

## EFFECT OF REFLECTION FROM THE REMOTE LOAD ON MODE COMPETITION IN MULTIMODE RESONANT ELECTRON OSCILLATORS

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Effect of reflection from the remote load on the mode competition in a two-mode electronic maser is considered. A system of two coupled equations for slowly varying amplitudes of the modes with time delayed coupling is investigated analytically and numerically. It is shown that the reflections can, under the certain conditions, strongly affect the operation regime. Special attention is paid to a gyrotron with build-in quasioptical mode convertor where the radiation after reflection from the window is converted into oppositely rotating mode.

*Keywords:* reflections, mode competition, maser, gyrotron, delayed feedback.

### 1. INTRODUCTION

Reflections from a remote load can strongly affect the dynamics of microwave oscillators, especially gyrotrons [1-7]. Not only reflection from the output window but also reflection from the plasma in electron-cyclotron plasma heating experiment may play a significant role [7]. Moreover, this effect is used in autodyne (self-mixing) detection. In particular, autodyne chaotic oscillators for noise radar systems had been developed in IRE NASU [8,9].

In Refs. [10-12], we studied a general model of a single-mode oscillator with delayed reflection from a remote load. The conditions of stability of steady states were derived analytically. Numerical simulation of transient processes was performed in the most interesting case of large delay and low reflections. Good agreement between theory and simulations was observed. However, most of modern microwave resonant masers, especially gyrotrons, utilize oversized resonators where excitation of different eigenmodes and strong mode-competition phenomena are typical. We believe that delayed reflections from the remote load should strongly affect the mode competition processes.

In this paper, we investigate two different models of resonant multi-mode oscillators with reflections. In Sec. 2, a system of two competing modes described by the well-known equations for slowly varying amplitudes is presented and the influence of additional delayed feedback due to reflections is investigated. In Sec. 3, a gyrotron with build-in quasioptical mode convertor is considered, where the radiation after reflection from the window is converted into oppositely rotating mode.

It is shown that the reflections can, under the certain conditions, strongly affect the operation regime. Complicated sequence of transitions from one regime to another which takes place with the increase of reflection factor is investigated.

### 2. REFLECTION EFFECT ON MODE COMPETITION IN A TWO-MODE MASER

Consider a model of a two-mode electronic oscillator, which is described by the equations for slowly varying complex amplitudes of the modes  $A_{1,2}$  in the

quasi-linear approximation of the electron susceptibility by cubic polynomial:

$$\dot{A}_1 = (\sigma_1 - \beta_1 |A_1|^2 - \gamma_1 |A_2|^2) A_1 + \rho_1 e^{i\psi_1} A_{1\tau}, \quad (1)$$

$$\dot{A}_2 = q (\sigma_2 - \beta_2 |A_2|^2 - \gamma_2 |A_1|^2) A_2 + \rho_2 e^{i\psi_2} A_{2\tau}. \quad (2)$$

In (1), (2) parameter  $q$  is a relation of start-oscillation currents of the two modes. Coefficients  $\sigma_j$ ,  $\beta_j$ ,  $\gamma_j$  are complex functions of electron transit angle; details of derivation are presented in [13]. For simplicity, we assume that these coefficients are real, i.e. the effects of reactive phase nonlinearity are negligible. The last terms in right-hand side of Eqs. (1), (2) describe an influence of delayed reflections,  $\rho_{1,2} \exp(i\psi_{1,2})$  are complex parameters of reflection,  $\tau$  is the delay time,  $A_{j\tau} \equiv A_j(t - \tau)$ . In the case of zero reflections, Eqs. (1), (2) are converting into well-known equations of competing modes [14].

Without reflections dynamics of the system is determined by parameters  $\chi_j = \gamma_j \sigma_i / \beta_i \sigma_j$ ,  $i \neq j$ . Consider the situation when  $\chi_1 < 1$ ,  $\chi_2 > 1$ . In that case, the first mode survives in the competition process and suppresses the second one. Let us investigate how the reflections affect the operation of the oscillator.

Consider a single-mode solution of Eqs. (1), (2):  $A_1 = A_0 \exp(i\omega t)$ ,  $A_2 = 0$  where amplitude and frequency obey the following equations

$$|A_0|^2 = (\sigma_1 + \rho_1 \cos \theta) / \beta_1', \quad (3)$$

$$\omega = -(\beta_1'' |A_0|^2 + \rho_1 \sin \theta), \quad (4)$$

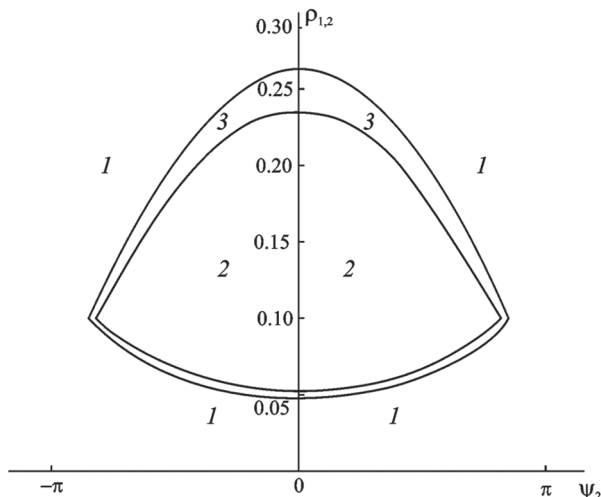
where  $\theta = \omega\tau - \psi_1$ . With growing of reflections, there appear new solutions of the transcendental characteristic equation (4), thus, higher-order steady-state modes arise [10-12]. Frequencies of these modes depend on reflections in such a way that  $\cos \theta \rightarrow 1$  with the increase of  $\rho_1$ .

Detailed analysis of stability of the single-mode solutions with respect to perturbations of the first mode is presented in [10-12]. However, apart from the mechanisms of instability described in [10-12], in the two-mode oscillator there exists one more mechanism caused by excitation of the second mode.

We performed numerical simulation of the mode-competition processes with the increase of

reflections assuming that both modes have equal reflection coefficients:  $\rho_1 = \rho_2$ . The most significant impact of reflections takes place when the phase of the first mode reflection parameter is close to  $\pi$ , and phase of parameter of second mode reflection is close to zero. In that case, reflections result in decrease of the amplitude of the first mode and increase of the second one. Thus, reflections facilitate excitation of the second mode.

In Fig. 1, domains of different regimes on the  $\psi_2 - \rho_{1,2}$  plane are presented. One can see, that even when rather small reflections are entered, transition to the regime of two-mode oscillations is observed. With the increase of  $\rho_{1,2}$ , the second mode completely suppressed the first one. However, with further increase of the reflections, excitation of higher-order steady states of the first mode becomes possible. Since frequencies of these solutions differ from that of the fundamental one (see [10–12]), the value of phase  $\psi_1 = \pi$  does not provide suppression of the first mode. Therefore, a backward sequence of transitions to the two-mode regime and then the regime of the first mode generation is observed.

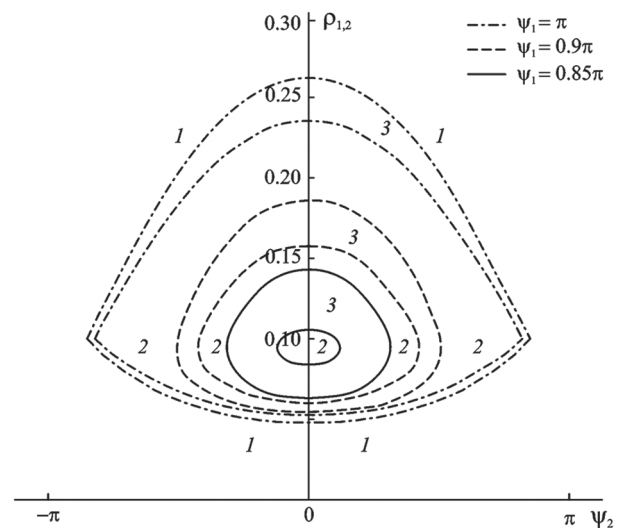


**Fig. 1.** Domains of regimes of generation of the first mode (1), second mode (2), and two-mode generation (3) on  $\psi_2 - \rho_{1,2}$  plane of parameters for  $\chi_1 = 0.9$ ,  $\chi_2 = 1.1$ ,  $\psi_1 = \pi$ ,  $\tau = 10$ ,  $q = 1$

However, the picture described above is quite sensitive to the value of  $\psi_1$ . Numerical simulation shows that when  $\psi_1$  shifts off the value  $\psi_1 = \pi$ , at which reflections suppresses the first mode, the domains of two-mode and second mode oscillations quickly decrease in size. This is illustrated by Fig. 2 where boundaries of the domains of the different regimes are plotted for three different values of  $\psi_1$ . On the contrary, with decreasing of the delay time  $\tau$ , domains of the second mode generation and of the two-mode oscillation grow.

We also studied the case  $\chi_{1,2} < 1$ , when without the reflections regime of two-mode generation is stable. When small reflections appear, the second mode suppresses the first one, since the chosen values of the phases  $\psi_{1,2}$  provide decrease of the amplitude of the first mode and increase of the amplitude of the second

one. However, with further increase of  $\rho_{1,2}$  excitation of a higher-order steady state of the first mode occurs. Frequency of this state shifts off the fundamental one, and, similar to the previous case, a backward transition to the two-mode regime takes place.



**Fig. 2.** Domains of regimes of generation of the first mode (1), second mode (2), and two-mode generation (3) for different values of the phase  $\psi_1$ . Other parameters are the same as in Fig. 1

### 3. REFLECTIONS IN A GYROTRON WITH RADIAL OUTPUT

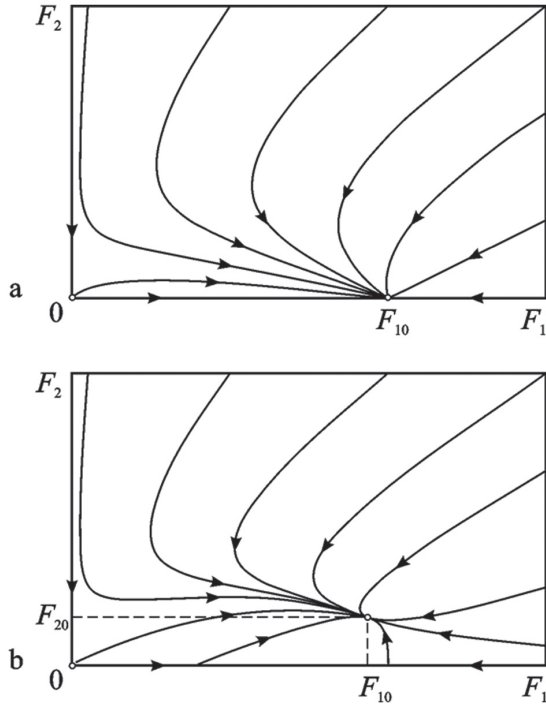
Modern gyrotrons designed for operation at high power levels and high frequencies, especially in sub-terahertz and terahertz region, are operating at very high-order modes,  $TE_{m,n}$ ,  $m, n \gg 1$ . For instance, the gyrotron developed for the International Thermonuclear Experimental Reactor (ITER), operates at the  $TE_{34,19}$  mode; here 34 and 19 are the azimuthal and radial indices, respectively (see e.g. [5,6]). For such gyrotrons, a build-in quasioptical mode convertor is used to convert high-order cavity mode radiation into a Gaussian wave beam. The radiation, after leaving the cavity and propagating through the output waveguide, hits the launcher in which an individual rotating mode loses its identity and with the use of a quasiparabolic reflector is converted into an approximately linearly polarized Gaussian wave beam. This beam is guided by means of phase correcting mirrors to the output window. When, for some reasons, the window is not perfectly matched for this wave beam, the reflected radiation follows the reverse path. On this way, the beam is transformed inside the launcher into the oppositely rotating mode, which returns to the cavity. This means that, in contrast with the situation considered in Sec. 2, in a gyrotron with a radial output one has to consider the competition between oppositely rotating modes with the same azimuthal and radial indices,  $TE_{m,n}^{\pm}$  [5,6]. Here the superscripts “+” and “-” denote positively and negatively rotating modes, respectively.

For the case of competition of the  $TE_{m,n}^{\pm}$  modes, Eqs. (1), (2) can be simplified. First, for the modes with the same azimuthal and radial indices, coefficients

$\beta_j$  and  $\gamma_j$  are equal. Moreover, one can show that  $\gamma_j = 2\beta_j$  [13,15]. Thus, Eqs. (1), (2) become

$$\dot{A}_1 = (\sigma_1 - \beta|A_1|^2 - 2\beta|A_2|^2)A_1, \quad (5)$$

$$\dot{A}_2 = q(\sigma_2 - \beta|A_2|^2 - 2\beta|A_1|^2)A_2 + \rho e^{i\psi} A_{1\tau}. \quad (6)$$



**Fig. 3.** Phase portraits without (a) and with (b) reflections

Here,  $A_1$  is the amplitude of the fundamental mode and  $A_2$  is that of the spurious mode with opposite rotation. It is assumed that the first mode, after leaving the cavity, is partly reflected from the window, changes the direction of its rotation and returns to the cavity affecting the second mode. The secondary mode with opposite rotation usually does not reach the window and the effect of its reflection on the primary mode can be neglected [5,6].

Introducing real amplitudes and phases,  $A_j = F_j/\sqrt{\beta'} \cdot \exp(i\varphi_j)$ , one can rewrite Eqs. (5), (6) as follows:

$$\dot{F}_1 = (\sigma'_1 - F_1^2 - 2F_2^2)F_1, \quad (7)$$

$$\dot{F}_2 = q(\sigma'_2 - F_2^2 - 2F_1^2)F_2 + \rho F_{1\tau} \cos(\varphi_{1\tau} - \varphi_2 + \psi), \quad (8)$$

$$\dot{\varphi}_1 = \sigma''_1 - \lambda(F_1^2 + 2F_2^2), \quad (9)$$

$$\dot{\varphi}_2 = q(\sigma''_2 - \lambda(F_2^2 + 2F_1^2)) + \rho(F_{1\tau}/F_2)\sin(\varphi_{1\tau} - \varphi_2 + \psi), \quad (10)$$

where  $\sigma'_j = \text{Re} \sigma_j$ ,  $\sigma''_j = \text{Im} \sigma_j$ ,  $\beta' = \text{Re} \beta$ ,  $\beta'' = \text{Im} \beta$ ,  $\lambda = \beta''/\beta'$ .

Consider the case when  $\sigma'_1 > 0$ ,  $\sigma'_2 < 0$ . In that case, excitation of the oppositely rotating mode in the gyrotron with perfectly matched window ( $\rho = 0$ ) is impossible. Steady-state solutions of the Eqs. (7)–(10) – are

$$F_j = F_{j0}, \quad \varphi_j = \Omega_j t + \varphi_{j0}, \quad (11)$$

where  $F_{j0}$ ,  $\varphi_{j0}$  and  $\Omega_j$  are constants. Without reflections there exist two steady-state solutions: the unstable zero solution  $F_{j0} = 0$  and the stable one

$$F_{10} = \sqrt{\sigma'_1}, \quad F_{20} = 0, \quad \Omega_1 = \sigma''_1 - \lambda\sigma'_1, \quad (12)$$

which corresponds to generation of the fundamental mode. Phase portrait on the  $F_1 - F_2$  plane is plotted in Fig. 3a.

When small reflections appear,  $\rho \ll \sigma'_1$ , they induce excitation of the secondary mode and two-mode oscillation arises. The stable fixed point on the  $F_1 - F_2$  plane shifts off the horizontal axis  $F_2 = 0$  (Fig. 3b). One can find approximate steady-state solution accurate within  $O(\rho)$ :

$$F_{10} \approx \sqrt{\sigma'_1}, \quad F_{20} \approx \frac{\rho\sqrt{\sigma'_1} \cos \vartheta_0}{q(2\sigma'_1 - \sigma'_2)}, \quad (13)$$

$$\Omega_1 = \Omega_2 \approx \sigma''_1 - \lambda\sigma'_1. \quad (14)$$

Note that frequencies of both modes in the steady state should be equal,  $\Omega_1 = \Omega_2 = \Omega$ . In Eq. (13)

$$\vartheta_0 = \varphi_{10} - \varphi_{20} - \psi - \Omega\tau \quad (15)$$

which can be found from (7)–(10), (13):

$$\text{tg} \vartheta_0 = \frac{\sigma''_1(1-q) - \lambda\sigma'_1(1-2q)}{q(2\sigma'_1 - \sigma'_2)}. \quad (16)$$

Since we consider the case of small reflections, higher-order steady-state solutions discussed in Sec. 2 do not appear.

Thus, even at very small reflections, at the same time with excitation of fundamental mode, in a gyrotron cavity stable forced oscillation of the mode of opposite rotation arise. We believe that this effect is responsible for distortion of the transverse pattern of radiation discovered in [16] where a gyrotron with modulated reflection from an oscillating membrane was studied.

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УДК 621.37

**Влияние отражений от удаленной нагрузки на конкуренцию мод в многомодовых резонансных электронных генераторах** / С.А. Усачева, М.М. Чумакова, М.Ю. Глявин, Ю.В. Новожилова, Н.М. Рыскин // Прикладная радиоэлектроника: науч.-техн. журнал. — 2013. — Том 12. — № 1. — С. 54–57.

Рассматривается влияние отражений от удаленной нагрузки на конкуренцию мод в двухмодовом электронном лазере. Аналитически и численно исследуется система двух связанных уравнений для медленно меняющихся амплитуд мод, в которую входят слагаемые, содержащие запаздывание. Показано, что при определенных условиях отражения сильно влияют на режим генерации. Особое внимание уделяется гиротрону со встроенным квазиоптическим преобразователем мод, где излучение после отражения от окна преобразуется в моду встречного вращения.

**Ключевые слова:** отражения, конкуренция мод, лазер, гиротрон, запаздывающая обратная связь.

Ил. 3. Библиогр.: 16 назв.

УДК 621.37

**Вплив відбиттів від віддаленого навантаження на конкуренцію мод в багатомодових резонансних електронних генераторах** / С.А. Усачова, М.М. Чумакова, М.Ю. Глявін, Ю.В. Новожилова, Н.М. Рискін // Прикладна радіоелектроніка: наук.-техн. журнал. — 2013. — Том 12. — № 1. — С. 54-57.

Розглядається вплив відбиттів від віддаленого навантаження на конкуренцію мод у двухмодовому електронному лазері. Аналітично і чисельно досліджується система двох зв'язаних рівнянь для повільно мінливих амплітуд мод, в яку входять доданки, що містять запізнювання. Показано, що при певних умовах відбиття сильно впливають на режим генерації. Особлива увага приділяється гиротрону з вбудованим квазіоптичним перетворювачем мод, де випромінювання після відбиття від вікна перетворюється в моду зустрічного обертання.

**Ключові слова:** відбиття, конкуренція мод, лазер, гиротрон, запізнений зворотний зв'язок.

Іл. 3. Бібліогр.: 16 найм.