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NONLINEAR OSCILLATIONS OF DIESEL ENGINE POWER MINIPLANT

The method for calculation of resonance nonlinear torsional vibrations in diesel engine power plant is suggested. Nonlinear torsional vibrations of the generating plant, which consists of diesel engine 3TD-4 and traction alternator, are investigated. As follows from the results of the calculations, a torsional resonance between braking moment of the generator and the engine transmission can occur. Therefore, the resonance vibrations must be tuned out.

Keywords: *diesel engine power miniplant, braking electromagnetic torque, nonlinear torsional vibrations, backbone curve*

Introduction. Reutilization of machinery units is an effective way to increase operation life of complex mechanisms in order to decrease financial expenditure on the system manufacturing. However, if the assemblies are designed using the old units, unexplored interaction between components can take place. Such interactions may lead to the system failure.

In the present paper it is discussed the use of the diesel engine 3TD-4, widely used in armored machines, to generate the mechanical energy for a diesel engine power plant. Traction generator GS 530AMU3 is used as the generating equipment. The interaction between the engine power transmission and the generating equipment is considered in this paper. The main nonlinear resonance between the engine power transmission torsional vibrations and the higher harmonics of the generator braking torque is investigated.

Outline of generating plant and equations of nonlinear torsional vibrations. The basis of the generating plant is three-cylinder diesel engine 3TD-4. The angular velocity of the crankshaft belongs to the range 900÷2800 min^{-1}. In order to produce alternating current with frequency 50 Hz, the frequency of the crankshaft rotation must be 1000 min^{-1}. The kinematic diagram of the engine 3TD-4 with electric generator is shown on Fig.1 [4]. The following notations are used in Fig.1: 1, 2 are hub and flywheel of inlet crankshaft damper; 3 is inlet crankshaft; 4 – 8 are gears of the main transmission; 9 is outlet crankshaft; 10 is elastic clutch of outlet crankshaft; 11 is spring of compressor drive; 12 is elastic clutch of the compressor drive; 13 is friction clutch; 14 is compressor rotor; 15 is spring of the turbine drive; 16 is turbine rotor; 17 is bolt coupling; 18 is electric generator. The nonlinear elastic clutch, which is described by a piecewise-linear spring, is mounted on the outlet crankshaft.

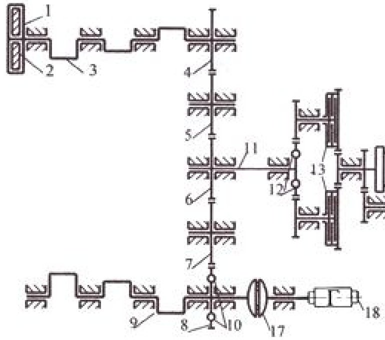


Fig. 1. – **Kinematic diagram of diesel engine power miniplant**

The generating equipment, which is coupled to the diesel engine, is shown on Fig.2. The following notations are used on Fig.2: 1, 4, 17 are coverage; 2 is bearing; 3 – thrust collar; 5 – rings; 6 – lubricator; 7 – fan; 8 – snail; 10 – stator coil; 11 – rotor; 12 – stator; 14 – insulator; 15 – brush arm; 16 – collectors; 19 – shaft; 21 – cage. The generator is a six pole synchronous electric machine with self-ventilation. It consists of the fixed part (stator) and the moving part (six pole rotor). Stator casing is a cylinder, which has supports to install generator. The stator sheets, which are clamped between the casing disk and the packing washer, are installed in the casing. The channels are made in the stator sheets to lay the stator winding.

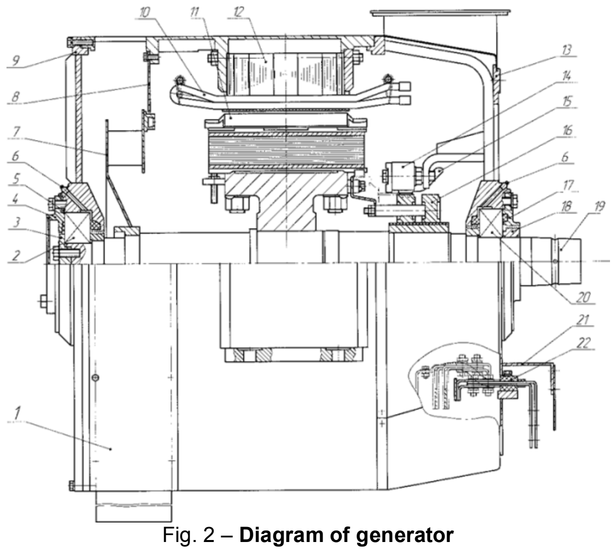


Fig. 2 – **Diagram of generator**

The dynamics of the engine power transmission in combination with the rotor of the electric generator is treated. The design model of the transmission’s nonlinear torsional vibrations is shown on Fig.3. The following notations are used on Fig.3: $I_1 \div I_3$ are inertia moments of inlet crankshaft; $I_4 \div I_6$ – inertia moments of outlet crankshaft; $I_7 \div I_{11}$ – inertia moments of the gears of the main transmission; I_{12} – inertia moment of the compressor; I_{13} – turbine inertia moment; I_{14} – inertia moment of hub of inlet crankshaft damper; I_{16} – moment inertia of the rotor generator. The stiffness coefficients of the model are denoted by $c_1 \div c_{15}$. The parameters $\beta_1 \div \beta_6$ are damping coefficients, obtained from the comparison of the experimental resonance vibrations and the calculations data. Time periodic perturbing torques, generated by the gas forces in the engine pistons, are denoted by $M_1 \div M_6$. The distributing torque takes the form of the third harmonic of the Fourier series, as this harmonic is the most dangerous for a three-cylinder engine.

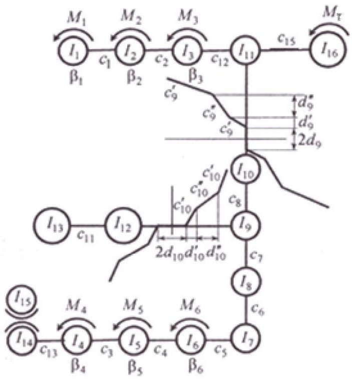


Fig. 3 – **Model of nonlinear torsional vibrations of power transmission**

The nonlinear torsional vibrations of the power transmission of the diesel engine power plant are described by the following system of sixteen nonlinear ordinary differential equations:

$$I_1\ddot{\theta}_1 + \beta_1\dot{\theta}_1 - c_1(\theta_2 - \theta_1) = M_1;$$
$$I_2\ddot{\theta}_2 + \beta_2\dot{\theta}_2 + c_1(\theta_2 - \theta_1) - c_2(\theta_3 - \theta_2) = M_2;$$
$$I_3\ddot{\theta}_3 + \beta_2\dot{\theta}_3 + c_2(\theta_3 - \theta_2) - c_2(\theta_{11} - \theta_3) = M_3;$$
$$I_4\ddot{\theta}_4 + \beta_4\ddot{\theta}_4 - c_{13}(\theta_{14} - \theta_4) - c_3(\theta_5 - \theta_4) = M_4;$$
$$I_5\ddot{\theta}_5 + \beta_5\dot{\theta}_5 - c_4(\theta_6 - \theta_5) + c_3(\theta_5 - \theta_4) = M_5;$$
$$I_6\ddot{\theta}_6 + \beta_6\dot{\theta}_6 - c_5(\theta_7 - \theta_6) + c_4(\theta_6 - \theta_5) = M_6;$$
$$I_7\ddot{\theta}_7 + c_5(\theta_7 - \theta_6) - c_6(\theta_8 - \theta_7) = 0;$$
$$I_8\ddot{\theta}_8 + c_6(\theta_8 - \theta_7) - c_7(\theta_9 - \theta_8) = 0; \qquad (1)$$

$$I_9\ddot{\theta}_9 + c_7(\theta_9 - \theta_8) - c_8(\theta_{10} - \theta l) - f_{10}(\theta_{12} - \theta_9) = 0;$$
$$I_{10}\ddot{\theta}_{10} + c_8(\theta_{10} - \theta_9) - f_9(\theta_{11} - \theta_{10}) = 0;$$
$$I_{11}\ddot{\theta}_{11} + c_{12}(\theta_{11} - \theta_3) - c_{15}(\theta_{16} - \theta_{11}) + f_9(\theta_{11} - \theta_{10}) = 0;$$
$$I_{12}\ddot{\theta}_{12} + f_{10}(\theta_{12} - \theta_9) - c_{11}(\theta_{13} - \theta_{12}) = 0;$$
$$I_{13}\ddot{\theta}_{13} + c_{11}(\theta_{13} - \theta_{12}) = 0;$$
$$I_{14}\ddot{\theta}_{14} + c_{13}(\theta_{14} - \theta_4) = 0;$$
$$I_{15}\ddot{\theta}_{15} + b_{14}(\dot{\theta}_{15} - \dot{\theta}_{14}) = 0;$$
$$I_{16}\ddot{\theta}_{16} + c_{15}(\theta_{16} - \theta_{11}) = -M_T,$$

where θ_i is a rotation angle of disk with the inertia moment $I_i, i = 1,...,16$; M_T is a braking torque of the generator, which consist of constant torque and variable electromagnetic torque. The last one is generated by non-uniform magnetic flow between stator and rotor. The mathematical model of diesel engine power miniplant torsional vibrations is essentially nonlinear with a lot of degrees of freedom.

Calculation of braking electromagnetic torque of the electric generator. The calculation of braking electromagnetic torque starts with the analysis of the generator magnetic flow. As the frequency of the rotor rotation is changed in time insignificantly, the quasi-static approach is used to simulate the magnetic field. Sequential calculations of the generator stationary magnetic field are performed for different values of the rotor rotation angle. This problem is solved by a 2D finite element method.

For the stationary magnetic field the following functional is minimized

$$F = \int_S \left(\int_0^{B_x} \frac{1}{\mu} B_x dB_x + \int_0^{B_y} \frac{1}{\mu} B_y dB_y \right) dS - \int_S \vec{A} \cdot \vec{\delta} dS,$$

where S is magnetic field area; B_x, B_y are components of magnetic induction vector; $\vec{A}$ is magnetic potential, which satisfy the following equation: $\vec{B} = \operatorname{rot} \vec{A}$. For the Cartesian coordinates the components of the magnetic induction vector are determined as follows:

$$B_x = \frac{\partial A}{\partial y}; \qquad B_y = -\frac{\partial A}{\partial x}.$$

The region S is divided on the triangular elements of different shapes and dimensions. Inside each finite element the solution is approximated by linear functions. Thus, the minimization of energy functional is transformed to the system of linear algebraic equations.

If the region S is divided on r triangular elements with n nodes, the energy functional is approximated by piecewise-linear functional as follows

$$F=\sum_{m=1}^{m=r}\int_{S_m}\left(\int_0^{B_x}\frac{1}{\mu}B_x\,dB_x+\int_0^{B_y}\frac{1}{\mu}B_y\,dB_y-A\,\delta_m\right)dS= \\ =\sum_{m=1}^{m=r}F^m=F^1+F^2+...+F^r, \tag{2}$$

where S_m is triangle area; δ_m is current density in the triangular elements. As the variables of functional are magnitudes in the vertex of the triangles, the functional minimum is obtained from the following system

$$\frac{\partial F^m}{\partial A_p}=0,$$

where $p=1,2,...,n$; $m=1,2,...,r$. The following equations are true in the vertexes of the triangles:

$$\frac{\partial F^m}{\partial A_i}=\frac{1}{4\,\mu\,S_m}\left[\left(c_i^2+b_i^2\right)A_i+\left(c_ic_j+b_ib_j\right)A_j+\left(c_ic_k+b_ib_k\right)A_k\right]-\frac{\delta\,S_m}{3};$$

$$\frac{\partial F^m}{\partial A_j}=\frac{1}{4\,\mu\,S_m}\left[\left(c_ic_j+b_ib_j\right)A_i+\left(c_i^2+b_i^2\right)A_j+\left(c_jc_k+b_jb_k\right)A_k\right]-\frac{\delta\,S_m}{3};$$

$$\frac{\partial F^m}{\partial A_k}=\frac{1}{4\,\mu\,S_m}\left[\left(c_ic_k+b_ib_k\right)A_i+\left(c_jc_k+b_jb_k\right)A_j+\left(c_k^2+b_k^2\right)A_k\right]-\frac{\delta\,S_m}{3},$$

where A_i, A_j, A_k are magnitudes of the magnetic potential A in the vertexes i,j,k of the triangles; δ is current density in the finite element with vertexes i,j,k; S_m is area of the triangular elements, which are calculated as $S_m=0.5\left(x_ib_i+x_jb_j+x_kb_k\right)$; μ is magnetic conductivity in the triangular elements; the coefficients a,b,c are calculated by the coordinates x,y of the vertexes i,j,k in the element m according to the formulas: $a_i=x_jy_k-x_ky_j$; $b_i=y_i-y_k$; $c_i=x_k-x_j$; $a_j=x_ky_i-x_iy_k$; $b_j=y_k-y_i$; $c_j=x_i-x_k$; $a_k=x_iy_j-x_jy_i$; $b_k=y_i-y_j$; $c_k=x_j-x_i$. As the energy functional is approximated by linear functions inside the triangular elements, the magnetic potential inside of an element is a linear function with respect to the potentials in the vertexes of this element. The magnetic potential takes the following form:

$$A = \frac{1}{S_m}\left[\left(a_i + b_i x + c_i y\right) A_i + \left(a_j + b_j x + c_j y\right) A_j + \left(a_k + b_k x + c_k y\right) A_k\right].$$

The magnetic induction is considered constant in every finite element. It is determined from the relation

$$B = \frac{1}{2\,S_m}\sqrt{\left(c_i A_i + c_j A_j + c_k A_k\right)^2 + \left(b_i A_i + b_j A_j + b_k A_k\right)^2}\ .$$

The system of equations, describing minimum of the functional (2), consists of the number of equations, equal to the number of the calculating points. Thus, the following system of linear algebraic equations is obtained

$$T \times A = \Delta\ ,$$

where T is a matrix, which elements consists of the coefficients of the matrix G. The matrix G has the form

$$G = \frac{1}{4\mu}\ \begin{array}{c|c} & \begin{array}{ccc} i & j & k \end{array} \\ \hline \begin{array}{c} i \\ j \\ k \end{array} & \begin{bmatrix} b_i^2 + c_i^2 & b_i b_j + c_i c_j & b_i b_k + c_i c_k \\ b_i b_j + c_i c_j & b_j^2 + c_j^2 & b_j b_k + c_j c_k \\ b_i b_k + c_i c_k & b_j b_k + c_j c_k & b_k^2 + c_k^2 \end{bmatrix} \end{array},$$

where i, j, k are numbers of matrix G elements.

The magnetic conductivity of the triangular elements varies in the nonlinear medium. It depends on the values of the magnetic potentials. Therefore, the solution of the system of nonlinear equations is performed by iterative methods. Vector Δ is composed from the vectors δ, determined as follows

$$\delta = \frac{J\ S_m}{3}\cdot\begin{bmatrix}1 & 1 & 1\end{bmatrix}^T ,$$

where J is current density in the triangular elements.

The design area is shown on Fig.4. As the combined excitation system has only central symmetry, the calculations are performed for all machine cross-sections. The geometry of the calculated region depends on the rotor position and the value of the current in the stator winding. The equipotential lines of the electromagnetic field in the stator and the rotor are shown on Fig.4b. As follows from the calculations, the electromagnetic moment of the generator rotor has significant variable component (Fig.5a). The amplitude of this moment is equal to 204 $N \cdot m$, which is 8.5 percent of the generator moment. As follows from the harmonic analysis (Fig.5b), the torque of the shaft generator has very complex spectrum with significant 12th harmonic.

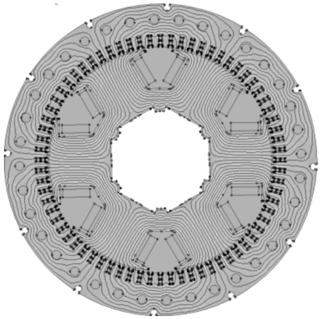
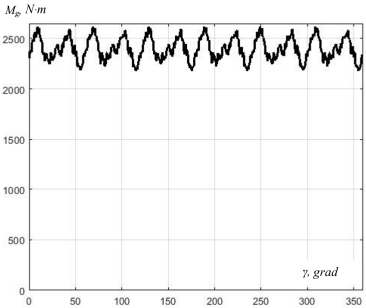
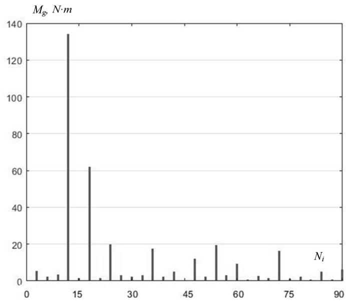


Fig. 4 – **Equipotential lines of the electromagnetic field**



a)



b)

Fig. 5 – **Calculation of the torque on the generator shaft;**
a) – dependence of the moment on the rotor rotation angle;
b) – harmonic analysis

Free nonlinear vibrations of power transmission. The analysis of the forced vibrations of essentially nonlinear system in the main resonance can be transformed to the backbone curve calculations. It is well-known, that the frequency response outlines the backbone curve [6], so frequency range of the crankshaft rotation, where the resonance of the forced vibrations is observed, is determined by backbone calculations. The parametric synthesis, which includes the shifting of the backbone curve, can be performed to detune the mechanical system.

The torsional vibrations due to the engine operation are excited by disturbing moment acting on the crankshaft which is approximated by the third harmonic (for a three-cylinder engine) of the Fourier series:

$$M(t) = M_S \sin\left(3\Omega t\right),$$

where Ω is angular velocity of the crankshaft rotation. As follows from the results of the calculations published in the previous section, the disturbing moment governed by the variable magnetic field takes the following form:

$$\tilde{M}(t) = \tilde{M}_S \sin(12\Omega t) .$$

The nonlinear torsional resonances between the transmission and above-mentioned moments are analyzed in this section.

The power transmission of the diesel engine 3TD-4 has two nonlinear elastic clutches (Fig.1), located between the crankshafts and in the compressor drive. Free nonlinear torsional vibrations of the power transmission with several piecewise linear clutches are described by the following system of the nonlinear ordinary differential equations in the vector form:

$$\ddot{x} + K(x) = 0 , \tag{3}$$

where x is an N dimensional vector of the generalized coordinates; $K(x) = [K_1, ..., K_N]$ $K(x) = [K_1, ..., K_N]$ is an N dimensional piecewise-linear vector-function of the generalized coordinates.

The approach based on the Galerkin method is suggested to analyze free nonlinear vibrations of the transmission. Every generalized coordinate is presented as the truncated Fourier series with an unknown frequency ω :

$$x_i = \sum_{j=0}^{N_C} C_{i,j} \cos(j\omega t) , \tag{4}$$

where i is a number of the generalized coordinate; j is a harmonic number; N_C is a quantity of terms in the expansion; $C_{i,j}$ are unknown coefficients. The solution (4) is substituted into (3) and Galerkin approach is applied, leading to the following equations:

$$\int_0^{2\pi/\omega} \left[\ddot{x}_i + K_i(x) \right] \cos(j\omega t) dt = 0, i = 1, ..., N; \ j = 0, ..., N_C ; \tag{5}$$

The equations (5) are transformed into the system of nonlinear algebraic equations with respect to the coefficients $C_{i,j}$. The dimension of this system is $(N_C + 1)N$. The Newton's method is applied to solve equations (5).

The suggested method is applied to analyze free nonlinear vibrations of the power shown in Fig.1, 3. Frequency of the crankshaft rotation required for achieving 50 Hz frequency of the alternating current, is equal to 16.75 Hz . As follows from previous analysis, the third and the 12th harmonics, resulting of the gas forces and the variable magnetic field respectively, are the dangerous ones that can cause intensive torsional vibrations. The backbone curves which describe dangerous regimes are shown in Fig.6.

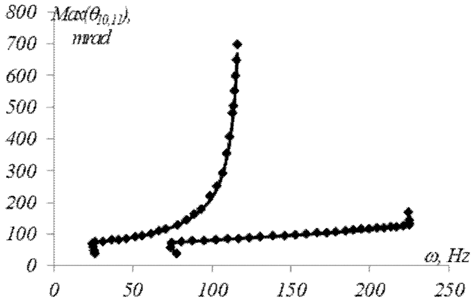


Fig. 6 – **Backbone curves of free torsional vibrations**

Conclusions. The method for analysis of nonlinear resonance interaction between the power transmission of the diesel engine and the generator equipment is suggested. The problem is divided into two uncoupled subproblems: analysis of the rotor electromagnetic braking torque and analysis of free torsional vibrations of the diesel engine power transmission. Analysis of the nonlinear resonance between these subsystems is performed by analysis of backbone curves calculated with respect to influence of several nonlinear elastic clutches in the transmission. The resonance of torsional vibrations between the power transmission and the higher harmonics of the electromagnetic braking torque is observed. This resonance can lead to the system failure; therefore, such an analysis should be performed while choosing the generator parameters.

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НЕЛИНЕЙНЫЕ КОЛЕБАНИЯ ДИЗЕЛЬНОЙ МИНИЭЛЕКТРОСТАНЦИИ

Предложен метод анализа резонансных нелинейных крутильных колебаний в дизельной миниэлектростанции. Рассмотрены нелинейные крутильные колебания генератора, состоящего из дизельного двигателя 3ТД-4 и тягового генератора ГС 530АМУ3. Результаты анализа показывают возможность возникновения резонанса между силовой передачей и ротором генератора, который требует отстройки силовой передачи.

Ключевые слова: *дизельный генератор, тормозной момент генератора, нелинейные крутильные колебания, скелетная кривая.*

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НЕЛІНІЙНІ КОЛИВАННЯ ДИЗЕЛЬНОЇ МІНІЕЛЕКТРОСТАНЦІЇ

Запропоновано метод аналізу резонансних нелінійних крутильних коливань у дизельній мініелектростанції. Розглянуто нелінійні крутильні коливання генератора, який складається з дизельного двигуна 3ТД-4 та тягового генератора ГС 530АМУ3. Результати аналізу свідчать про можливість виникнення резонансу між силовою передачею та ротором генератора, який потребує відлаштування.

Ключові слова: *дизельний генератор, тормозний момент генератора, нелінійні крутильні коливання, скелетна крива.*

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