ISSN 2079–1836
2017, вип. 26

UDC 539.3

K. Panin PhD (Phis.-Math.)

THIN-WALLED TUBE DRAWING

The problem of tube drawing from a hardening material through a rigid axisymmetric matrix is considered. To describe the behavior of the tube material, the equations of the theory of microdeformation are used. Mathematically, the problem is reduced to the Cauchy problem, which is solved by the Euler method with intermediate iterations. The results of the calculation of the stress variation along the tube length are given. It is shown that the loading path of the tube element in the stress space has considerable curvature.

Keywords: *tube drawing, theory of microdeformation, complex loading.*

Introduction. The solution of the problem of tube drawing was first proposed by Swift [1], who considered an ideal-plastic material, obeying the equations of flow theory. Grudnev in [2] investigated the influence of material hardening on the process of drawing. In [3], a solution was obtained for a tube of hardening material, the deformation of which is described by the relations of Khristianovich's theory.

When studying the process of tube drawing through a rigid matrix with a smoothly changing profile, we shall neglect the friction between the tube and the matrix. Elements of the tube as it moves along the matrix pass through a sequence of plane stressed states. The trajectory of the loading of tube elements is completely determined by the profile of the matrix. The problem is interesting in that the trajectory of the loading of a tube element in the stress space has considerable curvature and plastic deformation is accompanied by considerable partial unloading, so calculations by the classical formulas of the theory of flow turn out to be inaccurate.

Formulation of the problem. The drawing scheme of a thin-walled tube 1 through an axisymmetric rigid matrix 2 is shown in Fig. 1. The movement of the tube along the matrix occurs under the action of a force applied to the outlet edge of the tube. We assume that the drawing process is steady (this is the case for long tubes) and quasi-static. In such a process, each initially unloaded tube element, when moving along the matrix, successively passes through the same deformation stages.

We choose the system of curvilinear coordinates xyz, directing the x axis along the generator of the matrix, the y axis in the circumferential direction, and the z axis along the normal to its surface. We denote the radii of curvatures of the surface of the matrix in normal sections passing through the x and y axes, respectively, R_x and R_y. The tube is assumed to be thin, so that $h/D << 1$, where D is the diameter of the cross-section of the matrix normal to the axis, h is the tube thickness. In this approximation, from the condition of equilibrium of the forces applied to the tube element (Fig. 2) in the projection onto the z axis, we have for the pressure p acting on the side of the matrix on the tube

$$p = h\left(\sigma_x / R_x + \sigma_y / R_y\right),$$

where σ_x, σ_y are the normal stresses acting on the areas perpendicular to the x and y axes. For a smoothly varying profile of the matrix $R_y \cong D/2$, $\left(\sigma_x/\sigma_y\right)\left(R_y/R_z\right) << 1$, so $p = h\sigma_y/R_y$.

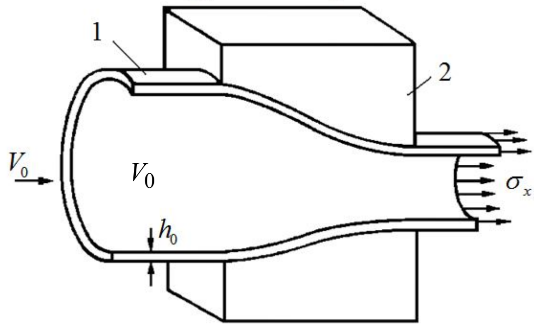

Fig.1 – **Scheme of tube drawing**

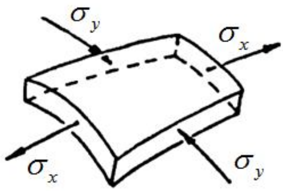

Fig.2. **– Tube element**

All quantities characterizing the state of the tube depend only on the coordinate x. The shape of the matrix is given in the form of the dependence of diameter D on x.

The equation of equilibrium in the projection on the axis of the tube

$$\frac{d}{dx}\left(\sigma_x R h\right) - h\sigma_y \frac{dR}{dx} = 0. \quad (1)$$

The geometric relationships connecting the deformations of the material with the kinematic and geometric characteristics of the tube have the form

$$\frac{d\varepsilon_x}{dx} = \frac{1}{u}\frac{du}{dx},$$

$$\frac{d\varepsilon_z}{dx} = \frac{1}{h}\frac{dh}{dx}, \qquad (2)$$

$$\frac{d\varepsilon_y}{dx} = \frac{1}{R}\frac{dR}{dx}$$

Here V is the velocity of the material in the section under consideration, h is the tube thickness, R is the channel width of the tool, which is a known function of x.

We believe that the material of which the tube is made is incompressible

$$d\varepsilon_x + d\varepsilon_y + d\varepsilon_z = 0 \qquad (3)$$

Let the deformation of the tube beyond the elastic limit be described by the relations of the theory of microdeformation [4], which for the problem under consideration is conveniently written in the following form:

$$\begin{cases} d\sigma_x = a_{11} d\varepsilon_x + a_{12} d\varepsilon_y, \\ d\sigma_y = a_{21} d\varepsilon_x + a_{22} d\varepsilon_y. \end{cases}, \qquad (4)$$

In this case

$$S_1 = \sqrt{\frac{2}{3}}\sigma_x - \sqrt{\frac{1}{6}}\sigma_y, \qquad S_2 = \sqrt{\frac{1}{2}}\sigma_y,$$

$$Э_1 = \sqrt{\frac{3}{2}}\varepsilon_x, \qquad Э_2 = \sqrt{\frac{3}{2}}\varepsilon_x - \sqrt{2}\varepsilon_y,$$

and coefficients a_{ij} in (4) have the form

$$a_{11} = \left(3A_{11} + \sqrt{3}A_{12} + A_{22}\right)/2,$$

$$a_{12} = a_{21} = \left(\sqrt{3} + 1\right)A_{12}, \qquad (5)$$

$$a_{22} = 2A_{22}.$$

In the stage of elastic deformation

$$A_{ij} = 2G\delta_{ij}, \qquad (6)$$

but in the stage of elasto-plastic deformation

$$A_{ij} = 2G\left(\delta_{ij} - G_{ij}^{(1)}\right),$$

$$G_{ij}^{(1)} = \left(\delta_{ik} + B_2 G_{ik}^{(2)}\right)^{-1} G_{kj}^{(2)}, \tag{7}$$

$$G_{kj}^{(2)} = \frac{1}{B_1}\left(G_{kj} - \frac{\mu_1}{1+\mu_1\Omega} F_k F_j\right).$$

Here

$$\mu = \frac{B_3}{B_1},$$

$$G_{11} = 2\pi \int_0^{\alpha_1} \cos^2\theta_1 \sin^3\theta_1 \left(\alpha_2(\theta_1) - \frac{1}{2}\sin 2\alpha_2(\theta_1)\right) d\theta_1,$$

$$G_{22} = \frac{1}{2}\pi \int_0^{\alpha_1} \sin^5\theta_1 \left(\alpha_2(\theta_1) - \frac{1}{2}\sin 4\alpha_2(\theta_1)\right) d\theta_1,$$

$$G_{12} = G_{21} = \pi \int_0^{\alpha_1} \cos\theta_1 \sin^4\theta_1 \left(\alpha_2(\theta_1) - \frac{1}{3}\sin 3\alpha_2(\theta_1)\right) d\theta_1, \tag{8}$$

$$F_1 = 2\pi \int_0^{\alpha_1} \cos\theta_1 \sin^3\theta_1 \left(\alpha_2(\theta_1) - \frac{1}{2}\sin 2\alpha_2(\theta_1)\right) d\theta_1,$$

$$F_2 = \pi \int_0^{\alpha_1} \sin^4\theta_1 \left(\sin\alpha_2(\theta_1) + \frac{1}{3}\sin 3\alpha_2(\theta_1)\right) d\theta_1,$$

$$\Omega = 2\pi \int_0^{\alpha_1} \sin^3\theta_1 \left(\alpha_2(\theta_1) - \frac{1}{2}\sin 2\alpha_2(\theta_1)\right) d\theta_1.$$

The parameter α_1 and the function $\alpha_2(\theta_1)$, which determine the direction of the active microplastic deformation, are found from the relations

$$\cos\alpha_1 = \frac{\varepsilon_s + \chi}{\sqrt{r_1^2 + r_2^2}}$$

$$\sin\theta_1 \cos\alpha_2 dr_2 + \cos\theta_1 dr_1 = d\chi,$$

$$d\chi = (F_1 dr_1 + F_2 dr_2)\mu, \tag{9}$$

$$dr_i = dЭ_i - B_2 dЭ_i^P$$

$$0 \le \alpha_2 \le \pi.$$

The initial conditions of the problem have the form

$$\sigma_x(0) = \sigma_y(0) = 0,$$

$$\varepsilon_x(0) = \varepsilon_y(0) = \varepsilon_z(0) = 0\,, \tag{10}$$

$$V(0) = V_0,\, h(0) = h_0.$$

The problem reduces to determining the components of stress and deformation tensors, the quantities h and V along the length of the matrix using equations (1) – (3) and the defining relations (4) – (9) under the initial conditions (10).

When solving the problem, it is convenient to take instead of the coordinate x the independent variable

$$s = \varepsilon_y = \ln\left(R/R_0\right).$$

Taking this into account, the problem of tube drawing in the mathematical plane reduces to the Cauchy problem for a system of differential equations

$$\begin{cases} \dfrac{d\varepsilon_x}{ds} = \dfrac{\sigma_y - a_{12}}{a_{11} - \sigma_x} \\ \dfrac{d\varepsilon_z}{ds} = -1 - \dfrac{\sigma_y - a_{12}}{a_{11} - \sigma_x} \\ \dfrac{d\sigma_x}{ds} = a_{11}\dfrac{\sigma_y - a_{12}}{a_{11} - \sigma_x} + a_{12} \\ \dfrac{d\sigma_y}{ds} = a_{12}\dfrac{\sigma_y - a_{12}}{a_{11} - \sigma_x} + a_{22} \end{cases} \tag{11}$$

under the following initial conditions

$$\varepsilon_x(0) = \varepsilon_y(0) = 0\,,\ \sigma_x(0) = \sigma_y(0) = 0 \tag{12}$$

Solution method. Since the deformations and stresses smoothly changing with increasing s, it is sufficient to use the Euler method with intermediate iterations to solve the Cauchy problem. At each step of the Euler method, the region of directions of the active microplastic deformation Ω, determined with the help of expressions (9), was found using an iterative procedure analogous to the simple iteration method. As a first approximation, at each step we used the value of Ω found in the previous step. Taking into account the nature of the change in the unknown quantities, 3 to 5 iterations were required to determine Ω.

Numerical realization and conclusion. When carrying out numerical calculations, consider tube made of steel St45. The constants of the theory of microdeformation, for this steel are the following:

$$E = 2\cdot 10^5\,МПа,\ \nu = 0.3,\ \varepsilon_S = 0.3\%,\ B_1 = 2.79,\ B_2 = B_3 = 0.$$

In Fig. 3 shows the calculation of the change in stress along the length of the tube, obtained in the framework of the theory of microdeformations (solid lines). As the matrix shrinks, the axial stress increases, and the magnitude of the circumferential stress first increases and then decreases. The same figure shows the results of similar calculations carried out in the framework of the Khristianovich's theory [3] (dotted lines). Their comparison indicates that there is a satisfactory coincidence of the results.

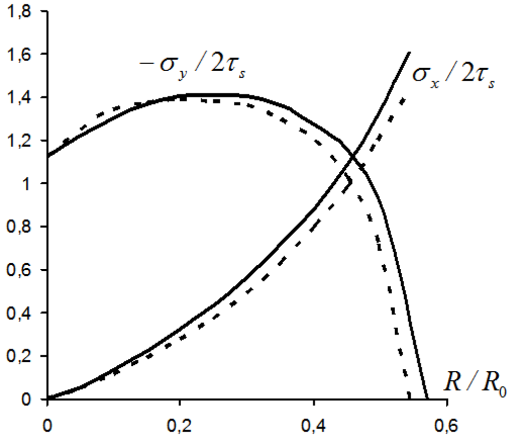

Fig. 3 – **Change in stress along the length of the tube**

The trajectory of the loading of the tube element in the stress space is shown in Fig. 4.

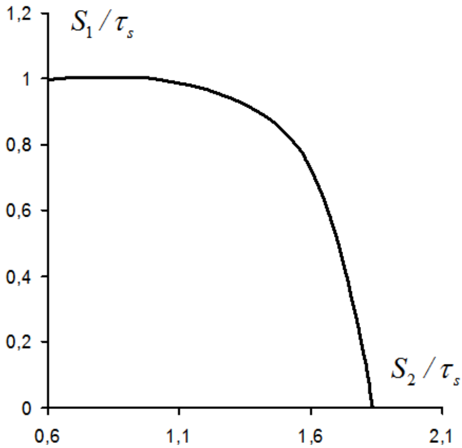

Fig. 4 – **Tube element loading trajectory**

It can be seen from the figure that the loading is complex. The solution showed that the deformation of the tube was accompanied by partial unloading, but occurred practically without repeated loading. Occurrence of partial discharges not taken into account by the simplest theories can lead to errors in the calculations within the framework of these theories.

REFERENCES

1. ***Swift H. W.*** Stresses and strains in tube-drawing/ H. W. Swift // Phil.mag. – 1949. – Vol. 40. – No 7. – P.75–95.
2. ***Grudnev M. D.*** Consideration of material hardening during pipe drawing / M. D. Grudnev // Strength and Plasticity. – Moscow. – 1971, P.47 – 63 (In Russian).
3. ***Mokhel A. I***. On the plastic deformation of hardening metals and alloys. Analysis of experimental data and solution of elastoplastic problems / A. I. Mohel, R. L. Salganik, S. A. Khristianovich // Izv. Academy of Sciences of the USSR. – MTT. – 1983. – No 5. – P.81 (In Russian).
4. ***Kadashevich, Yu. I.*** Theory of plasticity taking into account microstresses/ Yu. I. Kadashevich, Yu. A. Chernyakov // Advanced in Mechanics. – 1992. – Vol. 15. No 3 – 4. – P. 3 – 39.

УДК 539.3

К. В. Панін, канд. физ.-мат. наук

ВОЛОЧІННЯ ТОНКОСТІННОЇ ТРУБИ

Розглянуто задачу про волочіння труби з матеріалу, що зміцнюється, крізь жорстку осесиметричну матрицю. Для опису поведінки матеріалу труби використовуються рівняння теорії мікродеформації. В математичному плані поставлена задача зводиться до задачі Коші, яка розв'язується за допомогою методу Ейлера з проміжними ітераціями. Наведено результати розрахунку зміни напружень по довжині труби. Показано, що траєкторія навантаження елемента труби в просторі напружень має значну кривизну.

Ключові слова: *волочіння труби, теорія мікродеформації, складне навантаження.*

УДК 539.3

К. В. Панин, канд. физ.-мат. наук

ВОЛОЧЕНИЕ ТОНКОСТЕННОЙ ТРУБЫ

Рассмотрена задача о волочении трубы из упрочняющегося материала сквозь жесткую осесимметричную матрицу. Для описания поведения материала трубы используются уравнения теории микродеформации. В математическом плане поставленная задача сводится к задаче Коши, которая решается методом Эйлера с промежуточными итерациями. Приведены результаты расчета изменения напряжений по длине трубы. Показано, что траектория нагружения элемента трубы в пространстве напряжений имеет значительную кривизну.

Ключевые слова: ***волочение трубы, теория микродеформации, сложное нагружение.***

Dnipropetrovsk National University
named after Oles Honchar
Dnipro, Ukraine

Received in the editorial board 19.03.2017