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IMPLEMENTATION OF THE LINEAR ELASTIC STRUCTURE HALF-SPACE IN THE PLAXIS IN THE STUDY OF SETTLEMENTS

І.І. Солодей, Г.А. Затилюк. **Реалізація теорії лінійно-деформованого середовища в ПК Plaxis при дослідженні осідань основ**. Чисельне вирішення задач на основі методу скінченних елементів передбачає моделювання об'єктів як скінченної обмеженої області. При моделюванні системи «підземна споруда-ґрунтовий масив» завжди виникає питання обмеження нескінченного напівпростору ґрунтового масиву. Особливо гострим є питання вибору нижньої межі розрахункової моделі при дослідженні осідань, оскільки величини цього виду деформацій будуть стрімко зростати пропорційно до збільшення розмірів моделі по вертикалі. Ряд вчених вказують на можливість вирішити це питання, обмеживши розрахункову схему глибиною стисливої товщі, яка розраховується методом пошарового підсумовування. Однак часто, через особливості досліджуваних об'єктів, скористатися цією рекомендацією неможливо. Тому актуальним є питання розробки методик моделювання системи «підземна споруда-ґрунтовий масив» в програмних комплексах таким чином, щоб значення осідань були тотожними незалежно від обраної нижньої межі розрахункової моделі, а також відповідали б значенням, розрахованими за допомогою класичних аналітичних методів. В статті описано методику реалізації теорії лінійно-деформованого середовища, в програмному комплексі Plaxis 2D, який використовує метод скінченних елементів в якості своєї теоретичної бази, для дослідження осідань основ, незалежно від обраної нижньої межі моделі. На цій теорії базується метод пошарового підсумовування, який набув широкого поширення при розрахунку осідання основ. Нелінійна залежність між значенням осідань і глибиною в цьому методі досягається за рахунок введення коефіцієнта α. Виведені формули зміни модуля деформації з глибиною та отримані допоміжні коефіцієнти, представлені у табличній формі, які можуть бути використані при моделюванні систем «підземна споруда-ґрунтовий масив» в програмному комплексі Plaxis, задавши у вікні додаткових параметрів модуль деформації, що лінійно збільшується з глибиною. Практичне дослідження описаної методики показало збіжність значень осідань при різних глибинах нижньої межі моделі, що свідчить про можливість її використання при дослідженні систем «підземна споруда-ґрунтовий масив» в програмному комплексі Plaxis.
Ключові слова: підземна споруда, ґрунтова основа, осідання, ґрунтова модель, межа розрахункової області, теорія лінійно-деформованого середовища, метод скінченних елементів

I. Solodei, Gh. Zatyliuk. **Implementation of the linear elastic structure half-space in the Plaxis in the study of settlements**. Numerical problem solving based on the finite element method provides for objects modeling as finite bounded region. In modelling the "underground structure – soil mass" system always arises the question of limiting the infinite half-space of the soil mass. The problem is particularly acute for choosing the lower bound of the computing model through studies of settlement. It related to the fact that values of this strain regimes will increase in proportion to increase of the model dimensions vertically. Some scientists solve this problem in the following way. Limit the calculation scheme to the depth of the compression layer, which is calculated by the method of summation of the layers. However, it is often not possible to use this recommendation because of the features of the objects being studied. Therefore, the issue of developing methods for modeling the system "underground structure-ground massif" in software complexes. The value of the settlement must be identical and independent of the model dimensions. They should also correspond to the analytical calculation. The present review is concerned with linear elastic half-space implementing procedure in the Plaxis 2D, that uses the finite element method as its theoretical basis, to study settlements, regardless of the chosen lower bound of the model. This theory is based on the method of summation of layers, which was widespread in the calculation of settlement. The alpha-coefficient provides a non-linear relationship between settlement and depth. The derived formulas and auxiliary coefficients presented in tabular form. Using these formulas and coefficients, you can find the $E_{increment}$-value may be used, which is the increase of the Young's modulus per unit of depth and set it in the advanced features window. As well as given their practical use capability assessment in the "underground structure – soil mass" systems research in the Plaxis 2D. Settlement at different depths is in good agreement with each other.
Keywords: underground structure, soil mass, settlement, soil model, mesh dimension, linear elastic structure half-space, finite element method

Introduction. Correct modeling of the underground structure-soil array system using software systems that use FEM as its theoretical basis is a rather broad issue, which includes a number of problems covered in [1] (for example: determining the magnitude and nature of the distribution of permanent loads from the soil massif, the choice of soil model, etc.).

One of the most common issues in modeling the system "underground structure-soil array" is the choice of boundaries of the calculated area. Particularly acute is the question of choosing the lower

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bound of the model if the object of the study is the subsidence. It is known that the deformations of precipitation will increase along with the increase in the depth of the model (vertical dimensions).

Analysis of recent publications and problem statement. Perelmutter A.V, Slinker V. I. offer the choice of the sizes of the calculation model to solve by limiting its size in such a way that the influence of boundary conditions on the distribution of effort was minimal [2]. Such an approach was implemented in the works of Berezhnoi D.V, Sagdutullina M.K., Sultanova L.U. and Petrova D.N., Demenkov P.A., Potemkin D.O. [3, 4].

Since this technique consists in removing the limits of the calculation model from the underground structure, it is unacceptable in the study of sedimentation. An increase in the depth of the lower boundary of the model, on the one hand, reduces the effect on the distribution of forces, and on the other hand leads to an unjustified increase in the deformation of the subsidence.

In this case, Perelmutter A.V, Slivker V.I., Gorodetsky O.S., Yevzerov I.D. [2, 5] indicate the possibility to use another method – to limit the calculation scheme to the depth of the compression zone, which is calculated by the method of layer summing whose boundary is at a depth where the condition is fulfilled:

$$\sigma_{zp} \leq 0.2\sigma_{zg} , \tag{1}$$

where σ_{zg} – stress on the weight of the soil itself;

σ_{zp} – pressure from the building, taking into account the coefficient of damping in depth.

This approach is often used in design practice and is described by Ryabkov S.V and Solovyov R.A [6] when designing tunnel constructions. It should be noted that such an approach is possible with a static calculation. Methods of determining the limits of a dynamic calculation occupy a special place. They need to be modeled in such a way as to prevent the reflection of waves, that is, to ensure their passage or extinction [7, 8].

However, often, due to the features of the objects being studied, it is impossible to use the recommendation to limit the calculation scheme to the depth of the compressed zone. Therefore, the development of algorithms for modeling the system of “underground structure-soil massif” in software complexes is urgent in such a way that the values of settlements under the static calculation are identical, regardless of the selected lower boundary of the calculation model, and also correspond to the value of sediments calculated according to classical analytical methods.

The purpose and tasks of the research. The purpose of the work is to develop a methodology for modeling the system “underground structure-soil massif”, which would be deprived of a disadvantage of rapid proportional growth of sediments with increasing depth of the lower boundary of the model, using the possibility of introducing additional parameters for the Coulomb-Mora soil model in the Plaxis PC.

Presenting of the main material. In practice, the design for the determination of the value of precipitation was widely used by the layer-summing method, according to which the deposition is determined by the formula (2):

$$s = \beta \sum_{i=1}^{n} \frac{\sigma_{zp,i} h_i}{E_i} , \tag{2}$$

where β – dimensionless coefficient equal to 0.8;

$\sigma_{zp,i}$ – the average value of the vertical normal stress from the external load in the *i*-th layer of soil on the vertical passing through the center of the sole of the foundation;

h_i – the thickness of the *i*-th layer of soil, take no more than 0.4 width of the foundation;

n – the number of layers on which the base is divided;

E_i – deformation module of the *i*-th layer of soil along the line of initial load.

The vertical stress from the external load (p) decreases nonlinearly with increasing depth (z) and is determined by the formula (3):

$$\sigma_{zp} = \alpha p , \tag{3}$$

where α – coefficient taking according to the table;

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p – average pressure under the sole of the foundation.

This method is based on the theory of a linear deformed medium, according to which the normal stresses under the rectangular platform, which makes the pressure, vary with the depth according to the formula (4):

$$\sigma = \frac{2p}{\pi}\left(\operatorname{arctg}\left(\frac{\eta}{\zeta\sqrt{1+\zeta^2+\eta^2}}\right)+\frac{\zeta\eta(1+\eta^2+2\zeta^2)}{(\eta^2+\zeta^2)(1+\zeta^2)\sqrt{1+\zeta^2+\eta^2}}\right), \quad (4)$$

where η – the ratio of the sides of the platform, and ζ – depth relative to the width of platform:

$$\eta = \frac{l}{b}, \quad (5)$$

$$\zeta = \frac{2z}{b}, \quad (6)$$

where l and b – length and width of the site respectively;

z – the depth at which the stress is determined.

From the formula (3, 4) we conclude that the coefficient α varies with the depth z according to the law:

$$\alpha = \frac{2}{\pi}\left(\operatorname{arctg}\left(\frac{\eta}{\zeta\sqrt{1+\zeta^2+\eta^2}}\right)+\frac{\zeta\eta(1+\eta^2+2\zeta^2)}{(\eta^2+\zeta^2)(1+\zeta^2)\sqrt{1+\zeta^2+\eta^2}}\right), \quad (7)$$

Due to the coefficient α in formula (3), the vertical stress decreases nonlinearly with depth, which in turn, on the basis of formula (2), leads to a nonlinear decrease in the amount of sediment in each subsequent (i-th) layer of soil and the nonlinear relationship between the total values sediments of the base and depth.

On the other hand, formula (2) can be represented as:

$$s = \beta\sum_{i=1}^{n}\frac{p\,h_i}{E_{z,i}}, \quad (8)$$

where $E_{z,i}$ –a deformation module that is nonlinearly increasing with depth and is determined by the formula:

$$E_{z,i} = \frac{E_i}{\alpha} \ . \quad (9)$$

As already mentioned above, the deformation of the subsidence when calculating the “underground structure-base” systems in the software complexes that use FEM as its theoretical basis, increases linearly with the increase in the size of the calculation model vertically. However, when using a PC Plaxis is possible to specify additional options when using soil model of Colon-Mohr.

It is known that the hardness of soils in the natural state depends to a large extent on the level of stress, which means that it grows with the depth of their occurrence. In the general case, the use of the Colon-Mohr model does not imply a change in the stiffness of the soil and it is a constant value.

Additional parameters of the Coulomb-Mora model in the Plaxis PC include the ability to specify a deformation module that linearly increases with depth – $E_{\text{increment}}$ [kPa/m]. It is also necessary to specify a z_{ref} depth mark (in Plaxis – y_{ref}), above which the Young module has a normative value – E_i (in Plaxis – E_{ref}), and below the deformation module is a sum of normative value and gain with depth, that is, it becomes:

$$E_{z,i} = E_i + E_{\text{increment}}(z - z_{ref}) \ . \quad (10)$$

We can assume that there is a certain function $E_z = f(z)$, in which the value of the settles will coincide, regardless of the selected depth of the lower bound of the model.

Having a single normative value of the deformation module, let us express the increase of the rigidity module through the product of the normative value E_i and some coefficient k:

$$E_{\text{increment}}(z - z_{ref}) = E_i k\ . \qquad (11)$$

To determine this coefficient we use the formulas of the theory of linearly deformed media for the method of layer-summing, taking into account the formulas (9, 10, 11), we obtain:

$$\frac{E_i}{\alpha} = E_i + E_i k\ . \qquad (12)$$

Whereof:

$$k = \frac{1-\alpha}{\alpha}\ . \qquad (13)$$

Since the coefficient α varies nonlinearly depending on the relative depth, then k also changes nonlinearly with depth. In turn, this means that the module $E_{\text{increment}}$ and $E_{z,i}$ will also be nonlinear vary depending on the depth. Assuming that the rigidity module begins to grow immediately (i.e. $z_{ref} = 0$) we obtain:

$$E_{z,\text{increment}} = E_i \frac{k}{z}, \qquad (14)$$

$$E_z = E_i + E_{z,\text{increment}} z\ . \qquad (15)$$

However, as noted above, you can not specify the $E_{\text{increment}}$ deformation module described by the function in the Plaxis software package. To solve this problem, it is suggested to enter the average coefficient k_{avg} for different values of relative depth:

$$k_{avg} = \frac{1-\alpha_{avg}}{\alpha_{avg}}, \qquad (16)$$

where:

$$\alpha_{avg} = \frac{\int \alpha d\zeta}{\zeta}, \qquad (17)$$

or substituting (7):

$$\alpha_{avg} = \frac{\int \frac{2}{\pi}\left(\operatorname{arctg}\left(\frac{\eta}{\zeta\sqrt{1+\zeta^2+\eta^2}}\right) + \frac{\zeta\eta(1+\eta^2+2\zeta^2)}{(\eta^2+\zeta^2)(1+\zeta^2)\sqrt{1+\zeta^2+\eta^2}}\right)d\zeta}{\zeta}\ . \qquad (18)$$

The coefficient k_{avg} by analogy with the coefficient α can be represented in tabular form (Table), completing the table Д.1 in the ДБН В.2.1-10-2009. To use this coefficient, taking into account that a problem is solved in the software complex, and that the coefficient η is taken as a tape foundation, when $\eta \geq 10$, only one parameter must be found: the relative depth to the width of the site ζ by formula (6).

The Plaxis 2D PC also allows you to count axially symmetric problems, so the Table. 1 also shows the coefficient k_{cep} for the round foundations, which is searched by a similar method. In accordance with the theory of linearly deformed media, the coefficien α in this case varies according to the law:

$$\alpha = 1 - \left(\frac{1}{1+\frac{1}{\zeta^2}}\right)^{\frac{3}{2}}, \qquad (19)$$

and in formula (6) b is the diameter of the round foundation.

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Coefficient k_{avg}

ζ	Coefficient α for foundations								Coefficient k_{avg} for foundations	
	Round	Rectangular with sides ratio η=l/*b*, which is equal to						Tape (η≥10)	Tape (η≥10)	Round
		1	1.4	1.8	2.4	3.2	5			
0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0	0
0.4	0.949	0.960	0.972	0.975	0.976	0.977	0.977	0.977	0.006	0.014
0.8	0.756	0.800	0.848	0.866	0.876	0.879	0.881	0.881	0.037	0.083
1.2	0.547	0.606	0.682	0.717	0.739	0.749	0.754	0.755	0.092	0.203
1.6	0.390	0.449	0.532	0.578	0.612	0.629	0.639	0.642	0.162	0.352
2.0	0.285	0.336	0.414	0.463	0.505	0.530	0.545	0.550	0.238	0.519
2.4	0.214	0.257	0.325	0.374	0.419	0.449	0.470	0.477	0.319	0.696
2.8	0.165	0.201	0.260	0.304	0.349	0.383	0.410	0.420	0.401	0.879
3.2	0.130	0.160	0.210	0.251	0.294	0.329	0.360	0.374	0.483	1.066
3.6	0.106	0.131	0.173	0.209	0.250	0.285	0.319	0.337	0.566	1.256
4.0	0.087	0.108	0.145	0.176	0.214	0.248	0.285	0.306	0.648	1.447
4.4	0.073	0.091	0.123	0.150	0.185	0.218	0.255	0.280	0.729	1.641
4.8	0.062	0.077	0.105	0.130	0.161	0.192	0.230	0.258	0.810	1.835
5.2	0.053	0.067	0.091	0.113	0.141	0.170	0.208	0.239	0.890	2.031
5.6	0.046	0.058	0.079	0.099	0.124	0.152	0.189	0.223	0.969	2.227
6.0	0.040	0.051	0.070	0.087	0.110	0.136	0.173	0.208	1.048	2.423
6.4	0.036	0.045	0.062	0.077	0.099	0.122	0.158	0.196	1.126	2.620
6.8	0.031	0.040	0.055	0.064	0.088	0.110	0.145	0.185	1.203	2.817
7.2	0.028	0.036	0.049	0.062	0.080	0.100	0.133	0.175	1.280	3.015
7.6	0.024	0.032	0.044	0.056	0.072	0.091	0.123	0.166	1.356	3.213
8.0	0.022	0.029	0.040	0.051	0.066	0.084	0.113	0.158	1.431	3.411
8.4	0.021	0.026	0.037	0.046	0.060	0.077	0.105	0.150	1.506	3.609
8.8	0.019	0.024	0.033	0.042	0.055	0.071	0.098	0.143	1.580	3.808
9.2	0.017	0.022	0.031	0.039	0.051	0.065	0.091	0.137	1.653	4.006
9.6	0.016	0.020	0.028	0.036	0.047	0.060	0.085	0.132	1.727	4.205
10.0	0.015	0.019	0.026	0.033	0.043	0.056	0.079	0.126	1.799	4.404
10.4	0.014	0.017	0.024	0.031	0.040	0.052	0.074	0.122	1.871	4.602
10.8	0,013	0.016	0.022	0.029	0.037	0.049	0.069	0.117	1.943	4.801
11.2	0.012	0.015	0.021	0.027	0.035	0.045	0.065	0.113	2.014	5.000
11.6	0.011	0.014	0.020	0.025	0.033	0.042	0.061	0.109	2.085	5.200
12.0	0.010	0.013	0.018	0.023	0.031	0.040	0.058	0.106	2.155	5.399

The results of research. Now in the additional parameters window for the Colon-Mohr model in Plaxis, you can specify a rigidity module that linearly increases with depth, having previously found it by the formula (20):

$$E_{\text{increment}} = E_i \frac{k_{avg}}{z}, \qquad (20)$$

where E_i – soil deformation module;

z – depth of the bottom of the model;

k_{avg} – the coefficient, which depends on the relative depth of the lower bound of the model, is calculated by (16) or selected from the Table.

Possibilities of practical use of the described method were investigated by comparing the values of precipitation in solving problems with its use and in the usual setting, at different sizes of the model vertically. Also obtained sediments were compared with the solution obtained using the method of layer summing.

Experiments were carried out by increasing the vertical dimensions of the calculation model, with steps equal to the height of the compression zone. Dimensions of the model were taken horizontally according to the recommendations [2], that is, the distance from the object to the lateral faces of the model was taken equal to the height of the compression zone.

In addition, the accepted boundary conditions do not affect the stress-strain state.

The pressure on the base was transmitted through a rigid stove. The pressure and width of the plate varied. The type of boundary conditions in the nodes of the soil foundation model on its faces was taken on the basis of the recommendations given in [1]. The upper boundary of the model remained free for displacements, imposed on the boundary on the horizontal displacements of the lateral faces and horizontal and vertical for the lower limit. The calculation was made taking into account the vertical plane of symmetry by introducing into the model the corresponding ligaments. Fundamentals were simulated with the following physical and mechanical characteristics:

1) E=40 MPa, γ=20 kN/m^3, φ=29°, c=25 kN/m^2;

2) E=14.8 MPa, γ=18.74 kN/m^3, φ=23.4°, c=12.6 kN/m^2;

3) E=35.3 MPa, γ=17.46 kN/m^3, φ=34.9°, c=35.3 kN/m^2.

Analyzing the obtained results of the solution of a number of test problems (Fig. 1), it can be argued that the values of precipitation at different depths with the involvement of the above methodology are well consistent, whereas in the usual formulation, as expected, precipitation rapidly increases in proportion to the increase in vertical dimensions

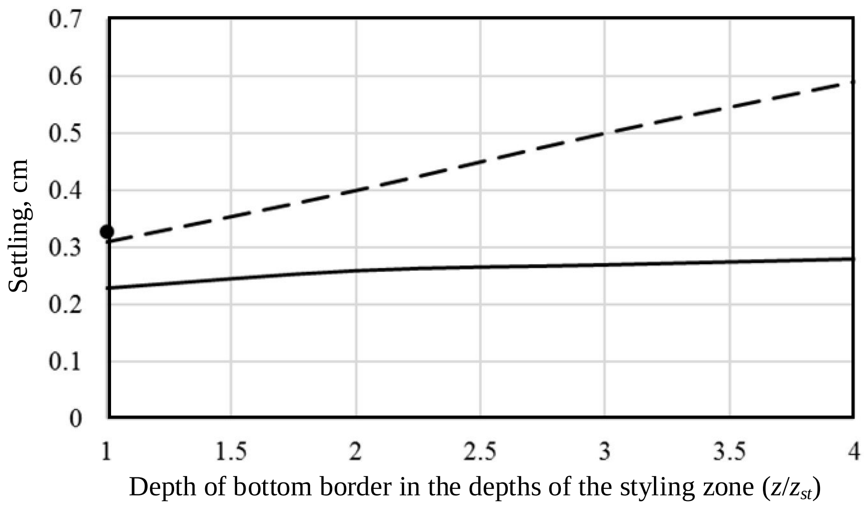

Fig. 1. The dependence of sediments on the vertical dimensions of the model (Dependence is typical for different output data. The sedimentation for soils with physico-mechanical characteristics of the first coin, the pressure p=100 kPa, the width of the site b=1 m) is presented: dashed line – with the usual solution; solid line - with the involvement of the described method; point – a subset, calculated by the method of layer summing

Conclusions The article describes a method for implementing the theory of linearly deformed media in the Plaxis PC. The formulas for modifying the deformation module with depth are derived and the auxiliary coefficients are presented, which are presented in tabular form. Problems solved by the algorithm described in the article are effectively deprived of a flaw in the rapid proportional growth of sediments to increase the depth of the lower boundary of the model. On the other hand, because of the overvalued deformation module, the obtained values of deposition are somewhat lowered compared to the values calculated by the layered summing method. Although the model of the basis and formulas of the method of layer-summing based on the theory of linear deformed media cannot reliably predict the value of sediment, the long-term practical application of this method can be considered as proof of the adequacy of calculated deformations. Therefore, the technique requires further refinement and verification in more difficult situations.

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