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VIBRATIONS OF LATERALLY STRENGTHENED ORTHOTROPIC CYLINDRICAL SHELLS WITH FLOWING LIQUID IN THE ELASTIC MEDIUM

The constructions in the form of plates and shells interacting with elastic rigid and liquid medium have found wide application in engineering and construction. In particular, the problems arising in designing underground and underwater reservoirs and pipelines, linings of underground tunnels and major workings, airfield pavements, solid fuel engine elements and ice covers etc are reduced to such calculation scheme.

Keywords**:** *eigen-vibrations, elastic medium, Lame equation, variational principle, filler.*

Introduction. Study of vibration processes is of great value for up-to-date engineering. Its development is connected with growth of velocity of motion, pressures, temperature, with continuous increase of power and speed of machines and mechanisms, growth of aerodynamic action of the flowing medium flow.

The eigen vibrations of isotropic cylindrical shells with flowing liquid and strengthened with crossed system of ribs in an infinite elastic medium were considered in paper [1]. Free vibrations of a laterally strengthened cylindrical shell with flowing fluid in an infinite elastic medium were investigated by [7]. The problems on stability of filled annular cylindrical shells strengthened with different systems of ribs under time changing different loads were solved in papers [4, 5].

Problem formulation. In the present paper, a problem on eigen-vibrations of a laterally strengthened orthotropic shell contacting with external medium and flowing liquid is solved by means of the variational principle. Influences of external medium have been taken into account by means of the system of Lame equations in displacements.

Differential equations of motion for a medium-contacting laterally strengthened orthotropic cylindrical shell are gotten on the base of Ostrogradsky-Hamilton variational principle. For applying the Ostrogrdsky-Hamilton principle, we write the potential and kinetic energies of the system beforehand.

The potential energy of elastic deformation of an orthotropic cylindrical shell is of the form [2]

$$V=\frac{hR}{2}\iint\left\{B_{11}\left(\frac{\partial u}{\partial x}\right)^2-2\left(B_{11}+B_{12}\right)\frac{w}{R}\frac{\partial u}{\partial x}+\frac{w^2}{R^2}\left(B_{11}+2B_{12}+B_{22}\right)+\frac{B_{22}}{R^2}\left(\frac{\partial \vartheta}{\partial\theta}\right)^2-\right.$$
$$\left.-2\left(B_{12}+B_{22}\right)\frac{w}{R^2}\frac{\partial\vartheta}{\partial\theta}+2B_{12}\frac{1}{R^2}\frac{\partial u}{\partial x}\frac{\partial\vartheta}{\partial\theta}\right\}dxd\theta, \qquad (1)$$

where

$$B_{11}=\frac{E_1}{1-\nu_1\nu_2};\quad B_{22}=\frac{E_2}{1-\nu_1\nu_2};\quad B_{12}=\frac{\nu_2E_1}{1-\nu_1\nu_2}=\frac{\nu_1E_2}{1-\nu_1\nu_2};$$

R is the radius of the median surface of the shell; h is the shell thickness; u, v, w are the components of displacements of median surface points of the shell.

The expressions for potential energy of elastic deformation of the j-th lateral rib are the followings [3]

$$\Pi_j = \frac{1}{2}\int\limits_{y_1}^{y_2}\left[E_jF_j\left(\frac{\partial \vartheta_j}{\partial y} - \frac{w_j}{R}\right)^2 + E_jJ_{xj}\left(\frac{\partial^2 w_j}{\partial x^2} + \frac{w_j}{R^2}\right)^2 + E_jJ_{zj}\left(\frac{\partial^2 u_i}{\partial y^2} - \frac{\varphi_{\kappa p j}}{\mathrm{R}}\right)^2 + G_jJ_{\kappa p j}\left(\frac{\partial \varphi_{\kappa p j}}{\partial y} + \frac{1}{R}\frac{\partial u_j}{\partial y}\right)^2\right]dy. \quad (2)$$

In expression (2) y_1, y_2 are the coordinates of curvilinear edges of the shell; $F_j, J_{zj}, J_{yj}, J_{kpj}$ are area and inertia moments of the cross-section of the j-th lateral bar, respectively with respect to the axis Oz and the axis parallel to the axis Oy and passing through the gravity center of the cross-section, and also its inertia moment under torsion; E_j, G_j are elasticity and shift module of the material of the j-th lateral bar; φ_{kpj} .

The potential energy of external surface and edge loads applied to the casing is determined as a work performed by these loads while taking the system from the deformed state to initial underformed state, and is represented in the form

$$A_0 = -\int\limits_{x_1}^{x_2}\int\limits_{y_1}^{y_2}\left(q_x u + q_y \vartheta + \left(q_{zm} + q_{zc}\right)w\right)dxdy - \int\limits_{y_1}^{y_2}\left(T_1 u + S_1 \vartheta + Q_1 w + M_1 \varphi_1\right)\Big|_{x=x_1}^{x=x_2} dy -$$
$$-\int\limits_{x_1}^{x_2}\left(S_2 u + T_2 \vartheta + Q_2 w + M_2 \varphi_2\right)\Big|_{y=y_1}^{y=y_2} dx. \quad (3)$$

Similarly, the potential energies of external edge loads applied to the end faces of the j-th lateral bar are defined by the following expressions (it is accepted that only edge loads are applied to the ribs)

$$A_j = -\left(S_j u_j + T_j \vartheta_j + Q_j w_j + M_j \varphi_j + M_{1j}\varphi_{zj} + M_{kpj}\varphi_{kpj}\right)\Big|_{y=y_1}^{y=y_2}. \quad (4)$$

The total potential energy of the system is equal to the sum of potential energies of elastic deformations of the shell and ribs, and to also the sum of potential energies of all external loads:

$$\Pi = \Pi_0 + \sum_{j=1}^{k_1}\Pi_j + \sum_{j=1}^{k_1}A_j + A_0. \quad (5)$$

Kinetic energies of the shell and ribs are written as

$$K_0 = \frac{\rho_0 h R^2}{2}\int\limits_0^{\xi_1}\int\limits_0^{2\pi}\left[\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial \vartheta}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2\right]d\xi d\theta; \quad (6)$$

$$K_j = \rho_j F_j \int_{y_1}^{y_2} \left[\left(\frac{\partial u_j}{\partial t} \right)^2 + \left(\frac{\partial \vartheta_j}{\partial t} \right)^2 + \left(\frac{\partial w_j}{\partial t} \right)^2 + \frac{J_{\kappa pj}}{F_j} \left(\frac{\partial \varphi_{\kappa pj}}{\partial t} \right)^2 \right] dx. \quad (7)$$

Here t is a time coordinate, ρ_0, ρ_j correspond to the density of materials of the shell, j to the lateral bar.

Kinetic energy of the ridge orthotropic shell

$$K = K_0 + \sum_{j=1}^{k_1} K_j. \quad (8)$$

Equations of motion of a medium contacting orthotropic ridge shell are obtained on the base of Ostrogradsky-Hamilton action stationarity principle:

$$\delta W = 0, \quad (9)$$

where $W - = \int_{t'}^{t''} L dt$ is Hamilton's action; $L = K - \Pi$ is the Lagrange function; t' and t'' are given the arbitrary moments.

Assuming that the main velocity of the flow equals to U and deviations from this velocity are small, we use the wave equation for the potential of perturbed velocities φ with respect to [3]

$$\Delta\varphi - \frac{1}{a_0^2} \left(\frac{\partial^2 \varphi}{\partial t^2} + 2U \frac{\partial^2 \varphi}{R \partial \xi \partial t} + U^2 \frac{\partial^2 \varphi}{R^2 \partial \xi^2} \right) = 0. \quad (10)$$

When harmonic vibrations are considered, the motion equations of medium are taken in the form

$$a_e^2 \,\mathrm{grad}\,\mathrm{div} \vec{u} - a_t^2 \,\mathrm{rot}\,\mathrm{rot} \vec{u} + \omega^2 \vec{u} = 0. \quad (11)$$

The potential and kinetic energies of the shell (5), (8), the motion equations of liquid (10) and medium (11) are complemented with contact conditions.

Continuity of radial velocities and pressures on the contact surface “shell-liquid” is observed. The condition of impermeability or smoothness of flow about the shell wall is of the form [3]

$$\vartheta_r \big|_{r=R} = \frac{\partial \varphi}{\partial r} \bigg|_{r=R} = -\left(\frac{\partial w}{\partial t} + U \frac{\partial w}{R \partial \xi} \right). \quad (12)$$

Equality of radial pressures of the liquid on the shell

$$q_{zm} = -p\big|_{r=R}. \quad (13)$$

Suppose that the contact between the shell and medium is sliding, i.e. for $r = R$

$$w = s_z, \quad (14)$$

$$q_x = -\sigma_{rx} = 0, \quad q_\theta = -\sigma_{r\theta} = 0, \quad q_{zc} = -\sigma_{rr}. \tag{15}$$

Complementing the expressions of the potential and kinetic energies of the shell (5), (8), the equation of motion of liquid (10) and medium (11) by contact conditions (12)–(15), we get a problem on eigen vibrations in an infinite elastic medium of a laterally strengthened orthotropic cylindrical shell with flowing liquid. In other words, the problem on eigen vibrations of a laterally strengthened cylindrical shell with flowing liquid in an infinite elastic medium is reduced to joint integration of equations of theory of shells, medium and liquid when the indicated conditions on their contact surface are fulfilled.

Solution method. We will look for the displacements of the shell in the form:

$$\begin{aligned} u &= u_0 \sin kx \cos n\theta \sin \omega_1 t_1; \\ \vartheta &= \vartheta_0 \cos kx \sin n\theta \sin \omega_1 t_1; \\ w &= w_0 \cos kx \cos n\theta \sin \omega_1 t_1. \end{aligned} \tag{16}$$

Here u_0, ϑ_0, w_0 are the unknown constants; k, n are wave numbers in longitudinal and peripheral directions, respectively.

The solution of the medium motion equation is of the form:

in the case of inertialess medium:

$$\begin{aligned} s_x &= \left[\left(-kr\frac{\partial K_n(kr)}{\partial r} - 4(1-\nu_s)kK_n(kr)\right)A_s + kK_n(kr)B_s\right]\cos n\varphi \cos kx \sin \omega t; \\ s_\theta &= \left[-\frac{n}{r}K_n(kr)B_s - \frac{\partial K_n(kr)}{\partial r}C_s\right]\sin n\varphi \sin kx \sin \omega t; \\ s_r &= \left[-k^2 r K_n(kr)A_s + \frac{\partial K_n(kr)}{\partial r}B_s + \frac{n}{r}K_n(kr)C_s\right]\cos n\varphi \sin kx \sin \omega t, \end{aligned} \tag{17}$$

in the case of inertial medium:

$$\begin{aligned} s_x &= \left[\tilde{A}_s k K_n(\gamma_e r) - \tilde{C}_s \frac{\gamma_t^2}{\mu_t} K_n(\gamma_t r)\right]\cos n\varphi \cos kx \sin \omega t; \\ s_\theta &= \left[-\frac{\tilde{A}_s n}{r}K_n(\gamma_e r) - -\frac{\tilde{C}_s nk}{r\mu_t}K_n(\gamma_t r) - \frac{\tilde{B}_s}{n}\frac{\partial K_n(\gamma_t r)}{\partial r}\right]\sin n\varphi \sin kx \sin \omega t; \\ s_r &= \left[\tilde{A}_s \frac{\partial K_n(\gamma_e r)}{\partial r} - \frac{\tilde{C}_s k}{\mu_t}\frac{\partial K_n(\gamma_t r)}{\partial r} + \frac{\tilde{B}_s n}{r}K_n(\gamma_t r)\right]\cos n\varphi \sin kx \sin \omega t. \end{aligned} \tag{18}$$

We look for the perturbed velocities potential φ in the form

$$\varphi(\xi, r, \theta, t_1) = f(r)\cos n\varphi \sin kx \sin \omega t. \tag{19}$$

Using (19) and conditions (12), for $p_0 = 0$ from (13) we have:

$$\varphi = -\Phi_{\alpha n}\left(\omega_0 \frac{\partial w}{\partial t_1} + U\frac{\partial w}{R\partial \xi}\right); \tag{20}$$

$$p=\Phi_{\alpha n}\rho_m\left(\omega_0^2\frac{\partial^2 w}{\partial t_1^2}+2U\omega_0\frac{\partial^2 w}{R\partial\xi\partial t_1}+U^2\frac{\partial^2 w}{R^2\partial\xi^2}\right),$$

where

$$\Phi_{\alpha n}=\begin{cases} I_n(\beta r)/I_n^{'}(\beta r), & M_1<1\\ J_n(\beta_1 r)/J_n^{'}(\beta_1 r), & M_1>1.\\ \dfrac{R^n}{nR^{n-1}}, & M_1=1\end{cases} \tag{21}$$

Here $M_1=\dfrac{U+\omega_0 R\omega_1/\alpha}{a_0}$; $\beta^2=R^{-2}\left(1-M_1^2\right)\chi^2, \beta_1^2=R^{-2}\left(M_1^2-1\right)\chi^2$, I_n is the modified Bessel function of the first kind of order n, J_n is the Bessel function of the first order of kind

n, $t_1=\omega_0 t,\ \omega_0=\sqrt{\dfrac{E_1}{\left(1-v^2\right)\rho_0 R^2}};\ \omega_1=\omega/\omega_0.$

Then in (3) as q_{zm} we should take the quantity $q_{zm}=-p$, where p is pressure with respect to (20). Allowing for (16), we can represent the pressure p as follows:

$$p=\frac{\rho_m\Phi_{\alpha n}}{\rho_0\omega_0^2 h}\left(\omega_0^2\omega_1^2+2\omega_0\omega_1\chi U+\chi^2U^2\right)w. \tag{22}$$

Using contact conditions (14) and (15), the solution of the medium motion equation (17) and (18), formulas for the stresses σ_{rr} [6], we can define the contact pressure q_{zc} of the medium on the shell

$$q_{zc}=\tilde{C}_{rr}w_0\cos\chi\xi\cos n\theta\sin\omega_1 t_1, \tag{23}$$

where in the case of inertialess medium, $\tilde{C}_{rr}$ has the form:

$$\tilde{C}_{rr}=-\mu_s\Delta^{-1}\left(q_{11}\Delta_1^{(3)}+q_{12}\Delta_2^{(3)}+q_{13}\Delta_3^{(3)}\right);$$

$$\Delta=k^{\bullet 2}n^2K_n^{''}\left(k^\bullet\right)I_n^2\left(k^\bullet\right)-k^{\bullet 4}K_n^{'3}\left(k^\bullet\right)+4n^2k^\bullet\left(1-\nu_s\right)K_n^3\left(k^\bullet\right)-$$

$$-4k^{\bullet 3}\left(1-\nu_s\right)K_n\left(k^\bullet\right)K_n^{'2}\left(k^\bullet\right)+k^{\bullet 4}K_n^2\left(k^\bullet\right)K_n^{'}\left(k^\bullet\right);$$

$$q_{11}=\left(2\left(1-2\nu_s\right)K_n\left(k^\bullet\right)+2k^\bullet K_n^{'}\left(k^\bullet\right)\right)k^{\bullet 2};\quad q_{12}=-2k^{\bullet 2}K_n^{''}\left(k^\bullet\right);\ q_{13}=2n\left(K_n\left(k^\bullet\right)-k^\bullet K_n^{'}\left(k^\bullet\right)\right);$$

$$\Delta_1^{(3)} = \begin{vmatrix} 0 & l_{12} & l_{13} \\ 0 & l_{22} & l_{23} \\ w_0 & l_{32} & l_{33} \end{vmatrix}; \ \Delta_2^{(3)} = \begin{vmatrix} l_{11} & 0 & l_{13} \\ l_{21} & 0 & l_{23} \\ l_{31} & w_0 & l_{33} \end{vmatrix}; \ \Delta_3^{(3)} = \begin{vmatrix} l_{11} & l_{12} & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & w_0 \end{vmatrix};$$

$$l_{11} = \left(k^{\bullet} K_n\left(k^{\bullet}\right) + k^{\bullet} K_n''\left(k^{\bullet}\right) + \left(5 - 4\nu_s\right) K_n'\left(k^{\bullet}\right)\right) k^{\bullet 2}; \ l_{12} = -2k^{\bullet 2} K_n'\left(k^{\bullet}\right); \ l_{13} = -nk^{\bullet} K_n\left(k^{\bullet}\right);$$

$$l_{21} = -nk^{\bullet 2} K_n\left(k^{\bullet}\right); \ l_{22} = 2n\left(k^{\bullet} K_n'\left(k^{\bullet}\right) - K_n\left(k^{\bullet}\right)\right); \ l_{23} = k^{\bullet 2} K_n''\left(k^{\bullet}\right) - k^{\bullet}\left(k^{\bullet}\right) + n^2 K_n\left(k^{\bullet}\right); k^{\bullet} = kR;$$

$$l_{31} = -k^{\bullet 2} K_n\left(k^{\bullet}\right); \ l_{32} = k^{\bullet} K_n'\left(k^{\bullet}\right); \ l_{33} = nK_n\left(k^{\bullet}\right).$$

After substituting (16), (22), (23) in (9), the problem is reduced to the homogeneous system of third order linear algebraic equations

$$a_{i1}u_0 + a_{i2}v_0 + a_{i3}w_0 = 0 \ \left(i = 1, 2, 3\right). \tag{24}$$

The elements $a_{i1}, a_{i2}, a_{i3}\left(i = 1, 2, 3\right)$ have a bulky form, therefore we don't cite then. The non-trivial solution of the system of linear algebraic equations (24) of third order is possible when ω_1 is the root of its determinant. The definition of ω_1 is reduced to a transcendental equation since ω_1 enters into the argument of the Bessel function J_n

$$\begin{vmatrix} \tilde{a}_{11} + \rho_j \omega_1^2 & a_{12} & a_{13} \\ a_{21} & \tilde{a}_{22} + \omega_1^2 & a_{23} \\ a_{31} & a_{32} & \tilde{a}_{33} - \varphi_1 \omega_1^2 - \varphi_2 \omega_1 \end{vmatrix} = 0. \tag{25}$$

Note that for $U = 0(\varphi_2 = 0)$ equation (25) passes to the frequency equation of free vibrations situated in an unbounded inertialess elastic medium filled with liquid at rest. The latter equation for $\tilde{C}_{rr} = 0$ passes to the equation of free vibrations of a laterally strengthened cylindrical shell filled with liquid at rest [6].

Numerical results. Consider some results of calculations carried out proceeding from the above-mentioned dependences, by means of ECM.

For geometrical and physical parameters characterizing the materials of the medium's shell we accept:

$$E_j = 6.67 \cdot 10^9 \frac{N}{m^2}; \ \nu = 0.3; \quad n = 8; \ h_j = 1.39\, mm; \ R = 160\ mm;$$

$$k_1 = 4; \ \rho_0 = \rho_j = 0.26 \cdot 10^4\ N \frac{s^2}{m^4}; \quad h = 0.45\ mm; \ I_{kp.j} = 0.48\ mm^4;$$

$$\frac{\rho_0}{\rho_m} = 0.105 l = 800\ mm; \quad F_j = 5.75\ mm^2; \ I_{xj} = 19.9\ mm^4;$$

$$a_l = 2.25 a_t; \ a_t = 308 \frac{m}{s}; \ m = 8.$$

The dependence of the frequency parameter ω_1 on relative velocity of flow $U^* = U/c,\ c = \omega_0 R$ for different values of $k^\bullet$ and n are shown in fig. 1. It is seen that the velocity increase reduces to frequency decrease. It is important to note the values of U^* at which oscillations frequency vanishes. Obviously, here the stability loss of the shell should occur.

At last, fig. 2 illustrates the influence of the amount of lateral ribs k_1 on the frequency parameter ω_1 of the system under consideration. It is seen that as k_1 increases, the frequency parameter ω_1 of the system vibrations at first increases and then at certain value of k_1 begins to decrease. This is explained by the fact that the bars weight increases according to increase of k_1 and this reduces to essential influence of inertia features.

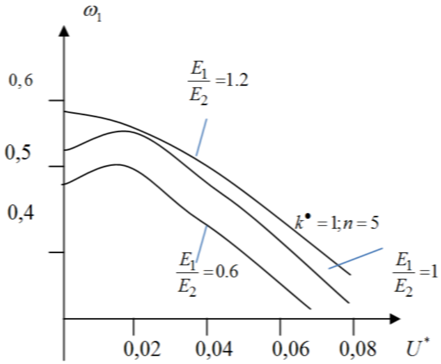

Fig. 1 – **Dependence of frequency parameter on flow’s velocity for a laterally strengthened shell in medium**

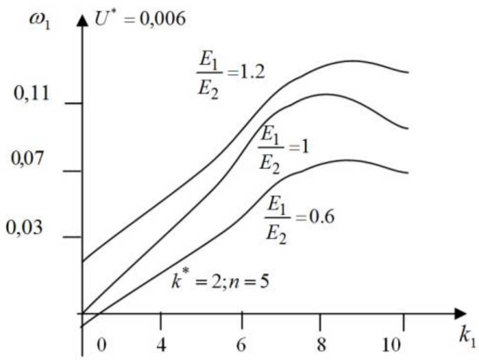

Fig. 2 – **Dependence of frequency parameter on the amount of lateral ribs of a laterally strengthened shell in medium with moving liquid**

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КОЛИВАННЯ ПОПЕРЕЧНО ПІДКРІПЛЕНИХ ОРТОТРОПНИХ ЦИЛІНДРИЧНИХ ОБОЛОНОК З ПРОТІКАЮЧОЮ РІДИНОЮ В ПРУЖНОМУ СЕРЕДОВИЩІ

Робота присвячена дослідженню коливань конструкцій у вигляді пластин і оболонок, що взаємодіють з пружним твердим і рідким середовищем, що знайшли широке застосування в техніці та будівництві. Розглянуті розрахункові схеми, що виникають при проектуванні підземних і підводних ємностей і трубопроводів, оздоблень тунелів метро і капітальних гірничих виробок, аеродромних покриттів, елементів твердопаливних двигунів і т. п.

Ключові слова: *власні коливання, пружне середовище, рівняння Ламе, варіаційний принцип, заповнювач.*

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КОЛЕБАНИЯ ПОПЕРЕЧНО ПОДКРЕПЛЕННЫХ ОРТОТРОПНЫХ ЦИЛИНДРИЧЕСКИХ ОБОЛОЧЕК С ПРОТЕКАЮЩЕЙ ЖИДКОСТЬЮ В УПРУГОЙ СРЕДЕ

Работа посвящена исследованию колебаний конструкций в виде пластин и оболочек, взаимодействующих с упругой твердой и жидкой средой, которые нашли широкое применение в технике и строительстве. Рассмотрены схемы, возникающие при проектировании подземных и подводных емкостей и трубопроводов, отделок тоннелей метро и капитальных горных выработок, аэродромных покрытий, элементов твердотопливных двигателей и т. п.

Ключевые слова: *собственные колебания, упругая среда, уравнения Ламе, вариационный принцип, заполнитель.*

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