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## ЕЛЕКТРОНІКА. РАДІОТЕХНІКА. ЗАСОБИ ТЕЛЕКОМУНІКАЦІЙ

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# THE PROPERTIES OF TIME-FREQUENCY LOCALIZATION OF BASIS FUNCTIONS USED IN OFTDM COMMUNICATION SYSTEMS

*А.Е. Брянський.* **Властивості частотно-часової локалізації базисних функцій, які використовуються в OFTDM системах зв'язку.** Досліджуються властивості частотно-часової локалізації (ЧЧЛ) різних базисних функцій: прямокутна функція, функція половина косинуса, функція алгоритму ізотропного ортогонального перетворення (Isotropic Orthogonal Transform Algorithm – IOTA) і розширена функція Гаусса, які використовуються в OFTDM системах зв'язку. Метою є дослідження властивостей частотно-часової локалізації базисних функцій, які використовуються в OFTDM системах зв'язку, що є необхідною умовою для створення системи передачі інформації на основі нових базисних функцій, добре локалізованих як в частотній, так і в часовій області, з хорошим придушенням міжсимвольної (МСІ) і міжканальної (МКІ) інтерференції. Представлені результати, отримані за допомогою методу математичного моделювання. Представлено кілька різних типів формуючих імпульсів, з подальшим використанням параметра Гейзенберга $\xi$ як індикатора властивостей ЧЧЛ. Чим більша $\xi$, тим краща спільна частотно-часова локалізація базисної функції. На ефективність розширеної функції Гаусса впливатимуть два параметра: $\alpha$, і кількість ланок фільтра *M*. Властивості частотно-часової локалізації, які описуються параметром Гейзенберга, функцією невизначеності, а також функцією перешкод і миттєвою функцією кореляції, дозволяють описати, яким чином сигнали на різних несучих частотах і з різними символами взаємодіють один з одним. За рахунок використання різних базисних функцій з різними властивостями ЧЧЛ, динамічний розподіл спектра може бути досягнуто більш природним чином, оскільки передавач і приймач швидко пристосовуються до різних умов каналу і середовища перешкод, і можна очікувати більш високої надійності і спектральної ефективності системи зв'язку. Також при спрощенні синхронізації можна очікувати меншої чутливості до часового і частотного зсуву.
*Ключові слова:* системи зв'язку, частотно-часова локалізація (ЧЧЛ), OFDM, OFTDM, IOTA функція, розширена функція Гаусса, МСІ/МКІ, WRAN, параметр Гейзенберга, перетворення Фур'є, QAM-16

*A.E. Bryanskiy.* **The properties of time-frequency localization of basis functions used in OFTDM communication systems.** This article investigates the properties of time-frequency localization (TFL) of various basis functions, such as rectangular function, the function of half cosine, isotropic orthogonal transform algorithm (IOTA) function and extended Gaussian function used in OFTDM communication systems. The aim is to study the properties of time-frequency localization of basis functions used in OFTDM communication systems which is a prerequisite for creating information communication system based on new basis functions that are well localized both in frequency and in time domain, with good suppression of intersymbol (ISI) and interchannel (ICI) interference. The results presented in the article are obtained by the method of mathematical modeling. The paper presents several different types of forming impulses with the further use of the Heisenberg parameter $\xi$ as an indicator of TFL properties. The higher $\xi$ is, the better the joint time-frequency localization of basis function. Two parameters affect the effectiveness of extended Gaussian function. One of them is $\alpha$ and the other is the number of filter *M* units. The properties of time-frequency localization described by Heisenberg parameter, uncertainty function and interference function and instantaneous correlation function allow to describe how signals at different carrier frequencies and with different characters interact with each other. By using different basis functions with different properties of TFL dynamic spectrum distribution can be achieved more naturally because the transmitter and receiver quickly adapt to different channel conditions and environmental obstacles. Thus, we can expect higher reliability and spectral efficiency of communications system. Also a lower sensitivity to time and frequency shift can be expected at simplifying the synchronization.
*Keywords:* communication systems, time-frequency localization (TFL), OFDM, OFTDM, IOTA function, extended Gaussian function, ISI/ICI, WRAN, Heisenberg parameter, Fourier transform, QAM-16

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**Introduction.** When using forming impulses in orthogonal frequency-time division multiplexing (OFTDM) systems [1] the reduction of intersymbol and interchannel interferences without adding cyclic prefix as compared to classical OFDM systems can be achieved. There have been proposed various basis functions with good properties of TFL [2, 3] and consider implementation based on different banks of filters. In contrast to classical OFDM scheme in which each subcarrier is modulated by complex-symbol in OFTDM each subcarrier is modulated by real character and thus it is allowed to use of well-localized time-frequency forming impulse with higher system requirements to TFL [4]. This enables a very efficient symbols packaging, increasing to a maximum, for example, bandwidth or stability in the communication lines. OFTDM was implemented to the technical standard for digital radio communication and its use has been considered in standard WRAN (IEEE 802.22).

**Aim of the Research** is to study the properties of time-frequency localization of basis functions used in OFTDM communication systems [1], which is a prerequisite for creating information communication system based on new basis functions that are well localized both in frequency and in time domain, with good suppression of intersymbol (ISI) and interchannel (ICI) interference [2].

**Materials and Methods.** The transmitted signals in OFTDM system can be written in the following analytical form:

$$s(t)=\sum_{n=-\infty}^{\infty}\sum_{m=0}^{N-1}a_{m,n}g_{m,n}(t), \tag{1}$$

where $a_{m,n}$ ($n\in\mathbb{Z}\, m=0,1,...,N-1$) are real symbols transmitted on subcarrier with index $m$ for symbol time with index $n$;

$g_{m,n}(t)$ is the forming impulse with indexes $(m,n)$ in a synthesized basis obtained from the converted time-frequency version of the basic functions $g(t)$ as follows [5]:

$$g_{m,n}(t)=e^{j(m+n)\frac{\pi}{2}}e^{j2\pi m\nu_0 t}g(t-n\tau_0),\ \ \nu_0\tau_0=\frac{1}{2}. \tag{2}$$

The paper presents several types of forming impulses with further use of Heisenberg parameter $\xi$ as an indicator of TFL properties.

Rectangular basic function is defined as follows:

$$g(t)=\begin{cases}\dfrac{1}{\sqrt{\tau_0}},\ \text{при}\ |t|\le\dfrac{\tau_0}{2},\\ 0,\ \text{при}\ |t|>\dfrac{\tau_0}{2},\end{cases}\qquad \text{i}\qquad G(f)=\frac{\sin(\pi\tau_0 f)}{\pi f\sqrt{\tau_0}}. \tag{3}$$

Fourier transform of rectangular basis function $G(f)$ is the function $\operatorname{sinc}$ that has a first side lobe – 13 dB and slowly fading along the frequency axis.

Function half cosine is defined as follows:

$$g(t)=\begin{cases}\dfrac{1}{\sqrt{\tau_0}}\cos\dfrac{\pi t}{2\tau_0},\ \text{при}\ |t|\le\tau_0,\\ 0,\ \text{при}\ |t|>\tau_0.\end{cases} \tag{4}$$

It is quite compact in the time domain and has the rapid decline in the frequency domain and, as well, can be used as a good basis function.

Gaussian function is defined as:

$$g_\alpha(t)=(2\alpha)^{1/4}e^{-\pi\alpha t^2},\ \alpha>0. \tag{5}$$

Extended Gaussian function is obtained from Gaussian function (5) and can be described using the following analytical expression:

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$$z_{\alpha,\nu_0,\tau_0}(t)=\frac{1}{2}\left[\sum_{k=0}^{\infty} d_{k,\alpha,\nu_0}\left[g_\alpha\left(t+\frac{k}{\nu_0}\right)+g_\alpha\left(t-\frac{k}{\nu_0}\right)\right]\right]\cdot\sum_{l=0}^{\infty} d_{l,1/\alpha,\tau_0}\cos\left(2\pi l\frac{t}{\tau_0}\right), \quad (6)$$

where $\tau_0\nu_0=\frac{1}{2}$, $g_\alpha$ is Gaussian function, and coefficients $d_{k,\alpha,\nu_0}$ are real.

It should be noted that the Fourier transform of the extended Gaussian function and Gaussian function hasthe same form as the functions itself, except the axles zoom factor:

$$\mathcal{F}z_{\alpha,\nu_0,\tau_0}(t)=z_{1/\alpha,\tau_0,\nu_0}(f),\ \mathcal{F}g_\alpha(f)=g_{1/\alpha}(f). \quad (7)$$

A special case of extended Gaussian function is a function $\zeta(t)=z_{1,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}}(t)$ that is called isotropic orthogonal transfer algorithm or IOTA-function because of its invariance to Fourier transform $\mathcal{F}\zeta(t)=\zeta(t)$.

Instead of deduce of system implementing structure from the filter bank theory let's find the realization by direct sampling of continuous time model [1] excluding the conditions of exact recovery.

Let $s(t)$ be the output signal of OFTDM modulator (for simplicity let's assume further that $\varphi_0=0$), which can be written as:

$$s(t)=\sum_{n=-\infty}^{\infty}\sum_{m=0}^{N-1}\left[a_{m,n}^{\mathrm{Re}}g_{m,2n}(t)+a_{m,n}^{\mathrm{Im}}g_{m,2n+1}(t)\right], \quad (8)$$

and the demodulated signal in branch $k$ for the duration of symbol $n$ can be written as follows:

$$\tilde{a}_{m,n}^{\mathrm{Re}}=\mathrm{Re}\left\{\int_{\mathbb{R}} s(t)g_{m,2n}^{*}(t)dt\right\},\quad \tilde{a}_{m,n}^{\mathrm{Im}}=\mathrm{Im}\left\{\int_{\mathbb{R}} s(t)g_{m,2n+1}^{*}(t)dt\right\}, \quad (9)$$

where $\mathrm{Re}$ and $\mathrm{Im}$ are real and imagine parts respectively.

While sampling $s(t)$ with speed $\frac{1}{T_s}$ during the time interval $[nT-\tau_0,\ nT+\tau_0\}$ can be obtained [1]:

$$s(nT+kT_s)=\sum_{l=-\infty}^{\infty}\sum_{m=0}^{N-1}\left[a_{m,l}^{\mathrm{Re}}g(nT+kT_s-lT)+ja_{m,l}^{\mathrm{Im}}g\left(nT+kT_s-lT-\frac{T}{2}\right)\right]e^{j\frac{\pi}{2}(m+2l)}e^{j2\pi\frac{mk}{N}}, \quad (10)$$

where $n\in\mathbb{Z}$ and $k=-\frac{N}{2},\dots,\ \frac{N}{2}-1$.

Let $s_k[n]=s[nN+k]=s(nT+kT_s)$ and making the replacement of variables $p=n-l$, let's rewrite (10) in the following way:

$$s_k[n]=\sum_{p=-\infty}^{\infty}g(pT+kT_s)\left[\sum_{m=0}^{N-1}a_{m,n-p}^{\mathrm{Re}}e^{j\frac{\pi}{2}(m+2n-2p)}e^{j2\pi\frac{mk}{N}}\right]+$$
$$+\sum_{p=-\infty}^{\infty}g\left(pT+kT_s-\frac{T}{2}\right)\left[\sum_{m=0}^{N-1}ja_{m,n-p}^{\mathrm{Im}}e^{j\frac{\pi}{2}(m+2n-2p)}e^{j2\pi\frac{mk}{N}}\right]=g_k[n]\cdot A_N^k(a_{m,n}^{\mathrm{Re}})+g_{k-\frac{N}{2}}[n]\cdot A_N^k(ja_{m,n}^{\mathrm{Im}}), \quad (11)$$

where

$$A_N^k(x_{m,n})=\sum_{m=0}^{N-1}x_{m,n}e^{j\frac{\pi}{2}(m+2n)}e^{j2\pi\frac{mk}{N}}, \quad (12)$$

$$g_k[p]=g[pN+k]=g(pT+kT_s),\ \ p\in\mathbb{Z}. \quad (13)$$

Therefore OFTDM modulator can be easily implemented using block of inverse fast Fourier transform (FFT) [5], defined in (12), with further use of bank of filters of forming impulse, defined in (13).

On the receiver side, by sampling the received signal $r(t)$ at the speed $\frac{1}{T_s}$ and integration approximation by addition, let's rewrite (9) as follows:

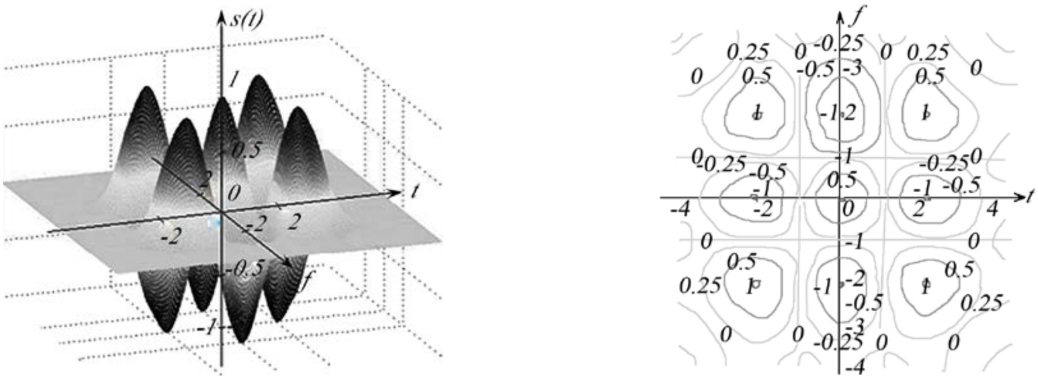
$$\tilde{a}^{\mathrm{Re}}_{m,n} \approx \mathrm{Re}\left\{ T_s\, j^{(m+2n)} \sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1} r_k[n]\cdot g_k[-n] e^{-j2\pi\frac{m\left(k+\frac{N}{2}\right)}{N}} \right\}, \tag{14}$$

$$\tilde{a}^{\mathrm{Im}}_{m,n} \approx \mathrm{Im}\left\{ T_s\, j^{-(m+2n)} \sum_{k=0}^{N-1} r_k[n]\cdot g_{k-\frac{N}{2}}[-n] e^{-j2\pi\frac{mk}{N}} \right\}, \tag{15}$$

where $g_k[-n] = g[-nN+k] = g(kT_s - NT)$. OFTDM demodulator can be implemented using a bank of filters $g_k[n]$ and $g_{k-\frac{N}{2}}[n]$, with further use of FFT block[4].

Assume that forming basic function $g(t)$ (or its truncated form) has a finite length on the interval $-M\tau_0 \le t < M\tau_0$, then its discrete form $g[n]$ is not empty on the interval $n = -\frac{MN}{2}, \ldots, \frac{MN}{2} - 1$. Therefore the length of each branch of the filter is $M$.

To illustrate how a basic function changes depending on the time and frequency distribution, we construct ambiguity function of the output signal of one branch of demodulation $\sum_{m,n} \left| A(\tau - 2n\tau_0, \nu - 2m\nu_0) \right|^2$. Three-dimensional graph is shown in Fig. 1 using basic functions IOTA. Axis are normalized by $\tau_0$ and $\nu_0$ respectively. For simplicity, data transmitted on each basis function are ignored and only neighboring points of spatial lattice to the same subset are consider.




*a* *b*

*Fig. 1. Distribution of IOTA basis functions in time-frequency plane in OFTDM system: IOTA basis function (a), contour graph (b)*

These impulses at points of spatial lattice with distance $2\tau_0$ or $2\nu_0$ have a negative envelope, thanks to phase factor $e^{j\frac{\pi}{2}(m+n)}$ equal to -1 when $|m| = 2$ either $|n| = 2$, but not both. IOTA function value is 0 on the edge of impulses so the neighboring impulses will be undertaken to interference until normalized time or frequency dispersion is less than 2.

In order to compare the properties of various impulses localization and obtaining quantitative understanding of this there was calculated Heisenberg parameter $\xi$ and direction parameter $\kappa$ for each impulse. It should be noted that for the rectangular impulse $\Delta f^2 = \int \sin^2(\omega f) df = \infty$ and, respectively, in theory $\xi = 0$ and $\kappa = 0$. For the extended Gaussian function and it will steadily increase to a maximum in proportion as $\alpha$ will be closer to 1 regardless of the direction [3]. Gaussian impulse reaches the maximum value $\xi$ and, therefore, has the best properties of TFL. IOTA impulse has good localization confirmed by the maximum value of parameter $\xi$ among extended Gaussian functions.

**Results.** Let's perform system simulation of OFTDM system in Matlab. Define the size of FFT / IFFT equal to 64 during the subsequent modeling. As it was mentioned above, a forming basis function $g(t)$ (or its truncated form) has a finite length at the interval $-M\tau_0 \le t < M\tau_0$.

Fig. 2 shows the signal constellation at the output demodulation of OFTDM system, when using half cosine function and extended Gaussian function as basis functions.

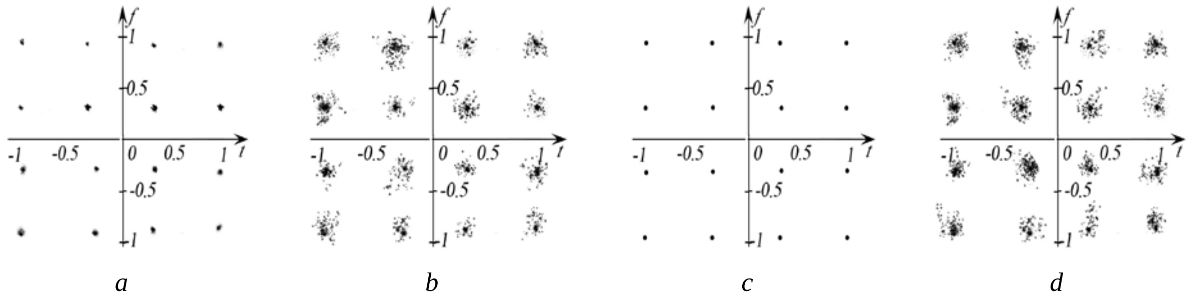


*Fig. 2. Signal constellation for QAM-16 modulation: half cosine function (a), half cosine function with frequency shift (b), extended Gaussian function (c), extended Gaussian function with frequency shift (d)*

Assume that $M = 5$, therefore, filters on each branch will have only $M = 5$ nonzero links. Extended Gaussian function can achieve almost ideal recovery when there is no noise or distortion, (Fig. 2, *c*) while half cosine function leads to some distortion (Fig.2, *a*). When there is a random frequency shift (for example $\Delta f = 2000$ Hz), as shown on Fig. 2,b and Fig. 2,d then there is a significant distortion of the signal constellation in both cases, and it is difficult to say which of the basis functions is better.

The effectiveness of extended Gaussian function is affected by two parameters. One of them is $\alpha$ and the other is the number of units of filter $M$. Fig. 3 shows the effect of these two parameters when using filters with different number of units ($M = 2$ and $M = 5$) and various values of parameter $\alpha$ ($\alpha = 1$ and $\alpha = 3$).

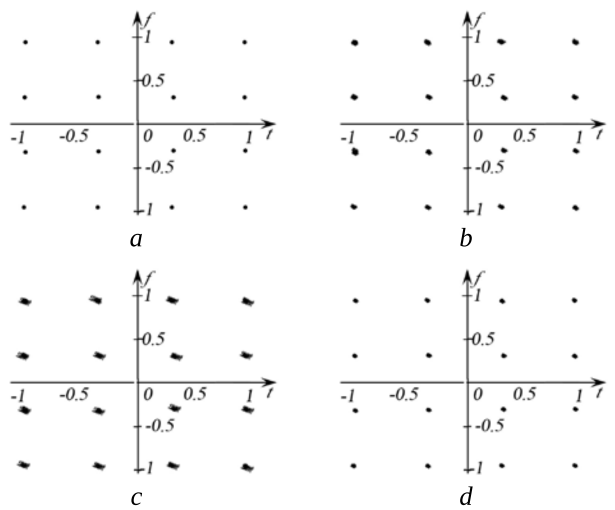


*Fig. 3. Signal constellation for extended Gaussian function during QAM-16 modulation: M=5, α=1, (a); M=5, α=3 (b); M=2, α=1 (c); M=2, α=3 (d)*

Fig. 3 shows that when the number of filter units is fairly large (for example $M = 5$), parameter $\alpha$ determines the overall efficiency. The most suitable form of TFL (α=1,with no distortion) provides the best efficiency (Fig.3, *a* compared to Fig. 3, *b*). If there is an insufficient number of filter units (for example $M$=2) a basis function with the largest value of $\alpha$ will be the least affected by truncation (Fig. 3, *c* compared to Fig. 3, *d*).

**Conclusions.** The properties of time-frequency localization described by Heisenberg parameter, ambiguity function as well as interference function and instantaneous correlation function allow describing how signals at different carrier frequencies and with different characters interact with each other. Since the transmitted signal consisting of basis functions places a copy of basis function at each point of the space lattice on the time-frequency plane, the smaller capacity of the basis function will be distributed to neighboring spatial lattice, the better reconstruction of the transmitted signal can be obtained after demodulation.

By using different basis functions with different TFL properties, dynamic spectrum distribution can be achieved more naturally because the transmitter and receiver quickly adapt to different channel conditions and environmental obstacles. Thus, higher reliability and spectral efficiency of communications system can be expect. In addition, less sensitivity to the time and frequency offset can be expect when simplifying the synchronization. Therefore, OFTDM system is a promising candidate for the future wireless communication.

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