SOLVING THE TASK OF MEASURING INFORMATION QUANTITY THROUGH MODEL IDENTIFICATION WITH MININIMISATION OF THE ESTIMATION ERROR

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INTRODUCTION

Accumulation and processing of information is a problem related to the development of human culture as a whole. Despite the length of theperiod of usage of information processing technology, the processes of accumulation of information in general and of education in particular still remain complex and inefficient. The variety and wealth of information create problems in the process of transmission and storage of information.

The desire to detail and organize content leads to significant difficulties in the establishment of storage and flow control algorithms . A significant influence is rendered by problems of language and formal differences between messages and stored data volumes.

Finally, there is the problem of measuring the amount of of information received by the storage system and contained in stored volumes. And the estimate of the maximum possible message volume of acceptable in the communication channel assessment does not hold any meaning in the case of accumulation of information.

FORMULATION OF THE PROBLEM

With the accumulation of information, whether it be training, formation of funds, simple storage of information, there are changes over time in the volume of useful or real information [1, 2]. A reflection of this is the representation of the model of the information accumulation process in the form of a dynamic system [3]. However, the complexity of evaluation of useful information, for example, during information accumulation in the learning process [4], makes it advisable to use an asymptotic observer procedure that guarantees an estimation of the process according to the model analogously to the processes described in [5,6]. Thus, the solution to the problem of information amount assessment requires solving the problem of identification of the model and the creation of an asymptotic observer procedure that allows to minimize the estimation error.

The aim of the article is the description of the recovery process for components of the information flow using the algorithm of search-less identification and asymptotic observation.

Statement of the base material. The approaches used in controlling the size of the accumulated information are based on two main methods:

1. Determination of the amount of information stored based on the assessment of the amount of input information.
2. Estimation of the amount of accumulated information based on its usefulness.

In the first case, the stored information is not estimated while the amount of the loaded media is, and the storage system is described as a communication channel element. In the second case, the usefulness of accumulated information is measured. For this purpose the information storage system is used (as a feedback loop or as associative memory). Therefore, an algorithm for determining the amount of accumulated information has to include the formation of task system, recreation of the real situation, and a method of estimating the accuracy of the recreation has to be known. This is monitoring and testing with an artificial storage system, and conduction of examinations or tests with people. Thus, one can imagine the procedure for validation of the amount and quality of accumulated knowledge as a deviation comparison with a certain model that serves as the base for the evaluation of the volume of accumulated knowledge. In this case, the main problem is to form an expert environment providing an adequate model of the problem. Based on the developed methods of formation of the expert environment [7], we assume that experts create a model of the dynamics of knowledge accumulation system:

$$\dot{I}_{m\varepsilon}(t-\tau) = A_m \boldsymbol{I}_{m\varepsilon}(t-\tau) + B\boldsymbol{I}_{mx}(t) \qquad (1)$$

By comparing the model with the real dynamics equation:

$$\dot{I}_{\varepsilon}(t-\tau) = AI_{\varepsilon}(t-\tau) + BI_x(t) \qquad (2)$$

Since the input message is controlled $I_{mx}=I_x$. But output is estimated with a certain error, which is caused by inaccurate measurements, subjective errors and other errors that are most rationally attributed to random variation and treated as random errors. Thus, there are two uncertainties: the state of I_x is observed with the error ξ and the matrix of A is unknown and needs assessment, as it determines the rate of change of information in the process of accumulation. The task of restoring the unknown state vector in the presence of a reserve of time is solved using the asymptotic observer algorithm. For a received message $I_x+\xi$ we construct a difference model:

$$\begin{aligned} \dot{I}_{\varepsilon}(t-\tau) - \dot{I}_{m\varepsilon}(t-\tau) &= A\boldsymbol{I}_{\varepsilon}(t-\tau) - A_m\boldsymbol{I}_{m\varepsilon}(t-\tau) + B\boldsymbol{I}_x(t-\tau) - B\boldsymbol{I}_{mx}(t) \\ \boldsymbol{I}_y &= \boldsymbol{I}_{m\varepsilon}(t-\tau) + \xi(t) \end{aligned} \qquad (3)$$

Since $I_{mx}=I_x$, and the matrix of the object is determined with an error up to the error matrix: $A\text{-}A_m=\Delta A$, we can say:

$$\begin{aligned} \dot{\varepsilon}(t-\tau) &= \dot{I}_{\varepsilon}(t-\tau) - \dot{I}_{m\varepsilon}(t-\tau) \\ \dot{\varepsilon}(t-\tau) &= A_m\varepsilon(t-\tau) + \Delta A\boldsymbol{I}_{\varepsilon}(t-\tau) \\ \boldsymbol{I}_y &= \boldsymbol{I}_{\varepsilon}(t-\tau) + \xi(t) \end{aligned} \qquad (4)$$

The object matrix is determined based on the error matrix and the model matrix:

$$A = A_m + \Delta A \quad (5)$$

Based on the possibility of multiple tests, we assume unbiased estimates for the disturbances:

$$M\{\xi(t)\} = 0 \quad (6)$$

Then, using an asymptotic observer algorithm, we restore the value of the state vector. Introducing the observer matrix D, we receive the observer equation:

$$\dot{\varepsilon}(t-\tau) = (A_m + D)\varepsilon(t-\tau) + \Delta A \boldsymbol{I}_\varepsilon(t-\tau) \quad (7)$$

It is obvious that when testing the system during the accumulation of knowledge the component associated with the uncertainty of the system matrix will cause a significant error in estimating the state. However, replacing the state vector of the object with itsestimate:

$$\boldsymbol{I}_\varepsilon(t-\tau) = M\{\boldsymbol{I}_{ij}\} = \bar{\boldsymbol{I}} \quad (8)$$

Proceeding to the current time, we obtain:

$$\dot{\varepsilon}(t) = (A_m + D)\varepsilon(t) + \Delta A \bar{\boldsymbol{I}}(t) \quad (9)$$

SinceinthisequationtheonlyunknownmatrixisΔA with $n \times n$ dimensions, weconduct $n \times n$ independenttestsandobtainthesystemofequations:

$$\Delta A \bar{I}_{ij}(t) = \dot{\varepsilon}_{ij}(t) - (A_m + D)\varepsilon_{ij}(t), \quad i = \overline{1,n},\ j = \overline{1,n} \quad (10)$$

The solution of this system allows us to find the estimate of the error matrix and construct an iterative procedure:

$$\Delta A_k \bar{I}^k{}_{ij}(t) = \dot{\varepsilon}^k{}_{ij}(t) - \big((A_m + \Delta A_{k-1}) + D\big)\varepsilon^k{}_{ij}(t), i = \overline{1,n}, j = \overline{1,n}, k = \overline{1,m}. \quad (11)$$

To improve the procedure, we discard the results that do not lead to a decrease in the error vector. Thus, we obtain a procedure for recovery of the state vector and determination of the matrix of the object. It is important to bear in mind that to get linear utility estimates we need quadratic estimates due to the norm and metric of the information space of the information accumulation system. Indeed, usingtheestimateμ=I^2weobtain

$$\mu\left(I_m\sqrt{\frac{f}{f_m}}\right)^2 = \frac{I_m^2}{f_m} f \quad (12)$$

Thus, we get a direct evaluation of the usefulness of the accumulated knowledge. When choosing matrix D, we use the procedure convergence condition: the matrix Am + D must satisfy the Hurwitz condition and provide a model velocity above the speed of subject's movement. An essential feature of the algorithm is adherence to the requirements of the fundamental theorem of identifiability towards models: the measurement and control points should not be in the same time, which guarantees the linear independence of the measurements. The second point which is taken into account in the procedure is the screening of measurements with low value of the deviation from the model object. This is due to computational difficulties and poor conditioning of the matrix system of equations being solved. Analyzing the capability of the identification procedure, consider, for example, a fourth-order object with object, model, and control matrices, table 1.

Таблица 1.

Model and object parameters

A	A_m	B
0 1 0 0	0 1 0 0	0 0
0 0 1 0	0 0 1 0	0 0
0 0 0 1	0 0 0 1	0 0
-1 -4 -6 -4	-1 -1 -1 -1	0 1

We regard the starting point of the object's trajectory as unknown, so we select zero as the starting point of the motion model. The task of monitoring the state of the object we lay on the asymptotic observer. The phase portrait and trajectories of the observer are shown in Fig. 1.

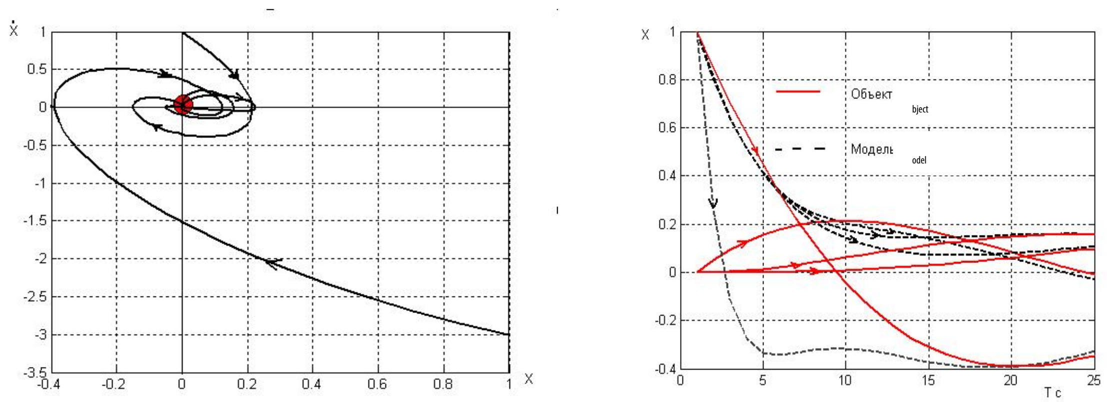

Fig. 1. *The phase portrait and trajectories of the observer*

But to accurately restore the state of the object we need to know the model. In the designed procedure the coordinate recovery alternates with the identification cycle (Fig. 2), which enables the use of the procedure for non-stationary objects, which are typical for tasks of accumulation of information and education.

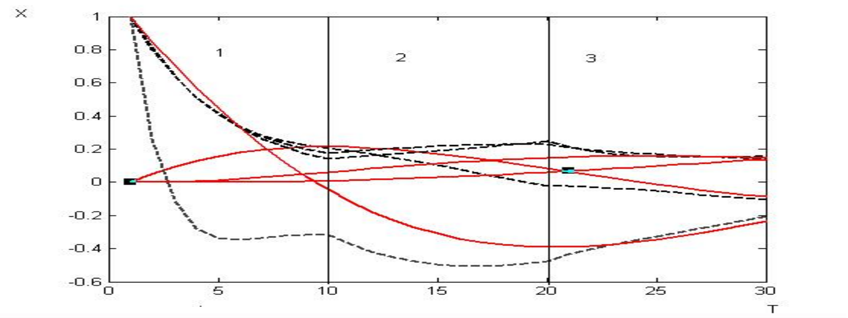

Fig. 2. Restoration cycles for states 1 and 3 and identification of 2.

When choosing an identification method the priority is given to the simplicity and speed of the convergence procedure. The considered procedure requires a solution of four equations. Thus, the solution to four systems of equations guarantees the determination of a correction matrix to the model matrix. At the same time, the identification operation is performed up until to the beginning of the state vector restoration operation, which allows you to complete the process in two cycles, Fig. 3.

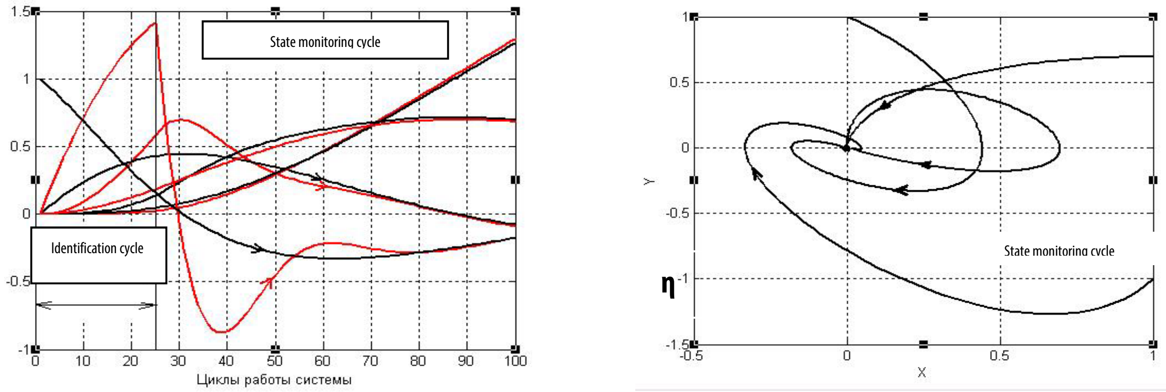

Fig. 3. Dynamics of the system's behavior and its phase portrait in the identification process.

Thus, the use of search-less identification algorithm together with the asymptotic observer algorithm allows us to determine the model parameters and to assess the state of the object at a minimum cost in system resources.

The essential advantage of the method is its independence from transport delays, as the management is applied to the object and the model at once. The estimation of the delay is not difficult, as it is sufficient to measure the time between exposure and response.

However, information technology of the accumulation of knowledge includes not only the storage of the data, but also a directed selection and information retrieval process, which requires implementation of control algorithms.

The stationary filtration problem involves the creation of an algorithm capable of the best, in the sense of a predetermined criterion, method to implement the data flow processing. This problem is generally identified with the Wiener problem [6].

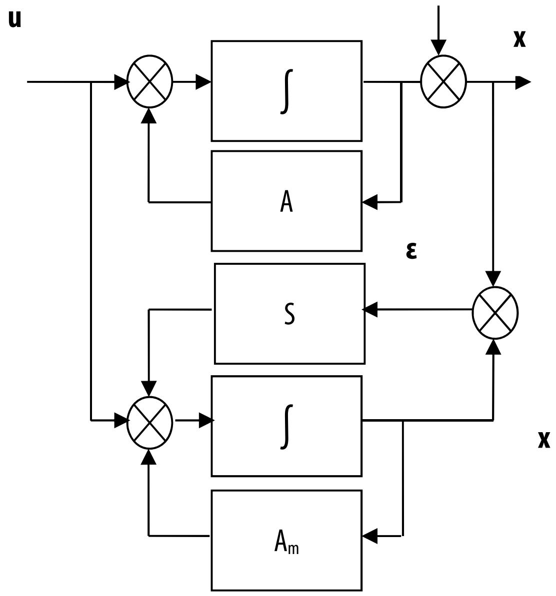

Fig. 4. An asymptotic observer.

The task of non-stationary filtration relies on the identification process and is implemented as an object model control algorithm. These taskssare combined into theKalman problem [7].

The most straightforward and flexible implementation of model-based filtering is the asymptotic observer [8]. Indeed, changing the feedback matrix in the observer's error chain, you can get results from the suppression of the deterministic component to calculating the mathematical expectation of the process.

Consider an asymptotic observer algorithm based on comparing the current process *x* in the dynamic system of object and process of model x_m (Fig. 4):

To measure the state estimation error *η* and the conditions of the exact model, which is achieved by identification, we'll write the equations of the object and model dynamics:

$$\begin{aligned} \dot{\boldsymbol{x}} &= Ax + Bu \\ \dot{\boldsymbol{x}}_m &= A_m x_m + Bu \end{aligned} \quad (13)$$

For the precise model $A=A_m$ and the error vector $\varepsilon=x-x_m$we get the observer dynamics equation

$$\dot{\varepsilon} = (A + S)\varepsilon \quad (14)$$

Since in this taskthe management costs are insignificant, the initial and edge conditions converge, and the objective functional integrand contains only the quadratic formof the error:

$$J = \int_0^t \varepsilon^T Q \varepsilon dt \quad (15)$$

The Hamiltonian function, in this case, has the form:

$$H = \varepsilon^T Q \varepsilon + \lambda^T (A + S)\varepsilon \quad (16)$$

Since neither the target's functional nor the Hamiltonian function depend on control:

$$\frac{\partial H}{\partial \mathbf{u}} = 0 \quad (17)$$

Therefore, management can be considered constant.

Assuming the convexity of the functional purpose, as determined by its structure, we can write the optimality conditions in the Bellman form:

$$\varepsilon^* \xrightarrow[u=u^*]{} minH = min\{\varepsilon^T Q \varepsilon + \lambda^T (A + S)\varepsilon\} \quad (18)$$

A special role in this case is played by the Lagrange multiplier. As is well known [9], in the Bellman problem, leading, in this case, to the synthesis of a Kalman filter, the Lagrange multiplier is determined by the type of the target's functional's integrand:

$$\lambda = \frac{\partial \mathrm{V}}{\partial \varepsilon} = 2\varepsilon^T Q^T = 2\varepsilon^T Q \quad (19)$$

Leading to:

$$\varepsilon^T Q \varepsilon + \lambda^T (A + S)\varepsilon = \varepsilon^T Q \varepsilon + 2\varepsilon^T Q (A + S)\varepsilon \quad (20)$$

Whichenables us to writethe optimality condition in the form:

$$\varepsilon^T Q (I + 2(A + S))\varepsilon \to min \quad (21)$$

Since the matrix Q is a matrix of a quadratic form, the the minimum condition is reached when the sum of the object matrix and the observer connection matrix is zero

$$S = -A \quad (22)$$

Consequently, the feedback loop in the observer must be negative. Given the analyticity of the target's functional, we can write a strong optimum condition

$$min \varepsilon^T Q \big(I + 2(A + S)\big)\varepsilon = 0 \quad (23)$$

Consequently, the observer feedback matrix is defined as

$$S = -\frac{1}{2} I - A \quad (24)$$

The result is interpreted simply. Indeed, with the observer at work the movements of the object and the model differ by the value ε and hence the error is:

$$\varepsilon = x + \eta - x_m \quad (25)$$

Assuming S=-A, we get

$$\dot{\varepsilon} = A\varepsilon + A\eta - A\varepsilon \quad (26)$$

Consequently, the relationship between the error and is determined by the differential equation:

$$\frac{d\varepsilon}{dt} = A\eta \quad (27)$$

Given that the error consists of the deviation of the states and the measuerd object's state error, we can write:

$$\frac{d\eta}{dt} = A_\eta - \frac{d\varepsilon}{dt} \quad (28)$$

Then for an object with a Hurwitz object matrix we obtain a system with administration converging to zero, and measurement error is reduced, at the same time with a decrease in the state estimation error (Fig. 5).

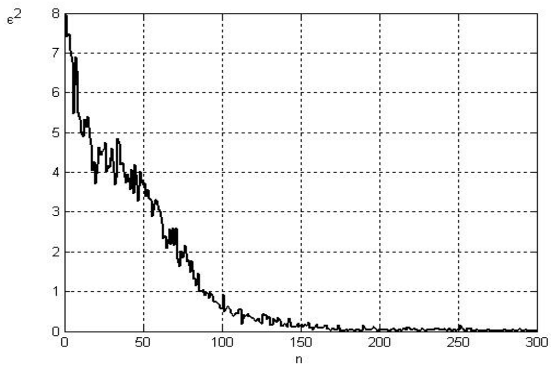

Fig. 5. The quadratic errorof an optimal observer.

CONCLUSIONS

Thus, by using the feedback for optimal filtering of measurement errors, it is possible to provide both the evaluation of the measurement object and the eliminationof errors. Therefore, we can draw the following conclusions:

1. Using identification procedures together with the restoration of the state of the procedure eliminates the uncertainty of the system model.
2. The proposed method of observer calculation allows us to remove the effect of measurement errors in the assessment of the information.

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