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O. R. Denysiuk, D. G. Zelentsov, Dr. Sci. (Tech.) USE OF GENETIC ALGORITHMS IN PROBLEMS OF CORRODING HINGED-ROD STRUCTURES OPTIMAL DESIGN

The paper proposes a new efficient algorithm to solve the problems of corroding hinged-rod structures optimal design, which involves obtaining solutions with given accuracy. The optimization algorithm is based on use of flexible tolerance strategy together with integer-valued real genetic algorithm. To ensure the required accuracy of the restrictions functions computation in the vicinity of an extremum an artificial neural network approximating the relationship between structure parameters, permissible error and parameters of computational procedures is used. The search for solution is made in a discrete non-metric space of varied parameters.

Keywords: optimal design, discrete optimization, genetic algorithm, flexible tolerance method, neural network, corrosive wear.

Introduction. Structures used in chemical industries are exposed to influence of technological environments corrosive to the metal during operation. It causes the destruction of the surface layer of metal (corrosive wear) and leads to decrease in geometric dimensions of the structural elements cross-sections that reduce bearing capacity of structures and their premature failure. There are three types of corrosion losses: the direct loss which is the cost of the metal destructed due to corrosion, indirect loss, related to the economic damage due to premature failure of structure, and loss caused by unsustainable design solutions. Losses of the third type are caused, according to the authors, by the lack of reliable, accurate and effective methods of structures optimal design considering corrosion processes in them.

In solving the problems of optimum design of structures exposed to corrosive environment, the most important criterion for selecting a numerical algorithm is its efficiency (the ability to produce solutions with minimal computational cost). The importance of this criterion is related to the cost of constraint functions (CF) calculation.

CF calculation assumes numerical solution of differential equation system (DES) describing the process of corrosion in structural elements. Significant influence of mechanical stresses on corrosion rate assumes solution of the problem of stress-strain state, for example by finite element method, in each node of the time grid [10]. Presence of feedback in the mathematical models of calculation and significant increase in the number of parameters which determine geometric size of a structure at any given time cause significant increase in computational cost of solving problems of this type in comparison with the «classical» optimization problems. It makes the problem of numerical algorithms efficiency particularly relevant for the problems of this class.

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In the search for optimal solutions varied parameters change at each iteration, and if the parameter of DES numerical solution is constant, it is impossible to control the error of CF calculation. Thus, the second relevant problem is the actual accuracy of the solution.

A review of researches on the use of non-linear mathematical programming methods for solving problems of optimal design of structures operating in aggressive environment is adequately presented in [14]. In the years after the review was published, various algorithms of flexible tolerance method (FTM) that reduce the computational costs due to changes in the accuracy of CF calculation at different stages of optimization problem solution were developed [8]. However, existing algorithms of this method do not guarantee obtaining the result with the required accuracy. It requires a calculation error control algorithm for the numerical solution of DES, simulating the process of geometrical damage accumulation in the structural elements.

In [12], apparently, for the first time, it was proposed to use artificial neural network (ANN) to determine the parameters of DES numerical solution, providing the specified accuracy of the solution for the current vector of varied parameters. Later this idea was used to create calculation error control algorithm for DES numerical solution.

Papers [9, 12] were devoted to solving corroding hinged-rod structures (HRS) optimization problems on a continuous set of varied parameters and this fact significantly reduces their practical application. HRS are usually made of rolled sections, and the dimensions their cross-sections are regulated by standards. Of much greater interest, in authors' opinion, are problem statements that involve changing the rod elements cross-section sizes discretely. In this formulation, type and standard size of the rolled section are the varied parameters. Thus, there is a discrete optimization problem of combinatorial type, solution of which is sought on non-metric set (index set).

In recent years, genetic algorithms (GA) are successfully used to solve discrete optimization problems [2, 3, 6, 7]. In GA only information about the objective function and the constraints functions is used, therefore their efficiency is objectively lower than efficiency of mathematical programming methods which use the derivatives of these functions. One way to improve the efficiency of GA is to minimize the computational cost of the CF calculation.

The purpose of this paper is to provide an efficient algorithm for solving the problem of corroding structures optimal design based on genetic algorithms together with flexible tolerance method and CF calculation error neural network control algorithm.

Problem statement. In this paper, hinged-rod structures operating in highly aggressive technological environments are the object of study. Such environments are characterized by a high rate of corrosion process (more than 0.05 cm/year), as well as significant impact of stress on the process rate. Some of the most common models describing this process are given in [1], and so is the foundation of the model used in further study:

$$\frac{d\delta}{dt} = v_0 \left(1 + k\sigma \right), \tag{1}$$

where δ is depth of corrosive damage (damage parameter); t is time; v_0 is corrosion rate in the absence of stress; k is coefficient of stress impact; σ is absolute value of stress.

Statement of problem of corroding HRS optimal design may be formulated the next way. It is required to determine the parameters of the elements cross-sections in such way that the volume of structure is minimal and within the specified operational life it maintains its bearing capacity, or, in other words, it satisfies the constraints on the strength and stability.

$$F(\overline{x}) = \sum_{i=1}^{N} L_i A_i(\overline{x}) \to \min; \quad \overline{x} \in X_D;$$
(2)

$$X_{D}: \{\overline{x} \in E^{n} \mid g(\overline{x}) = \sigma_{i} * (\overline{x}, t^{*}) - \sigma_{i}(\overline{x}, t^{*}) \ge 0; i \in \overline{1, N}\},$$
(3)

where L_i , A_i are length and cross-section area of *i*-th element; *N* is number of structural elements; \bar{x} is vector of varied parameters; σ_i and σ^*_i are current and ultimate stress in *i*-th element; t^* is specified operating time of structure.

Earlier, dimensions of the rods cross-sections were considered the varied parameters; it was assumed that the sectional form was known. For example, in [9, 12] rods of circular and annular cross-sections are used. Varied parameters change continuously within certain limits.

In this paper the HRS, elements of which are made of standard rolled sections (I-beam, C-beam, angle bars), are used. In this case, the cross-section dimensions, firstly, may be changed only discretely, and secondly, may not be changed independently. Therefore, the vector of varied parameters will be a set of indices that determine type and standard size of the section. The search space of the optimization problem solution, thus, is discrete and non-metric.

When modeling the structure behavior in aggressive environment, the following assumptions are made:

- at joints corrosion process is the same as in the entire structure;

- there are no mounting stresses in structure; the values of stresses in the elements are determined only by external loads and rods' own weight.

Constraint functions calculation. As noted above, the most important in the development of the optimization algorithms are two mutually contradictory criteria which are accuracy and efficiency. In this case, the accuracy of solution involves not only finding the global minimum of the function (2), but, first of all, the accuracy of constraint functions calculation. As noted in [5], the question of how exactly a structure was destroyed due to corrosion wear has a purely theoretical value. The main criterion is its durability, and for its calculation the corrosive wear model is used.

The behavior of structure in aggressive environment is simulated by the numerical solution of Cauchy problem for DES of the form

$$\frac{d\delta_i}{dt} = v_0 \left[1 + \sigma_i \left(\overline{\delta} \right) \right]; \, \delta_i \Big|_{t=0} = 0; \quad i = \overline{1, N} ,$$
(4)

where δ_i and σ_i are depth of corrosive damage and stress in *i*-th element.

Calculation of stress functions requires solution of the equations of mechanics, consisting of the equilibrium and compatibility of strains equations, Cauchy relations and Hooke's law. In terms of the finite element method (FEM), this system has a form:

$$\overline{R} = K \cdot \overline{u}; \qquad \overline{\varepsilon} = D \cdot \overline{u}; \qquad \overline{\sigma} = E \cdot \overline{\varepsilon}, \qquad (5)$$

where *K*, *D*, *E* are stiffness, differentiation and elasticity matrices; \overline{R} , \overline{u} , $\overline{\varepsilon}$, $\overline{\sigma}$ are vectors of external loads, nodal displacements, strains and stresses. The stiffness matrix coefficients depend on the cross-section areas, and therefore change over time.

Thus, in general case a solution of (4) can be obtained only numerically. Obviously, the computational cost of the solution of this problem is much higher than of the solution of optimal design problem in a traditional statement.

Parameters of DES numerical solution usually remain constant in the process of solution. At the same time the geometric characteristics of the rods cross-sections vary within the limits defined by boundaries of varied parameters change. Calculation error of the result in this case is unpredictable. Appointment of numerical solution parameters which, with acceptable probability, will allow determining durability of the structure with a permissible error on the entire solution space leads to excessive computational cost. To successfully resolve this problem, the parameters of DES numerical solution should be determined on the basis of information about the structural parameters (variable and constant), aggressive environment parameters, and the value of permissible error. In other words, it is necessary to construct the approximating function which formalizes this dependency. To do this, is necessary to select the algorithm of DES solution and parameter of error control, to determine the relevant parameters and the method of approximation.

It is proposed to approximate the dependency between the parameter of DES numerical solution algorithm, the parameters of rod element, aggressive environment and the permissible calculation error using artificial neural network (ANN). Fig. 1 shows the architecture of the neural network for stretched and compressed rods. They differ in the input parameters.

For stretched rod the relevant parameters are area and perimeter of cross-section, initial stress and corrosion rate. Since the shape of cross-section does not matter, it is proposed to use an annular cross-section as the equivalent one. The input parameters of the neural network are outer radius *R*, ratio of inner radius to external one *g*, initial stress σ_0 , rate of corrosion in the absence of stress v_0 and permissible solution error ε^* .

For compressed rod the shape of cross-section is the most important parameter as it determines rules for calculating the moment of inertia. For each type of section (I-beams, C-beams, equal and unequal angle bars) a separate network is trained. All sizes of rolled sections are regulated by standards, so instead of the radii used in the case of stretched rod, the appropriate standard size number *n* is used as input. Rod length has a significant effect on the value of the critical buckling stress. It can be calculated using the initial value of initial critical buckling stress σ_0^* , which is an additional input parameter for this ANN.



Fig. 1 – Architecture of the neural network for stretched (a) and compressed (b) rods

The detailed description of the neural network calculation error control algorithm is given in [9].

Optimization problem solution algorithm. To solve the problem an algorithm which, according to the authors, allows significantly reduce the computational cost, while fulfilling the conditions of accuracy of the optimal solution, was proposed and justified.

To transform the original problem (2) - (3) to an unconstrained optimization problem the exterior penalty function method is used. The initial problem is transformed to the following form:

$$P(\overline{x}) = F(\overline{x}) + \sum_{i=1}^{N} H_i \left[\sigma_i * (\overline{x}, t^*) - \sigma_i (\overline{x}, t^*) \right];$$

$$H_i = \begin{cases} 0, & \text{if } \sigma_i * (\overline{x}, t^*) \ge \sigma_i (\overline{x}, t^*) \\ H^*, & \text{if } \sigma_i * (\overline{x}, t^*) < \sigma_i (\overline{x}, t^*) \end{cases}$$
(6)

where H^* is penalty coefficient.

To solve the problem of unconstrained optimization it is proposed to use an integer-valued genetic algorithm. The main ideas of GA construction are adequately described, for example, in [1], so only the methods of solution space formation and chromosomes encoding used in this paper are mentioned below.

Chromosome (analogue of the varied parameters vector) is a set of indices that determine the position of section sizes in the three-dimensional array of sizes, where the layer number (section type) is determined by the odd indices, and the line number (standard section size) is determined by even ones. Thus, the amount of genes (used as varied parameter analogue) in chromosome is equal to 2N, where N is a number of rod elements, optimum parameters are to be determined. Fig. 2 and 3 show the solution space of optimization problem and the method of chromosome encoding.



Fig. 2 – The solution space of the problem

Fig. 3 – Example of the chromosome encoding

Cross-sections considered in the paper are described by the different number of sizes (from 4 for equal angle bar to 7 for I-beam). In this case, the construction of a real corroding cross-section model is associated with considerable complications, so cross-sections models composed of rectangular pieces are used in research. Such cross-sections are defined by four sizes, regardless of their type. The sizes are determined in such way that changes of geometrical characteristics (area and minimal moment of inertia) occur in the same manner as in the real sections. Methods of determining those sizes of model sections are presented in [11].

The procedure for decoding of data contained in the chromosome consists of extraction of sizes located in the layer and row numbers that correspond to the pairs of genes from the array of cross-sections numbers.

Description of evolution and population models used to solve the problem, as well as utilized genetic operators are presented below in numerical illustration.

The ability to control constraint functions calculation will significantly improve the efficiency of optimization algorithm. For this purpose it is proposed to use the flexible tolerance method together with the genetic algorithm [4].

When using the flexible tolerance method, the system of constraints (3) can be represented as

$$X_{D}: \{ \overline{x} \in E^{n} \mid g_{1}(\overline{x}) = Y(k) - T(\overline{x}, t^{*}) \ge 0 \} ,$$
(7)

where Y is criterion of flexible tolerance (CFT), a decreasing function of iteration number k, T is a functional upon all the set of constraint functions.

It is proposed to take permissible CF calculation error as Y and relative CF calculation error as T. The solution is sought both on the boundary of accessible region of the solution space, and outside it at a distance determined by the criterion of flexible tolerance. The point of the solution space can be classified as acceptable, almost acceptable or unacceptable.

In this case, the CF calculation error on initial Iterations of the search for solution may be high enough to minimize the computational cost, while in the vicinity of an extremum the error does not exceed a permissible value determined by the customer. The general scheme of the optimization problem solution algorithm is shown in Fig. 4.



Fig. 4 – Scheme of flexible tolerance method algorithm

On Fig. 4: OF is the objective function calculation module; SSS is the module of solution of strain-stress state problem; ANN is the module of defining parameters of numerical solution using ANN; DES is the module of solution of differential equation system simulating corrosion in structural elements; NLP is the module of non-linear programming problem solution.

In the process of problem solution, according to the idea of FTM, the criterion of flexible tolerance (the permissible CF calculation error) should decrease in the vicinity of an extremum. It is proposed to reduce CFT based on GA epoch number:

$$Y(k) = \varepsilon_k = \varepsilon_{\max} - \frac{\varepsilon_{\max} - \varepsilon_{\min}}{n} \cdot \operatorname{int}(\frac{k \cdot n}{k_{\max}}), \qquad (8)$$

where k_{\max} is a maximal epoch number; *n* is a number of CFT change steps; ε_{\max} , ε_{\min} are permissible values of errors in the beginning of the search and in the vicinity of an extremum.

Penalty terms in the function (4) are determined by the formula:

$$H = H^* \cdot \left(Y(k) - \frac{\left| t^* - t \left[\overline{x}, h_t(\overline{x}, Y(k)) \right] \right|}{t^*} \right), \tag{9}$$

where h_t is DES (5) numerical calculation step, which depends on values of varied parameters and permissible error on *k*-th GA epoch.

Numerical illustration. To illustrate the proposed algorithm, the solution of optimal design problem for statically indeterminate five- and fifteenelement trusses is considered (fig. 5).

The initial population of 250 individuals was used to test the optimization algorithm. A five-rod HRS shown on fig. 5 was an object of research. The structure had to maintain its bearing capacity for at least 2.5 years.

The optimization problem was solved over 100 epochs, or until full convergence of the population is achieved. At the same time, according to the algorithm of the flexible tolerance method the function calculation error reduced after every 20 epochs. In this case, the maximum restrictions function calculation error on in the initial search iterations was 5%, the minimum error on the last iterations was 1%.



In the implemented genetic algorithm de Vries' evolution model characterized by high probability of mutation is used. At the same time the possibility of mutation has been provided only for even genes that determine section sizes. Single-point crossover operator and tournament selection were used in the algorithm.

Fig. 6 shows variation of current stresses and critical buckling stresses in the compressed rods in time. Optimum design is characterized by the fact that durability of all elements is in the vicinity of a predetermined value and does not exceed it by more than 7.5%.



Fig. 6 - Variation of stresses in elements of optimal structure

Optimal sections of HRS elements obtained by solving the problem are presented in Table 1.

The volume of the resulting structure was 34604.42 cm³.

Nº	Section type	Standard size	t, years
1	Angle bar	125x125x9	2.52948
2	Angle bar	125x80x8	2.56995
3	Angle bar	125x80x8	2.57016
4	Angle bar	140x140x10	2.69019
5	Angle bar	125x125x9	2.55430

Table 1 - Optimal sections of rods in five-element truss

The number of calls to the procedure of finite element method in the search for optimal solution was used as criterion of developed algorithm efficiency. Table 2 shows the results of testing the algorithm efficiency.

Algorithm	Number of calls to FEM procedure
GA	3 375 508
NN+GA	1 618 484
FEM+NN+GA	528 043

Table 2 – Analysis of algorithm efficiency

Lines of Table 2 show the number of FEM problem solutions:

- using fixed step providing error now exceeding ε_{min} on all set of varied parameters for CF calculation;

- using neural network module to determine the distance between the nodes of time grid based on information about current structure parameters, aggressive environment parameters and permissible error ε_{min} ;

- using neural network module together with flexible tolerance method.

Table 3 shows the results of optimization problem solution for fifteenelement truss in two different statements.

Statement 1		Statement 2			
Nº	Туре	Standard type	Nº	Туре	Standard type
1	Angle bar-2	160x100x10	1	Angle bar-2	180x110x12
2	Angle bar-1	125x125x9	2	Angle bar-2	180x110x12
3	Angle bar-1	100x100x8	3	Angle bar-2	180x110x12
4	Angle bar-2	160x100x10	4	Angle bar-1	140x140x10
5	Angle bar-2	160x100x10	5	Angle bar-1	140x140x10
6	Angle bar-2	100x63x8	6	Angle bar-1	140x140x10
7	Angle bar-2	90x56x6	7	Angle bar-2	100x63x8
8	Angle bar-2	90x56x6	8	Angle bar-2	100x63x8
9	Angle bar-2	100x63x8	9	Angle bar-2	100x63x8
10	Angle bar-1	100x100x8	10	Angle bar-1	110x110x8
11	Angle bar-1	100x100x8	11	Angle bar-1	110x110x8
12	Angle bar-1	110x110x8	12	Angle bar-1	110x110x8
13	Angle bar-2	90x56x6	13	Angle bar-1	110x110x8
14	Angle bar-2	100x63x8	14	Angle bar-1	110x110x8
15	Angle bar-2	110x70x8	15	Angle bar-1	110x110x8

Table 3 - Optimal sections of rods in fifteen-element truss

In the first statement the sections of all rods were varied, in the second one the sections of rods were combined into four groups. Chromosomes consisted of 30 and 8 genes, respectively.

The volumes of optimal structures were 54908.86 and 72454.71 cm³.

Conclusions. The use of genetic algorithms for solving problems of corrosive hinged-rod structures discrete optimization was proposed and justified. The neural network algorithm of constraint function calculation error control designed by authors allowed to use the concept of flexible tolerance method and substantially reduces the computational cost of the search for optimal solutions. With optimization algorithm new solutions of optimization problems, representing the scientific and practical interest, were obtained.

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ИСПОЛЬЗОВАНИЕ ГЕНЕТИЧЕСКИХ АЛГОРИТМОВ ПРИ РЕШЕНИИ ЗАДАЧ ОПТИМАЛЬНОГО ПРОЕКТИРОВАНИЯ КОРРОДИРУЮЩИХ ШАРНИРНО-СТЕРЖНЕВЫХ КОНСТРУКЦИЙ

Предложен новый эффективный алгоритм решения задач оптимизации корродирующих шарнирно-стержневых конструкций, предполагающий получение решения с заданной точностью. Алгоритм построен на использовании стратегии скользящего допуска совместно с целочисленным генетическим алгоритмом. Для обеспечения требуемой точности вычисления функций ограничений в окрестности экстремума использована нейронная сеть, аппроксимирующая зависимость между параметрами конструкции, допустимой погрешностью и параметрами вычислительных процедур. Поиск решения осуществлён на дискретном неметрическом пространстве варьируемых параметров.

Ключевые слова: оптимальное проектирование, дискретная оптимизация, генетический алгоритм, метод скользящего допуска, нейронная сеть, коррозионный износ.

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ВИКОРИСТАННЯ ГЕНЕТИЧНИХ АЛГОРИТМІВ ПРИ РОЗВ'ЯЗАННІ ЗАДАЧ ОПТИМАЛЬНОГО ПРОЕКТУВАННЯ КОРОДУЮЧИХ ШАРНІРНО-СТЕРЖНЕВИХ КОНСТРУКЦІЙ

Запропоновано новий ефективний алгоритм розв'язання задач оптимізації кородуючих шарнірно-стержневих конструкцій, що передбачає отримання розв'язку із заданою точністю. Алгоритм заснований на використанні стратегії ковзного допуску сумісно з цілочисельним генетичним алгоритмом. Для забезпечення необхідної точності обчислення функцій обмежень в околі екстремуму використовується нейронна мережа, що апроксимує залежність між параметрами конструкції, допустимою похибкою і параметрами обчислювальних процедур. Пошук розв'язку здійснюється на дискретному неметричному просторі варійованих параметрів.

Ключові слова: оптимальне проектування, дискретна оптимізація, генетичний алгоритм, метод ковзного допуску, нейронна мережа, корозійний знос.

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