyapunov's exponents (LE). The LE are the dynamical invariants of the nonlinear system. A negative exponent indicates a local average rate of contraction while a positive value indicates a local average rate of expansion. In the chaos theory, the spectrum of LE is considered a measure of the effect of perturbing the initial conditions of a dynamical system. Note that both positive and negative LE can coexist in a dissipative system, which is then chaotic. Since the LE are defined as asymptotic average rates, they are independent of the initial conditions, and therefore they do comprise an invariant measure of attractor. In fact, if one manages to derive the whole spectrum of the LE, other invariants of the system, i.e. Kolmogorov entropy and attractor's dimension can be found. The Kolmogorov entropy, *K*, measures the average rate at which information about the state is lost with time. An estimate of this measure is the sum of the positive LE. The inverse of the Kolmogorov entropy is equal to an average predictability.

Estimate of dimension of the attractor is provided by the Kaplan and Yorke conjecture. There are a few approaches to computing the LE. One of them computes the whole spectrum and is based on the Jacobi matrix of system [27]. In the case where only observations are given and the system function is unknown, the matrix has to be estimated from the data. In this case, all the suggested methods approximate the matrix by fitting a local map to a sufficient number of nearby points.

In our work we use the method with the linear fitted map proposed by Sano and Sawada [27] added by the neural networks algorithm [7-10].

3. Chaotic elements in dynamics of the grid of two autogenerators and conclusions

Here we present results of non-linear analysis of the chaotic oscillations in a grid of two autogenerators. Dynamics of this systems has intensively studied from the viewpoint of the corresponding differential equations solutions (e.g. [2,14,15]). In Refs.[2,14,15] the time series for the characteristic vibration amplitude are presented in a case of two semiconductors lasers connected through general resonator. We have studied the time series in a regime of the hyper chaos (input data contain 4096 points). Firstly we have computed the variations of the autocorrelation coefficient for the amplitude level. Autocorrelation function exhibits some kind of exponential decay up to a lag time of about 100 time units. Such an exponential decay can be an indication of the presence of chaotic dynamics in the process of the level variations. On the other hand, the autocorrelation coefficient failed to achieve zero, i.e. the autocorrelation function analysis not provides us with any value of t. Such an analysis can be certainly extended to values exceeding 1000, but it is known that an attractor cannot be adequately reconstructed for very large values of t. The correlation dimension is computed on the basis of the correlation integral scheme.

To verify the results obtained by the correlation integral analysis, we use surrogate data method. The method of surrogate data is an approach that makes use of the substitute data generated in accordance to the probabilistic structure underlying the original data. This means that the surrogate data possess some of the properties, such as the mean, the standard deviation, the cumulative distribution function, the power spectrum, etc., but are otherwise postulated as random, generated according to a specific null hypothesis. We have evaluated the percentage of false nearest neighbours that was determined for the amplitude level series, for phase-spaces reconstructed with embedding dimensions from 1 to 20. In Table 1 we list the computed values of the correlation dimension d_2, embedding dimension d_N, which are computed on the basis of the false nearest neighbouring points algorithm with noting (%) of false points for different values of the lag time t. Accordingly in Table 2 we list the computed values of the Kaplan-York attractor dimension (d_L), LE (l_i, i=1-3) and the Kolmogorov entropy (K_{entr}).

Table 1

The correlation dimension d_2 , embedding dimension d_N , which are computed on the basis of the false nearest neighbouring points algorithm with noting (%) of false points for different values of the lag time t

τ	d_2	(d_N)
64	7.9	10 (12)
10	7.1	8 (1.2)
12	7.1	8 (1.2)

Table 2

The Kaplan-York attractor dimension (d_L), LE (l_i, i=1-3) and the Kolmogorov entropy (K_{entr}) for the system of two semiconductors lasers connected through general resonator (the hyperchaos regime)

λ_1	λ_2	λ_3	d_L	K_{entr}
0.515	0.198	-0.146	6.9	0.745

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Abstract

Chaos-geometric approach that consistently includes a number of new or improved known methods of analysis (the correlation integral, the fractal analysis, algorithms of the average mutual information, and false nearest neighbors, the Lyapunov exponents analysis, the Kolmogorov entropy, the method of surrogate data, a set of the spectral methods, a neural network algorithms, etc. .) is used to solve the problem of quantitative modeling and analysis of chaotic dynamics of a chain of two quantum autogenerators. There are theoretically studied a chaos scenario generation and obtained quantitative data on the dynamic and topological invariants of the system in the chaotic regime.

Keywords: chain of quantum autogenerators, dynamics, chaos, nonlinear analysis

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Г. П. Препелиця, С. В. Брусенцева, А. В. Дуборез, О. Ю. Хецелиус, П. Г. Башкарьов

НОВЫЙ ПОДХОД НА ОСНОВЕ НЕЛИНЕЙНОГО АНАЛИЗА, ТЕОРИИ ХАОСА И ИНФОРМАЦИОННЫХ ТЕХНОЛОГИЙ К ИЗУЧЕНИЮ ДИНАМИКИ КВАНТОВЫХ ГЕНЕРАТОРОВ И ЛАЗЕРНЫХ СИСТЕМ

Резюме

Хаос-геометрический подход, который единообразно включает ряд новых или усовершенствованных известных методов анализа (корреляционный интеграл, фрактальный анализ, алгоритмы средней взаимной информации, ложных ближайших соседей, показатели Ляпунова, энтропия Колмогорова, метод суррогатных данных, спектральные методы, нейросетевые алгоритмы и т.д.) использован для решения задачи количественного моделирования и анализа хаотической динамики цепочки двух квантовых автогенераторов. Теоретически изучен сценарий генерации хаоса и получены количественные данные по динамическим и топологическим инвариантам системы в хаотическом режиме.

Ключевые слова: цепочка квантовых автогенераторов, динамика, хаос, нелинейный анализ

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Г. П. Препелиця, С. В. Брусенцева, А. В. Дуборез, О. Ю. Хецелиус, П. Г. Башкарьов

НОВИЙ ПІДХІД НА ОСНОВІ НЕЛІНІЙНОГО АНАЛІЗУ, ТЕОРІЇ ХАОСУ ТА ІНФОРМАЦІЙНИХ ТЕХНОЛОГІЙ ДО ВИВЧЕННЯ ДИНАМІКИ ЛАНЦЮЖКА КВАНТОВИХ АВТОГЕНЕРАТОРІВ

Резюме

Хаос-геометричний підхід, що одноманітно включає низку нових або удосконалених відомих методів аналізу (кореляційний інтеграл, фрактальний аналіз, алгоритми середньої взаємної інформації, хибних найближчих сусідів, показники Ляпунова, ентропія Колмогорова, сурогатних даних, нелінійний прогноз, спектральні методи, нейромережеві алгоритми тощо) використаний для вирішення задач кількісно моделювання та аналізу хаотичної динаміки ланцюжка квантових автогенераторів. Теоретично вивчений сценарій генерації хаосу, отримані кількісні дані по динамічним та топологічним інваріантам системи у хаотичному режимі.

Ключові слова: ланцюжок квантових автогенераторів, динаміка, хаос, нелінійний аналіз