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AXISYMMETRIC STRESSED STATE OF THE ADHESIVE JOINT OF TWO CYLINDRICAL SHELLS UNDER AXIAL TENSION

С.С. Курєннов, К.П. Барахов, Д.В. Дворецька. **Осесиметричний напружений стан клейового з’єднання двох циліндричних оболонок при осьовому розтяганні.** Розглянута задача дослідження напружено-деформованого стану конструкції, складеної з двох склеєних коаксіальних циліндричних труб. Ціллю роботи є уточнення класичної моделі напруженого стану з’єднання та дослідження точності запропонованої моделі і умов її застосування. Труби розглядаються як тонкостінні осесиметричні оболонки, які з’єднані за допомогою клейового шару ненульової товщини. Дотичні напруження в клеї вважаються постійними за товщиною клейового шару та діючими лише в його серединній поверхні. Нормальні напруження покладаються лінійно залежними від радіальної координати. Дотичні напруження у клейовому шарі пропорційні різниці поздовжніх переміщень сторін оболонок, які обернені до клейового шару. Нормальні напруження – пропорційні різниці радіальних переміщень оболонок. Задачу знаходження напружено-деформованого стану з’єднання зведено до системи чотирьох звичайних диференційних рівнянь відносно радіальних та поздовжніх переміщень шарів. Систему розв’язано матричним методом. Переміщення шарів за межами склейки знаходяться за класичною теорію осесиметричних оболонок. Задоволення крайових умов та умов спряження призводить до системи двадцяти двох лінійних рівнянь із двадцяти двома невідомими коефіцієнтами. Розв’язано модельну задачу, результати порівняно з розрахунками, виконаними за допомогою методу скінчених елементів. Дотичні та нормальні напруження у клеї досягають максимальних значень на краях клейового шва. Показано, що запропонована модель з високою точністю описує напружений стан з’єднання, яке має напливи залишків клею на кінцях шва але не може бути застосована у випадку відсутності напливів клею. Тому що у такому разі дотичні напруження внаслідок закону парності досягають максимальних значень не на краю, а на деякій відстані від краю шва. Внаслідок цього розподіл нормальних напружень у краю шва також суттєво змінюється і відрізняється від розрахунків за запропонованою моделлю. Таким чином, математична модель з’єднання за певних обмежень має достатню для інженерних задач точність і може бути використана для розв’язання задач проектування конструкцій.

Ключові слова: клейове з’єднання, циліндричні осесиметричні оболонки, аналітичний розв’язок, система звичайних диференційних рівнянь, симетричні матриці

S. Kurennov, K. Barakhov, D. Dvoretskaya. **Axisymmetric stressed state of the adhesive joint of two cylindrical shells under axial tension**. The research of the deflected mode of the construction, composed of two coaxially-glued cylindrical pipes, is done. Pipes are considered as thin-walled axisymmetric shells, which are joined by adhesive layer of a certain thickness. The shearing stresses in the glue are considered to be constant over the thickness of the adhesive layer, and normal stresses are linearly dependent on the radial coordinate. The shearing stresses in the adhesive layer are considered to be proportional to the difference in the longitudinal displacements of the shell sides that are faced to the adhesive layer. Normal stresses are proportional to the difference in radial displacement of the shells. It is supposed that the change in the adhesive layer thickness under deformation does not affect the stress, that is, the linear model is considered. The problem of the joint deflected mode finding is reduced to the system of four ordinary differential equations relative to the radial and longitudinal displacements of the layers. The system is solved by the matrix method. Displacements of layers outside of the adherent area can be found by the classical theory of axisymmetric shells. Satisfaction of boundary conditions and conjugation conditions leads to a system of twenty two linear equations with twenty two unknown coefficients. The model problem is solved; the results are compared with the computation made by the finite element method. The tangential and normal stresses in the glue reach the maximum values at the edges of the adhesive line. It is shown that the proposed model describes the stressed state of the joint with high accuracy, and this joint has an influx of glue residues at the ends of the adhesive line but cannot be applied in the absence of adhesive influxes. Because in this case, the tangential stresses due to the parity rule reach maximum values not on the edge, but at some distance from the edge of the line. As a result, the distribution of normal stresses at the edge of the line also substantially changes. Thus, the proposed model with certain restrictions has sufficient accuracy for engineering problems and can be used to solve design problems.

Keywords: adhesive joint, cylindrical axisymmetric shells, analytical solution, system of ordinary differential equations, symmetric matrices

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Introduction. Adhesive joints of coaxial cylindrical pipes are common elements in the design of aviation, rocket and space technology. Such joints have several advantages compared with other classical types of compounds. Such advantages are tightness, high aerodynamic efficiency, manufacturability, low weight, etc. However, the control of adhesive joints is difficult, and the operating experience of composite structures shows that the loss of the bearing capacity of the structure often occurs due to the destruction of the connecting nodes. This is due to the fact that it is difficult to achieve uniform transmission of forces from one structural element to another in an overlap joint. As a rule, stress concentration occurs in the joint seam at the edges of the gluing, which reduces the strength of the structure. Therefore, the calculation of connecting elements is an important component of the calculation of aggregates for strength.

Analysis of publications. The task of determining in an analytical form the stress-strain state (SSS) of the connections of cylindrical pipes in the general formulation has not yet been solved. For the description of the coaxial pipe connections, in the analytical form of the SSS, exclusively axisymmetric models are used. Consider connections transmitting torque [1] or longitudinal load [2]. Mathematical models of compounds transmitting longitudinal load can be divided into two types. In the first case, the connected pipes are considered as thin-walled cylindrical shells. In this case, the stress distribution over the thickness of the layers is set a priori, and in the carrier layers the stress distribution is assumed to be linear, and in the adhesive layer, uniform across the layer thickness [3, 4]. Models of the second type consider joints of thick-walled pipes, the local bending of which can be neglected. In order to be able to build an analytical solution, it is assumed in [5, 6] that the normal stresses in the axial direction in the pipes being joined are constant in thickness, and the normal stresses in the radial direction are equal to zero. However, it is obvious that in a real construction the distribution of normal stresses across the thickness of the layers differs from the uniform distribution. A model [7] was also proposed, according to which normal stresses in the radial direction are variable, and stresses in the circumferential and axial directions are constant in thickness. Of course, the finite element method (FEM) is also used to study the SSS of adhesive joints of coaxial pipes, as for example, in [2, 8, 9] and experiments [10]. This approach allows you to explore a wide range of tasks, however, it complicates parametric studies, complicates solving problems for complex structures, solving design and optimization problems, etc. Comparison of the results of numerical calculations performed using FEM with analytical models showed that for thin-walled shells the model [3, 4] shows good results.

Purpose of the study. One of the drawbacks of the mathematical model SSS of the adhesive layer, which was used in [3, 4], is the hypothesis of a uniform distribution of stresses across the thickness of the adhesive layer. Obviously, this leads to an imbalance of the layers, since the areas of the outer and inner surfaces of the adhesive layer are different. In the joints of flat plates or beams of this imbalance does not occur. The purpose of this work is to create a refined mathematical model of the compound, which relieves it of the indicated disadvantage. It is proposed to consider the tangential stresses acting only in the middle surface of the adhesive layer, and the normal (tear-off) stresses in the adhesive – evenly distributed throughout the thickness of the adhesive layer. This approach was previously used to model the SSS of flat adhesive joints [11]. This approach is used for the first time in modeling the SSS of connections of cylindrical coaxial shells.

It should be noted that the new models SSS of the adhesive layer created in recent years make it possible to describe the adhesive SSS with high accuracy and take into account the peculiarities of the adhesive state at the ends of the adhesive joint, taking into account the boundary conditions on the external border of the adhesive layer [12, 13]. However, these mathematical models to describe the SSS of cylindrical compounds have not yet been used.

Formulation of the problem. The joint scheme and its dimensions are shown in Fig. 1.

The origin is placed in the middle of the gluing area. The radii of the middle surfaces of the cylindrical shells are denoted by R_1 and R_2 respectively. External and internal diameters of pipes facing the adhesive layer D_1 and D_2. The radius of the middle surface of the adhesive layer is R_0. The thickness of the bearing layers and the adhesive layer is denoted by δ_1, δ_2 and δ_0.

Let us consider in more detail the area of gluing. The differential elements of the connection and the acting forces are shown in Fig. 2

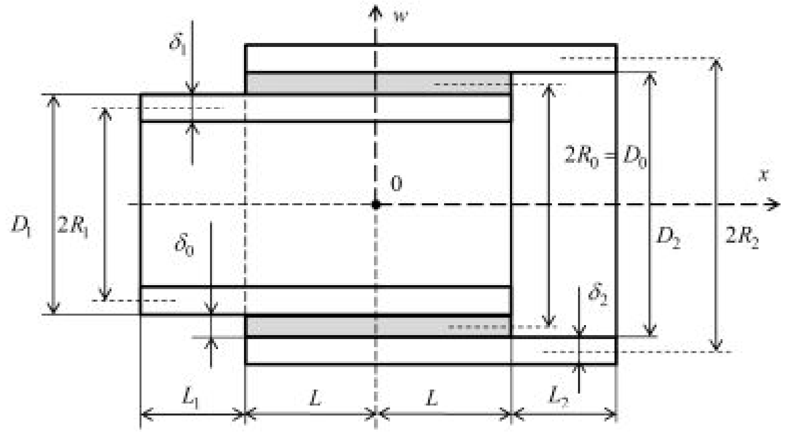

Fig. 1. Scheme of joint

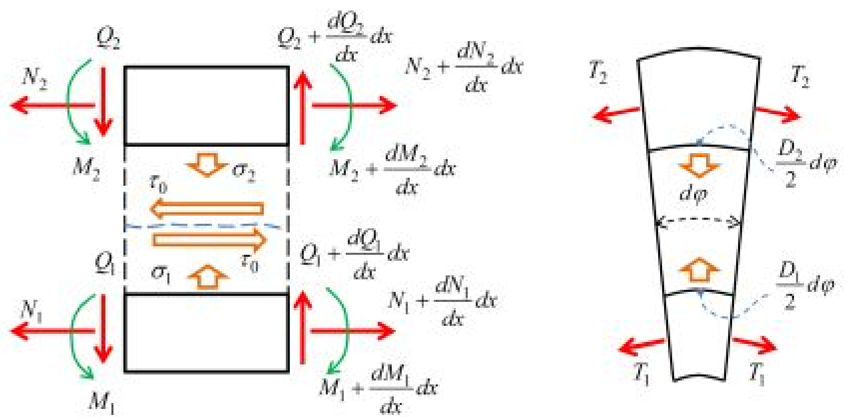

Fig. 2. Differential element of joint

The equilibrium equations of differential elements of a compound are:

$$R_1 \frac{dN_1}{dx} + R_0\tau_0 = 0; \quad R_2 \frac{dN_2}{dx} - R_0\tau_0 = 0, \tag{1}$$

$$R_1 \frac{dQ_1}{dx} + \frac{D_1}{2}\sigma_1 - T_1 = 0; \quad R_2 \frac{dQ_2}{dx} - \frac{D_2}{2}\sigma_2 - T_2 = 0, \tag{2}$$

$$R_1 \frac{dM_1}{dx} - R_1 Q_1 + R_0\left(s_1 + \frac{\delta_0}{2}\right)\tau_0 = 0; \quad R_2 \frac{dM_2}{dx} - R_2 Q_2 + R_0\left(s_2 + \frac{\delta_0}{2}\right)\tau_0 = 0. \tag{3}$$

Here, the values of s_1, s_2 are the distance from the neutral axis when bending to the outer surface of the base layer facing the adhesive layer. In the case of symmetric (homogeneous) layers, this distance is equal to half the thickness of the base layer.

The equations of physical law:

$$N_i = B_x^{(i)} \frac{du_i}{dx} + B_y^{(i)}\mu_{yx}^{(i)} \frac{w_i}{R_i}; \quad T_i = B_y^{(i)} \frac{w_i}{R_i} + B_x^{(i)}\mu_{xy}^{(i)} \frac{du_i}{dx}, \tag{4}$$

where $i = 1,\ 2$; $B_x^{(i)} = \dfrac{E_x^{(i)}\delta_i}{1-\mu_{xy}^{(i)}\mu_{yx}^{(i)}}$, $B_y^{(i)} = \dfrac{E_y^{(i)}\delta_i}{1-\mu_{xy}^{(i)}\mu_{yx}^{(i)}}$ - membrane stiffness of the bearing layers in the longitudinal and circumferential directions. In this case, we assume that the materials of the cylindrical shells being joined are orthotropic;

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$E_x^{(i)}$ и $E_y^{(i)}$ - the elastic moduli of the carrier layer with the number $i=1,\ 2$ in the corresponding direction.

The equations of bending shells:

$$\frac{d^2 w_i}{dx^2} = -\frac{M_i}{J_i}\,. \tag{5}$$

Where in the case of homogeneous bearing layers bending stiffness $J_i = \dfrac{E_x^{(i)}}{1-\mu_{xy}^{(i)}\mu_{yx}^{(i)}} I_i$, and where in turn $I_i = \delta_i^3/12$ - moment of inertia of the corresponding carrier layer.

The equilibrium equation of the differential element of the adhesive in the radial direction is:

$$\frac{d\tau_0}{dx} = \frac{D_2\sigma_2 - D_1\sigma_1}{\delta_0 D_0}\,. \tag{6}$$

Normal stresses in the glue will be presented in the form:

$$\sigma_2 = \frac{D_0}{D_2}\left(\sigma_0 + \frac{\delta_0}{2}\frac{d\tau_0}{dx}\right); \quad \sigma_1 = \frac{D_0}{D_1}\left(\sigma_0 - \frac{\delta_0}{2}\frac{d\tau_0}{dx}\right). \tag{7}$$

This form allows you to turn the equilibrium equation (6) into an identity. When this stresses σ_0 will be considered proportional to the difference of the transverse displacements of the carrier layers:

$$\sigma_0 = K(w_2 - w_1)\,, \tag{8}$$

where $K = E_0\delta_0^{-1}$ - stiffness of the adhesive layer in tension-compression in the transverse direction. The tangential stresses in the glue will be considered proportional to the difference of the longitudinal displacements of the inner, facing the adhesive layer, sides of the carrier layers:

$$\tau_0 = P\left(u_2 - u_1 + s_2\frac{dw_2}{dx} + s_1\frac{dw_1}{dx}\right), \tag{9}$$

where $P = \left(\dfrac{\delta_0}{G_0} + \dfrac{\delta_1}{2G_1} + \dfrac{\delta_2}{2G_2}\right)^{-1}$ - refined stiffness of the adhesive layer in shear [14].

Build a solution. The system of equations (1) – (9) can be reduced to a system of four differential equations for the longitudinal and transverse (radial) displacements of the carrier layers, which in matrix form can be written as follows:

$$\mathbf{A}_4\frac{d^4\vec{\mathbf{V}}}{dx^4} + \mathbf{A}_2\frac{d^2\vec{\mathbf{V}}}{dx^2} + \mathbf{A}_1\frac{d\vec{\mathbf{V}}}{dx} + \mathbf{A}_0\vec{\mathbf{V}} = 0\,, \tag{10}$$

where $\vec{\mathbf{V}} = (u_1;\ \ u_2;\ \ w_1;\ \ w_2)^T$ - vector function of displacement of layers.

$$\mathbf{A}_2 = \begin{pmatrix} \dfrac{R_1B_x^{(1)}}{R_0P} & \cdot & \cdot & \cdot \\ \cdot & \dfrac{R_2B_x^{(2)}}{R_0P} & \cdot & \cdot \\ \cdot & \cdot & -s_1^2 & -s_1s_2 \\ \cdot & \cdot & -s_1s_2 & -s_2^2 \end{pmatrix}; \quad \mathbf{A}_0 = \begin{pmatrix} -1 & 1 & \cdot & \cdot \\ 1 & -1 & \cdot & \cdot \\ \cdot & \cdot & \dfrac{B_y^{(1)}}{R_0R_1P} + \dfrac{K}{P} & -\dfrac{K}{P} \\ \cdot & \cdot & -\dfrac{K}{P} & \dfrac{B_y^{(2)}}{R_0R_2P} + \dfrac{K}{P} \end{pmatrix};$$

$$\mathbf{A}_1 = \begin{pmatrix} \cdot & \cdot & \beta_{yx}^{(1)} + s_1 & s_2 \\ \cdot & \cdot & -s_1 & \beta_{yx}^{(2)} - s_2 \\ \beta_{xy}^{(1)} + s_1 & -s_1 & \cdot & \cdot \\ s_2 & \beta_{xy}^{(2)} - s_2 & \cdot & \cdot \end{pmatrix}; \quad \mathbf{A}_4 = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \dfrac{R_1 J_1}{R_0 P} & \cdot \\ \cdot & \cdot & \cdot & \dfrac{R_2 J_2}{R_0 P} \end{pmatrix}.$$

Here $\beta_{yx}^{(i)} = \dfrac{\mu_{yx}^{(i)}}{R_0} \dfrac{B_y^{(i)}}{P}$; $\beta_{xy}^{(i)} = \dfrac{\mu_{xy}^{(i)}}{R_0} \dfrac{B_x^{(i)}}{P}$. According to the theory of elasticity of an orthotropic body, the equality is $\mu_{xy} E_x = \mu_{yx} E_y$. Consequently, the matrices of equation (10) are symmetric.

We look for the solution of equation (10) in the form $\vec{\mathbf{V}} = \vec{\mathbf{h}}\, e^{\lambda x}$, substituting in (10) and simplifying, we obtain the equation:

$$\mathbf{A}_s \vec{\mathbf{h}} = 0, \tag{11}$$

where $\mathbf{A}_s = \mathbf{A}_4 \lambda^4 + \mathbf{A}_2 \lambda^2 + \mathbf{A}_1 \lambda + \mathbf{A}_0$.

Characteristic equation $\det(\mathbf{A}_s) = 0$ has a root $\lambda = 0$ of multiplicity of the second degree, and ten non-zero roots. We find vectors $\vec{\mathbf{h}}_k$ from equation (11) up to an arbitrary factor C_k. Therefore, the general solution (10) can be written as:

$$\vec{\mathbf{V}} = \sum_{k=1}^{10} C_k \vec{\mathbf{h}}_k\, e^{\lambda_k x} + \vec{\mathbf{H}}_0 + \vec{\mathbf{H}}_1 x,$$

where $\vec{\mathbf{H}}_0$ and $\vec{\mathbf{H}}_1$ - eigenvectors corresponding to $\lambda = 0$;

$\vec{\mathbf{h}}_k$ - eigenvectors corresponding to eigenvalues $\lambda \neq 0$.

$$\vec{\mathbf{H}}_1 = \begin{pmatrix} C_{12} \\ C_{12} \\ 0 \\ 0 \end{pmatrix}; \quad \vec{\mathbf{H}}_0 = \begin{pmatrix} C_{11} \\ C_{11} \\ C_{12}\alpha_1 \\ C_{12}\alpha_2 \end{pmatrix}; \quad \alpha_1 = -R_1 \frac{K R_0 R_2 (\mu_{xy}^{(1)} B_x^{(1)} + \mu_{xy}^{(2)} B_x^{(2)}) + B_x^{(1)} \mu_{xy}^{(1)} B_y^{(2)}}{K R_0 (R_2 B_y^{(1)} + R_1 B_y^{(2)}) + B_y^{(1)} B_y^{(2)}},$$

$$\alpha_2 = -R_2 \frac{K R_0 R_1 (\mu_{xy}^{(1)} B_x^{(1)} + \mu_{xy}^{(2)} B_x^{(2)}) + B_y^{(1)} \mu_{xy}^{(2)} B_x^{(2)}}{K R_0 (R_2 B_y^{(1)} + R_1 B_y^{(2)}) + B_y^{(1)} B_y^{(2)}}.$$

Thus, the formula for displacement includes 12 constants C_k. From formulas (1) – (9) we find the efforts N_i, Q_i and T_i in the bearing layers, bending moments M_i and stresses τ_0 and σ_i in the adhesive layer.

Longitudinal and transverse movements of tips we denote u_3, w_3 ($x \in [-L_1 - L; -L]$) and u_4, w_4 ($x \in [L; L + L_2]$).Outside the area of gluing, displacements will be described by classical equations of an axisymmetric cylindrical shell:

$$\frac{R_1^2 J_1}{B_y^{(1)}} \frac{d^4 w_3}{dx^4} + (1 - \mu_{xy}^{(1)} \mu_{yx}^{(1)}) w_3 = -\frac{\mu_{xy}^{(1)} F_0}{2\pi B_y^{(1)}}, \quad \frac{R_2^2 J_2}{B_y^{(2)}} \frac{d^4 w_4}{dx^4} + \left(1 - \mu_{xy}^{(2)} \mu_{yx}^{(2)}\right) w_4 = -\frac{\mu_{xy}^{(2)} F_0}{2\pi B_y^{(2)}}, \tag{12}$$

$$\begin{aligned} N_3 &= B_x^{(1)} \frac{du_3}{dx} + B_y^{(1)} \mu_{yx}^{(1)} \frac{w_3}{R_1}; \qquad T_3 = B_y^{(1)} \frac{w_3}{R_1} + B_x^{(1)} \mu_{xy}^{(1)} \frac{du_3}{dx}; \\ N_4 &= B_x^{(2)} \frac{du_4}{dx} + B_y^{(2)} \mu_{yx}^{(2)} \frac{w_4}{R_2}; \qquad T_4 = B_y^{(2)} \frac{w_4}{R_2} + B_x^{(2)} \mu_{xy}^{(2)} \frac{du_4}{dx}, \end{aligned} \tag{13}$$

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where F_0 - longitudinal force applied to the joint; $N_3 = \frac{F_0}{2\pi R_1} = \text{const}$ и $N_4 = \frac{F_0}{2\pi R_2} = \text{const}$.

Solving the differential equations (12) and (13), we find the dependencies describing the displacements on the outer parts of the structure. Shell forces are described by dependencies:

$$\frac{dQ_3}{dx} = \frac{T_3}{R_1}, \quad \frac{dQ_4}{dx} = \frac{T_4}{R_2}, \quad \frac{dM_3}{dx} = Q_3, \quad \frac{dM_4}{dx} = Q_4, \quad \frac{d^2 w_3}{dx^2} = -\frac{M_3}{J_1}, \quad \frac{d^2 w_4}{dx^2} = -\frac{M_4}{J_2}.$$

Edge and pairing conditions:

$$u_3\big|_{x=-L-L_1} = \frac{dw_3}{dx}\bigg|_{x=-L-L_1} = Q_3\big|_{x=-L-L_1} = 0; \; N_2\big|_{x=-L} = Q_2\big|_{x=-L} = M_2\big|_{x=-L} = 0; \; u_3\big|_{x=-L} = u_1\big|_{x=-L};$$

$$w_3\big|_{x=-L} = w_1\big|_{x=-L}; \; \frac{dw_3}{dx}\bigg|_{x=-L} = \frac{dw_1}{dx}\bigg|_{x=-L}; \; N_3\big|_{x=-L} = N_1\big|_{x=-L}; \; Q_3\big|_{x=-L} = Q_1\big|_{x=-L}; \; M_3\big|_{x=-L} = M_1\big|_{x=-L};$$

$$N_1\big|_{x=L} = Q_1\big|_{x=L} = M_1\big|_{x=L} = 0; \; u_4\big|_{x=L} = u_2\big|_{x=L}; \; w_4\big|_{x=L} = w_2\big|_{x=L}; \; \frac{dw_4}{dx}\bigg|_{x=L} = \frac{dw_2}{dx}\bigg|_{x=L}; \; N_4\big|_{x=L} = N_2\big|_{x=L};$$

$$Q_4\big|_{x=L} = Q_2\big|_{x=L}; \; M_4\big|_{x=L} = M_2\big|_{x=L}; \; Q_4\big|_{x=L+L_2} = M_4\big|_{x=L+L_2} = 0.$$

The boundary conditions lead to a system of linear equations for the unknown coefficients C_k and coefficients arising from the integration of differential equations (12), (13).

Numerical example. Consider a structure having the following parameters: $E_x^{(i)} = E_y^{(i)} = 70$ GPa; $\mu_{xy}^{(i)} = \mu_{yx}^{(i)} = 0.3$; $i = 1, 2$; $L = 30$ mm; $L_1 = L_2 = 50$ mm; $\delta_1 = \delta_2 = 3$ mm; $G_0 = 0.36$ GPa; $E_0 = 0.9$ GPa; $\delta_0 = 0.1$ mm; radii $R_1 = 50$ mm; $R_0 = R_1 + 0.5\delta_1 + 0.5\delta_0$; $R_2 = R_0 + 0.5\delta_2 + 0.5\delta_0$. A longitudinal force is applied to the joint, uniformly distributed along the circumferential coordinate.

Fig. 3 shows graphs of tangential (a) and normal (b) stresses in the middle surface of the adhesive layer.

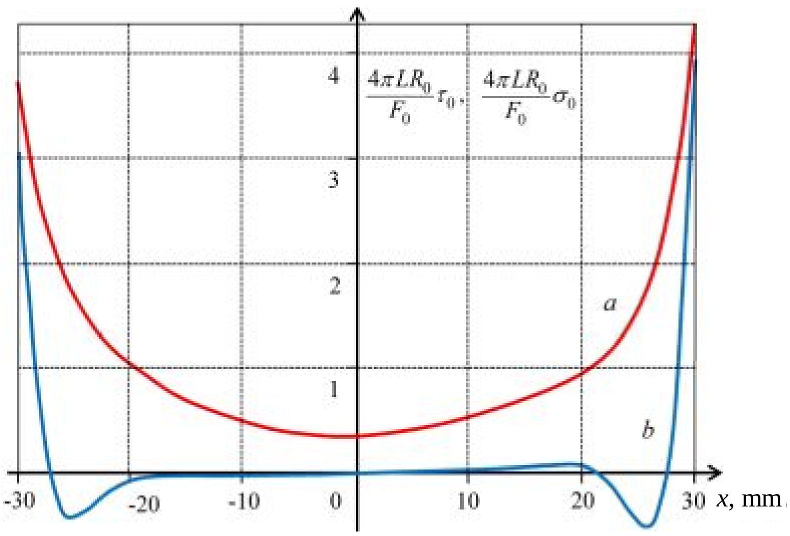

Fig. 3. Stresses in the adhesive layer

Stresses are shown in dimensionless form, as the ratio of the acting stresses τ_0 and σ_0 to the tangential stresses $F_0/(4\pi R_0 L)$, which would occur if the stresses were evenly distributed over the middle surface of the adhesive layer.

The stresses σ_1 and σ_2 in this case differ little from the stresses σ_0 and are almost indistinguishable in the graph, and therefore are not shown in the figure, but their average value σ_0 is shown.

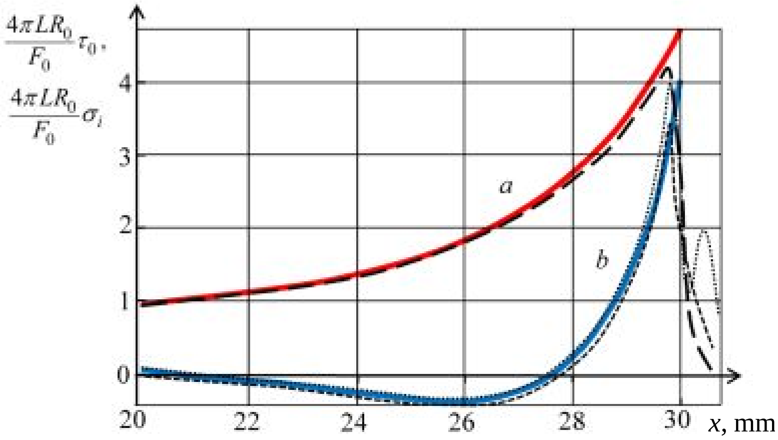

Fig. 4. Stresses in the adhesive layer at the seam edge

To verify the computational model, the stress state of the structure under consideration was calculated using FEM in the Comsol Multiphysics 5.2 system. The finite element model of the structure (Fig. 1) is supplemented with an influx of glue at both ends of the seam in the form of a quarter of a circle with a radius $5\delta_0$, and the edge of the part being joined is supplemented with chamfers with a side $2\delta_0$. The characteristic size of the final element in the adhesive layer is $0.2\delta_0$. Fig. 4 shows the stresses in the vicinity of one of the seam edges. Continuous lines show the graphs of tangent (*a*) and normal (*b*) stresses in the middle surface of the adhesive layer, calculated from the proposed model. Dotted lines show stress graphs τ_0, σ_1 and σ_2, calculated using FEM. Power surges outside the area of bonding are due to changes in the geometry of the adhesive layer, such as chamfer and adhesive flow. Jumps have a local character, not exceeding the values at the edge of the adhesive layer.

If there is no adhesive influx, then due to the conditions of tangential stress pairing, the tangential stresses at the seam edge are zero and reach a maximum at a distance of the order of the thickness of the adhesive layer from the edge of the adhesive seam [15, 16]. At the same time, the normal stresses σ_1 and σ_2 in the glue differ significantly (even by the sign) at the edges of the glue line, but they practically coincide in the depth of the region. In the presence of adhesive glue, which exceeds the thickness of the glue layer several times in thickness, as in this case, the shear stresses reach a maximum at the edge of the glue line. A voltage σ_1 and σ_2 differ slightly from each other. Therefore, it can be concluded that the proposed model better describes the SSS of the adhesive layer in the presence of adhesive flow.

Conclusions. A mathematical model of the axisymmetric stress state of the adhesive joint of two coaxial cylindrical shells is proposed. The solution was obtained in analytical form. Solved the model problem. It is shown that the proposed model has good accuracy, although it has some limitations. It can be used to solve problems of connection design. Further development of this model can be aimed at clarifying the mathematical model of the adhesive layer, using the theory of Tymoshenko's shells to describe the SSS bearing layers, taking into account the forces of inertia, temperature deformations, etc. One of the possible ways to generalize this approach is to create a mathematical model, the so-called double-shear connection. This is a combination of three carrier layers, in which the central carrier layer is connected to the two linings on the outer and inner side. Such a constructive solution allows significantly reduce the bending moments and tearing stresses in the adhesive.

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