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GEOMETRİCAL NONLİNEAR VİBRATİONS OF A MOVİNG FLUİD-CONTACTİNG FUNCTİONALLY GRADED CYLİNDRİCAL SHELL

İn the present paper we study geometrical nonlinear vibrations of a moving fluid-contacting functionally-graded cylindrical shell. Using the Hamilton-Ostrogradsky variational principle, the finding of vibration frequencies of the considered system is reduced to the solution of the system of differential equations and is realized by the numerical method.

Keywords: *functionally-graded material, cylindrical shell, geometrical nonlinear vibration, Ostrogradsky – Hamilton action.*

Introduction. Functionally-graded materials are widely used in manufacturing numerous objects, in particular aerocosmic objects. The objects made of these materials are used in contact with high temperature media. Therefore, application of these constructions with regard to liquid medium is of great importance. The problem of vibrations and stability of shells made of functionally-graded materials ignoring the influence of liquid medium found their solutions in the [1 – 6]. The [7] deals with geometrical nonlinear vibrations of a rectangular plate made of functionally-graded material. The [10] was devoted to study of linear vibrations of a functionally-gradient shell.

Problem statement. Let us consider a fluid-contacting cylindrical shell made of mixture of ceramics and metal. As in the [10] it is assumed that the fraction of ceramic material in the total volume changes by the law

$$V = \left(\frac{2z + h}{2h} \right)^k . \quad (1)$$

Here h is the shell's thickness, k is the power index of the fraction of the ceramical material in the volume and $0 \le k \le \infty$. If $k = 0$, the structure of the shell consists only of ceramics, if $k = \infty$, will consist of metal. We will assume that mechanical characteristics of materials (the Young's modulus, density, etc.) change by the following law [4, 5]:

$$P_j = P_0 \left(P_{-1} T^{-1} + 1 + P_1 T + P_2 T^2 + P_3 T^3 \right),$$

here, the coefficients $P_0, P_{-1}, P_1, P_2, P_3$ are specifically defined for each material. The values of these coefficients for some materials were given in the [1, 4, 5]. The mechanical properties of the mixture consisting of two parts are determined by the following formula

$$P(z,T)=\left(P_c(T)-P_m(T)\right)\left(\frac{z}{h}+\frac{1}{2}\right)^k+P_m(T). \quad (2)$$

By formula (2) we can calculate the elasticity modulus E of the mixture, the Poisson's ratio ν and density ρ. Here $P_c(T)$ and $P_m(T)$ are the characteristics of ceramics and metal.

The system of motion equations of medium-contacting composite cylindrical shell is found from the stationarity condition of Hamilton – Ostrogradsky action

$$\delta W=0. \quad (3)$$

Here $W=\int_{t'}^{t''}Ldt$ is the Hamilton's action, $L=K-\Pi$ is the Lagrange function, t' and t'' are the given any moments of time.

The potential and kinetic energies of the system are as follows:

$$U=\frac{1}{2}\iint_{\Omega}\left(N_{11}\varepsilon_{11}+N_{22}\varepsilon_{22}+N_{12}\varepsilon_{12}+M_{11}\chi_{11}+M_{22}\chi_{22}+M_{12}\chi_{12}\right)d\Omega+$$

$$+\frac{1}{2}\iint_{\Omega}\left(\varrho_x\left(w_{,x}+\psi_x\right)+\varrho_y\left(w_{,y}+\psi_y\right)\right)d\Omega;$$

$$T=\frac{1}{2}\iint_{\Omega}\left[I_0\left(u_{,t}^2+\nu_{,t}^2+w_{,t}^2\right)+2I_1\left(u_{,t}\,\psi_{x\,,\,t}+\nu_{,t}\,\psi_{y\,,\,t}\right)+I_2\left(\psi_{x\,,t}^2++\psi_{y\,,t}^2\right)\right]dxdy; \quad (4)$$

$$I_0=\left(\rho_m+\frac{\rho_c-\rho_m}{k+1}\right)h; \quad I_1=\int_{-\frac{h}{2}}^{\frac{h}{2}}\rho(z)zdz=\frac{(\rho_c-\rho_m)k}{2(k+1)(k+2)}h^2;$$

$$I_2=\int_{-\frac{h}{2}}^{\frac{h}{2}}\rho(z)z^2dz=\left(\frac{\rho_m}{12}+(\rho_c-\rho_m)\left(\frac{1}{k+3}-\frac{1}{k+2}+\frac{1}{4(k+4)}\right)\right)h^3;$$

$$A_0=-\iint_{\Omega}pwdxdy.$$

In expressions (4)

$$\varepsilon_{ij}=\varepsilon_{ij}^L+\varepsilon_{ij}^{ND} \qquad (i\,,j=1,2);$$

$$\varepsilon_{11}^{L} = u,_x + \frac{w}{R_x}; \quad \varepsilon_{22}^{L} = \nu,_y + \frac{w}{R_y}; \quad \varepsilon_{12}^{L} = u,_y + \nu,_x; \quad \varepsilon_{11}^{ND} = \frac{1}{2} w,_x^2;$$

$$\varepsilon_{22}^{ND} = \frac{1}{2} w,_y^2; \quad \varepsilon_{12}^{ND} = w,_x + w,_y; \quad \varepsilon_{13} = w,_x + \psi_x; \quad \varepsilon_{23} = w,_y + \psi_y;$$

$$\chi_{11} = \psi_{x\,,\,x}; \quad \chi_{22} = \psi_{y\,,y}; \quad \chi_{12} = \psi_{x\,,\,y} + \psi_{y,\,x};$$

$$N = \{N_{11}; N_{22}; N_{12}\}^T = \frac{1}{1-\nu^2}[C](E_1\varepsilon + E_2\chi);$$

$$M = \{M_{11}; M_{22}; M_{12}\}^T = \frac{1}{1-\nu^2}[C](E_2\varepsilon + E_3\chi);$$

$$[C] = \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}; \quad E_1 = \left(E_m + \frac{E_c - E_m}{k+1}\right)h; \quad E_2 = \frac{(E_c - E_m)kh^2}{2(k+1)(k+2)};$$

$$E_3 = \left(\frac{E_m}{12} + (E_c - E_m)\left(\frac{1}{k+3} - \frac{1}{k+2} + \frac{1}{4(k+4)}\right)\right)h^3; \quad \rho = \left(\rho_m + \frac{\rho_c - \rho_m}{k+1}\right)h.$$

Cutting forces Q_x and Q_y are determined from the expressions $Q_x = K_S^2 A_{33}\varepsilon_{13}$, $Q_y = K_S^2 A_{33}\varepsilon_{23}$. The coefficient K_S^2 is called the regularizing coefficient. In the calculation process we accept $K_S^2 = 5/6$. A_0 is the inverse sign work done by pressure force p on the shell in displacement w of the shell. Pressure force p is determined from the motion equation of ideal fluid moving with velocity U

$$\Delta\tilde{\phi} - \frac{1}{a_0^2}\left(\frac{\partial^2\tilde{\phi}}{\partial t^2} + 2U\frac{\partial^2\tilde{\phi}}{R\partial\xi\partial t} + U^2\frac{\partial^2\tilde{\phi}}{R^2\partial\xi^2}\right) = 0. \tag{5}$$

İn shell-fluid contact, in radial direction the equality of velocity and pressure is satisfied:

$$\vartheta_r\big|_{r=R} = \frac{\partial\tilde{\phi}}{\partial r}\bigg|_{r=R} = -\left(\omega_0\frac{\partial w}{\partial t^1} + U\frac{\partial w}{R\partial\xi}\right); \tag{6}$$

$$q_z = -p\big|_{r=R}. \tag{7}$$

Look for the $\tilde{\phi}$ – potetial of perturbations in the form

$$\tilde{\phi}(\xi, r, \theta, t_1) = f(r)\cos n\theta \sin kx \sin \omega t. \tag{8}$$

Here n, k are wave numbers in the direction of coordinate axes, ω is an unknown frequency, $f(r)$ is an unknown function. Using expressions (6), (7) and (8), we get:

$$\tilde{\phi} = -\Phi_{\alpha n}\left(w_0 \frac{\partial w}{\partial t^1} + U \frac{\partial w}{R\partial\xi} \right);$$

$$p = \Phi_{\alpha n}\rho_m \left(\omega_0^2 \frac{\partial^2 w}{\partial t_1^2} + 2U \frac{\partial^2 w}{R\partial\xi\partial t_1} + U^2 \frac{\partial^2 w}{R^2\partial\xi^2} \right). \qquad (9)$$

Here

$$\Phi_{\alpha n} = \begin{cases} \dfrac{I_n(\beta r)}{I_n(\beta r)}, & M_1 < 1; \\ \dfrac{J_n(\beta_1 r)}{J_n(\beta_1 r)}, & M_1 > 1; \\ \dfrac{R^m}{nr^{n-1}}, & M = 1. \end{cases} \qquad (10)$$

In (10) $M_1 = \dfrac{U + \omega / m}{a_0}$, $\beta^2 = R^{-2}\left(1 - M_1^2\right)\chi^2$, $\beta_1^2 = R^{-2}\left(M_1^2 - 1\right)\chi^2$, J_n is the first kind n-th order the Bessel's function, $J_n^{'}$ is its derivative with respect to variable r, I_n is the n-th order modified the Bessel's function, $I_n^{'}$ is its derivative with respect to variable r, a_0 is the rate of sound propagation in fluid.

We shall accept that the cylindrical shell was hingely supported, i.e. in the sections $x = 0$ and $x = L$ the following conditions are satisfied:

$$u = 0, \quad w = 0, \quad T_1 = 0, \quad M_1 = 0. \qquad (11)$$

Here T_1, M_1 is the force and moment acting on cross section of the cylindrical shell.

Using the stationarity condition of the Ostrogodsky – Hamilton action, if we realize the variation process in the equality $\delta W = 0$ and take into account arbitrariness and independence of variations $\delta u, \ \delta v, \ \delta w$, we get a frequency equation of a cylindrical shell dynamically contacting with fluid.

Thus, the solution of problem of geometrical nonlinear vibrations of a cylindrical shell dynamically contacting with fluid is reduced to integration of total energy of a construction consisting of a cylindrical shell with fluid-filled inner domain.

Problem solution. We look for displacements of the shell as follows:

$$u = u_0(t)\cos\chi\xi\cos n\theta;$$
$$\vartheta = \vartheta_0(t)\sin\chi\xi\sin n\theta; \qquad (12)$$
$$w = w_0(t)\sin\chi\xi\cos n\theta.$$

Here u_0, ϑ_0, w_0 are unknown functions, χ, n are generator and wave number of the cylindrical shell in peripheral direction, $\xi = x/L$.

Allowing for (8), substituting expressions (12) in (4), from the stationarity condition of the Hamilton-Ostrogradsky action, with respect to the unknown functions u_0, ϑ_0, w_0 we get the system of second order differential equations. As the system is of bulky form, we do not cite it here. İf we look for the solution of this system in a first approximation in the form $u_0 = u_1 \sin\omega t$, $\vartheta_0 = \vartheta_1 \sin\omega t$, $w_0 = w_1 \sin\omega t$, we find the dependence between the sought-for frequency ω and u_1, ϑ_1, w_1. By means of this dependence we can construct the skeleton curve. In calculations for the parameters the following estimations [7] were taken:

$$\frac{E_m}{E_c} = 70380; \quad \nu_m = \nu_c = 0,3; \quad \frac{\rho_m}{\rho_c} = \frac{2707}{3800}; \quad \frac{\rho_0}{\rho} = 0,115; \quad a_0 = 1350\ m/sek.$$

The dependence of the ratio of frequency of nonlinear vibrations on the curvature of the shell is depicted in Fig. 1. The dependence of linear frequency parameter $\Omega_x = \omega h\sqrt{\rho_c / E_c}$ on the power index k of fraction of the ceramic material in the volume was given in Fig. 2.

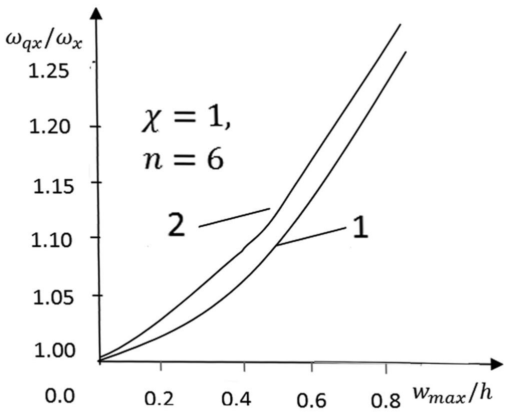

Fig. 1 **– Dependence of frequencies of nonlinear vibrations of the system on shell's curvature**

As is seen from Fig. 1, as the shell's curvature increases, the frequencies of nonlinear vibrations also increase. As the power index k of the fraction of ceramic material in the volume increases, as is seen from Fig. 2, the frequency of linear vibrations decreases.

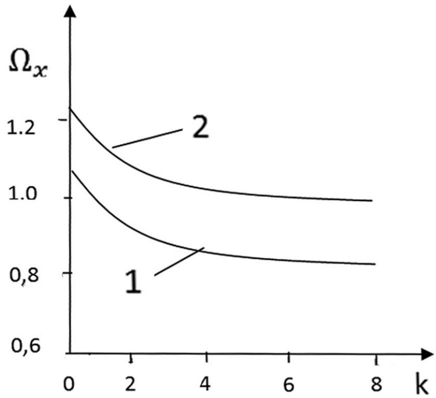

Fig. 2 **– Dependence of frequencies of linear vibrations of the system on power index**

İn both graphs, curve 1 corresponds to account of influence of fluid on vibration process, curve 2 to no fluid cases. As is seen, account of the influence of fluid reduces to decrease of frequencies of natural vibrations of the system.

REFERENCES

1. ***Alijani F.*** Nonlinear vibrations of functionally graded doubly curved shallow shells / F. Alijani, M. Amabili., K. Karagiozis, F. Bakhrtiari-Nejad // Journal of Sound and Vibrartion. – 2011. – Vol. 330. – P. 1432–1454.

2. ***Reddy J. N.*** Vibration of functionally graded cylindrical shells / J. N. Reddy, C.T. Loy, K. Y. Lam // Int J Mech Sci. – 1999. – Vol. 41. – P. 309–324.

3. ***Matsunaga H.*** Free vibration and stability of functionally graded shallow shells according to a 2D higher-order deformation theory / H. Matsunaga // Composite Structures. – 2008. – Vol. 84. – P. 132–146.

4. ***Reddy J. N.*** Analysis of functionally graded plates / J. N. Reddy // International Journal for numerical methods in engineering. – 2000. – Vol. 47. – P. 663–684.

5. ***Shen H. S.*** Functionally Graded Materials of Plates Shells / H. S. Shen. – Florida: CPC Press, 2009. – 266 p.

6. ***Chorfi S. M.*** Non-linear free vibration of a functionally graded doubly-curved shallow shell of elliptical plan-form / S. M Chorfi, A. Houmat // Composite Structures. – 2010. – Vol. 92. – P. 2573–2581.

7. ***Kurpa L. V.*** Studying geometrically nonlinear vibrations of functionally-graded shallow shells with complex plane-form / L. V. Kurpa, T. V. Shmatko // Vestnik Zaporozhskogo natsionalnogo universiteta – 2015. – No 1. – P. 89–97. (in Russian).

8. ***Kurpa L. V.*** Nonlinear free vibrations of sandwich shells of symmetric structure with complex plan-form / L. V. Kurpa // Math. met. and phis.-mech. filds. – 2008. – Vol. 51. – No 2. – P. 75–85. (in Ukrainion).

9. ***Kurpa L. V.*** The R –function method for solving linear problems of bending and vibrations of hollow shells / L. V. Kurpa // Kharkov: NTU «KhPU», 2009. – 408 p.

10. ***Kurpa L. V.*** Free vibrations of functionally graded shells with complex form-plane / L. V. Kurpa // Teoret. i prikl. mekhanika. – 2014. – Iss. 8 (54) – P. 77-85. (in Russian).

11. ***Rvachev V. R.*** Theory of R-function and some applications / V. R. Rvachev. – Kiev: Naukova Dumka, 1982. – 552 p. (in Russian).

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ГЕОМЕТРИЧНО НЕЛІНІЙНІ КОЛИВАННЯ ФУНКЦІОНАЛЬНО-ГРАДІЄНТНОЇ ЦИЛІНДРИЧНОЇ ОБОЛОНКИ, ЯКА КОНТАКТУЄ З РУХОМОЮ РІДИНОЮ

Вивчаються геометричні нелінійні коливання функціонально-градієнтної циліндричної оболонки, яка контактує з рухомою рідиною. За допомогою варіаційного принципу Гамільтона – Остроградського, знаходження частот коливань розглянутої системи зводиться до розв'язування системи диференціальних рівнянь чиселовим методом.

Ключові слова: *функціонально-градуйований матеріал, циліндрична оболонка, геометрично нелінійні коливання, принцип Остроградського – Гамільтона.*

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ГЕОМЕТРИЧЕСКИ НЕЛИНЕЙНОЕ КОЛЕБАНИЕ ФУНКЦИОНАЛЬНО-ГРАДИЕНТНОЙ ЦИЛИНДРИЧЕСКОЙ ОБОЛОЧКИ, КОНТАКТИРУЮЩЕЙ С ПРОТЕКАЮЩЕЙ ЖИДКОСТЬЮ

Изучены геометрически нелинейные колебания функционально-градиентной цилиндрической оболочки, контактирующей с протекающий жидкостью. Используя вариационный принцип Гамильтона – Остроградского, нахождение частот колебаний системы сводится к решению системы дифференциальных уравнений численным методом.

Ключевые слова: *функционально-градуированный материал, цилиндрическая оболочка, геометрические нелинейные колебания, принцип Остроградского – Гамильтона.*

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