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# RANDOM NOISE SIGNAL GENERATION

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## NONLINEAR DYNAMICS OF DELAYED FEEDBACK MICROWAVE OSCILLATORS

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An overview of research activity on nonlinear dynamics of delayed feedback microwave oscillators in Saratov State University during the ten years passed since the first NRT Workshop is presented. The paper covers a broad range of problems. First, general picture of nonlinear dynamics in delayed feedback oscillators is described. Recent advance in generation of robust hyperbolic chaos in a klystron-type microwave oscillator is discussed. The problem of controlling chaos in delayed feedback oscillators, as well as forced synchronization of such oscillator by external harmonic driving is considered.

*Keywords:* Delayed feedback oscillator, microwaves, klystron, traveling wave tube, hyperbolic chaos, controlling chaos, synchronization.

### 1. INTRODUCTION

Development of sources of high-power noise-like microwave radiation with a relatively wide band is important for noise radar technology, chaos-based communication systems, microwave plasma heating, and a number of other applications [1-3]. The most common schematic of a source of chaotic microwave radiation is a ring-loop oscillator consisting of a power amplifier which output power is partly fed to input through an external delayed feedback transmission line. The first chaotic generator of such kind had been developed as far back as in 1960-ies by V.Ya. Kislov *et al.* using a wide-band traveling wave tube (see e.g. [4]). Note that delayed feedback systems are of great importance not only in electronics but also in nonlinear optics, biophysics, geophysics, etc.

In this paper, we summarize the results of research on nonlinear dynamics of delayed feedback microwave oscillators in Saratov State University during the ten years passed since the first NRT Workshop. The paper is organized as follows. In Sec. 2, general picture of nonlinear dynamics in delayed feedback oscillators is reviewed. Sec. 3 presents a new idea for generation of robust hyperbolic chaos in microwave band by a system of two coupled klystrons. In Sec. 4 application of controlling chaos technique for suppression of spurious self-modulation oscillations using an additional feedback loop is considered. Finally, forced synchronization of a delayed-feedback oscillator driven by an external harmonic signal is studied in Sec. 5.

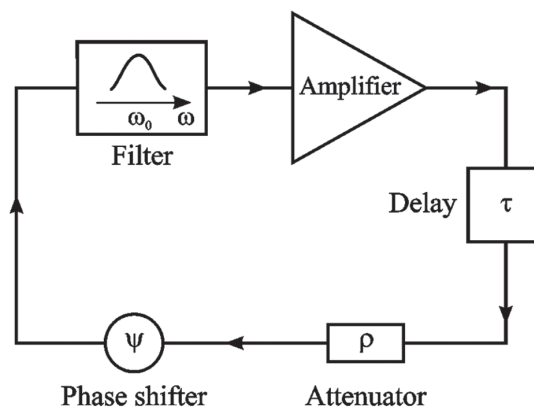
### 2. MODEL OF A DELAYED-FEEDBACK OSCILLATOR

Consider a model of a ring-loop oscillator consisting of a nonlinear power amplifier, a bandpass filter and a feedback leg which contains a delay line, a variable attenuator and a phase shifter which provide control of the amplitude and the phase of the feedback signal (Fig. 1).

Since the filter is assumed to have narrow bandwidth it is convenient to use the slowly varying amplitude approximation. In this approximation dynamics of the oscillator obeys the following equation [5]:

$$\frac{dA}{dt} + \gamma A = \alpha e^{i\nu} F(A_\tau). \quad (1)$$

Here  $A$  is the slow complex amplitude,  $\gamma$  is the parameter of losses,  $\alpha$  is the parameter of excitation proportional to the gain factor of the amplifier,  $F(A)$  is nonlinear transfer function of the amplifier,  $A_\tau = A(t - \tau)$ . Henceforth the delay time is accepted equal to unit that always can be achieved by renormalization of the variables.



**Fig. 1.** Scheme of the delayed feedback oscillator

Consider oscillator with cubic nonlinearity,  $F(A) = (1 - |A|^2)A$ . Seeking for single-frequency solutions of (1),  $A = A_0 \exp(i\omega t)$ , we obtain the following equation for the eigenfrequencies:

$$\omega = -\gamma \operatorname{tg}(\omega\tau - \psi). \quad (2)$$

Here  $\omega$  is the detuning from the central frequency of the system passband  $\omega_0$ . Eq. (2) has infinite number of complex roots, i.e. the time-delayed system with infinite-dimensional phase space has an infinite number of eigenmodes.

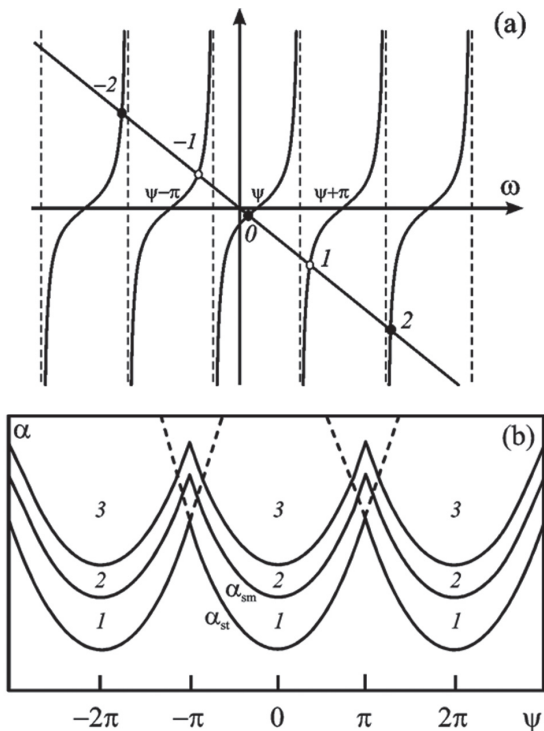
It is convenient to solve Eq. (2) graphically. If the roots are numbered as shown in Fig. 2a one can show that all the solutions can be divided into two classes. For the roots  $\omega_n$  with even numbers  $n = 2k$  the amplitude of oscillations satisfies the equation

$$|A_0|^2 = 1 - \frac{\sqrt{\gamma^2 + \omega_{2k}^2}}{\alpha}. \quad (3)$$

These solutions exist when the parameter  $\alpha$  exceeds the self-excitation threshold  $\alpha > \alpha_{st} = \sqrt{\gamma^2 + \omega_{2k}^2}$ . The solutions with odd numbers  $n = 2k + 1$  exist at any values of parameters, however they are always unstable. Nevertheless, they play an important role because self-modulation is caused by their excitation on the background of the fundamental mode with high amplitude [5]. Further we refer to them as self-modulation modes.

The self-excitation threshold is  $2\pi$ -periodic in  $\psi$  and has a form of discrete domains called “oscillation zones” (Fig. 2b). In the centers of the zones, at  $\psi = 2\pi k$ ,  $\omega_{2k} = 0$  and threshold value of  $\alpha$  is minimal. Near the boundaries of two adjacent zones, at  $\psi = (2k + 1)\pi$ , there is a region of bistability and oscillation hysteresis, where either of the two eigenmodes can survive as a result of a mode competition process, depending on the initial conditions. In Fig. 2b, these domains are bounded by dashed lines.

When the parameter  $\alpha$  increases well above  $\alpha_{st}$  the single-frequency regime becomes unstable and self-modulation arises. Lowest self-modulation threshold  $\alpha_{sm}$  is attained in the centre of a generation zone [5]. Further increase of  $\alpha$  above the self-modulation threshold result in transition to chaos through a sequence of period doubling bifurcations [5].



**Fig. 2.** (a) Graphical solution for eigenfrequencies. (b) Phase diagram on  $\psi$ - $\alpha$  plane: 1 — single-frequency oscillation; 2 — self-modulation; 3 — chaos

Delayed-feedback oscillators show the great variety of different chaotic regimes which are easy to control either by feedback parameters or by an external driving signal. Therefore, recently klystron [6] and traveling wave tube [7] oscillators driven by external signal have been considered as promising sources of chaotic radiation for chaotic-based communication systems at microwave frequencies.

### 3. KLYSTRON-TYPE MICROWAVE GENERATOR OF ROBUST HYPERBOLIC CHAOS

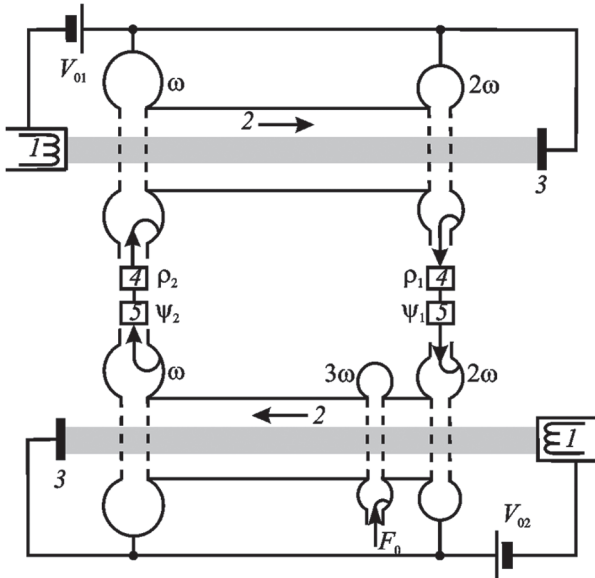
The hyperbolic chaos is known as the strongest type of the chaotic behavior when the strange attractor does not contain any stable periodic orbits and only comprises trajectories of saddle type [8]. These attractors possess the property of structural stability that implies insensitivity of the system dynamics and the attractor structure to variations of parameters and functions describing the system. Recently an approach for design of radio frequency oscillator with hyperbolic attractor has been proposed [9]. The operation principle of such system is alternating excitation of two coupled oscillators so that the transformation of the signal phase is described by chaotic Bernoulli map, which is a classical example of system with hyperbolic attractor [8]. Evidently, such generators are of practical interest for communication and radar systems using chaotic signals.

In [10, 11] we extended this principle to the microwave band using two coupled klystrons. Scheme of the oscillator is presented on Fig. 3. The input cavity of the first klystron is tuned to the frequency of  $\omega$ , while the output one is tuned to frequency of  $2\omega$ . Thus, the first klystron doubles the frequency of the input signal. The output signal of the first klystron is fed to the input cavity of the second klystron via a wide-band dispersionless transmission line containing a phase shifter and an attenuator, which allow the signal phase and amplitude to be adjusted. In the second klystron, this signal is mixed with a reference signal, which represents a periodic sequence of pulses with carrier frequency of  $3\omega$ . Thus, in the second klystron there is a mixing of the signals of the second and third harmonics. In the output cavity of the second klystron a signal on a difference frequency of  $\omega$  is separated and fed to the input cavity of the first klystron, thus closing the feedback circuit.

The dynamics of the oscillator is described by the following system of dimensionless delay-differential equations (DDEs) [10, 11]

$$\begin{aligned} \dot{F}_1^\omega + F_1^\omega &= \rho_2 e^{i\nu_2} F_2^\omega / \sqrt{2}, \\ \dot{F}_1^{2\omega} + \delta F_1^{2\omega} &= 4\alpha_1 J_2 \left( 2 \left| F_{\tau,1}^\omega \right| \right) e^{2i(\varphi_{\tau,1}^\omega - \theta_0)}, \\ \dot{F}_2^{2\omega} + \delta F_2^{2\omega} &= \sqrt{2} \rho_1 e^{i\nu_1} F_1^{2\omega}, \\ \dot{F}_2^\omega + F_2^\omega &= 2\alpha_2 e^{-i\theta_0} \sum_{m=-\infty}^{\infty} i^m J_{3m+1} \left( \left| F_{\tau,2}^{2\omega} \right| \right) \times \\ &\times J_{2m+1} \left( \left| F_2^{3\omega}(t) \right| \right) e^{-i(3m+1)\varphi_{\tau,2}^{2\omega}}. \end{aligned} \quad (4)$$

Here  $F_j^{k\omega}(t)$  are dimensionless slowly varying complex amplitudes of the signals in corresponding cavities,  $\varphi_j^{k\omega} = \arg(F_j^{k\omega})$ , the subscripts  $j = 1, 2$  henceforth indicate the number of klystrons, the superscripts  $\omega, 2\omega$  denote the resonance frequencies of the cavities;  $\tau$  is the normalized delay parameter;  $\theta_0$  is the unperturbed electron transit angle in the drift space; parameter  $\delta = 2Q^\omega/Q^{2\omega}$  defines the ratio of  $Q$ -factors of the cavities operating at frequencies  $\omega$  and  $2\omega$ ; parameters  $\rho_j$  and  $\psi_j$  are attenuations and phase shifts in the coupling transmission lines, respectively;  $J_n$  is  $n^{\text{th}}$  order Bessel function of the 1<sup>st</sup> kind. The excitation parameters  $\alpha_j$  which can be treated as the normalized dc electron beam currents most significantly influence the oscillator dynamics. The equations (4) are derived in a similar way as for other klystron-type delayed feedback oscillators (see e.g. [12]).



**Fig. 3.** Scheme of the proposed chaos generator based on coupled drift klystrons. 1 – electron guns, 2 – electron beams, 3 – collectors, 4 – variable attenuators, 5 – phase shifters

The reference signal at the third harmonic frequency is supplied from an external driving source in the form of a sequence of pulses with constant amplitude  $F_0$  and a repetition period equal to the time of signal passage via the feedback circuit,  $2\tau$ .

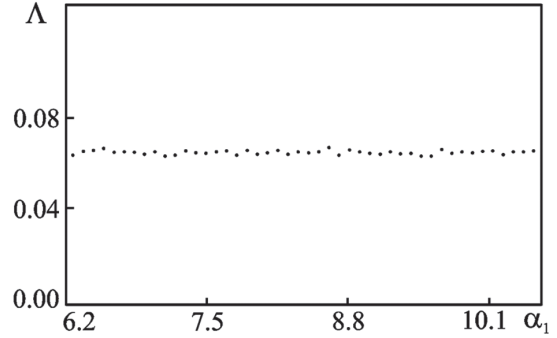
Assuming that the oscillation build-up in the cavities is fast in comparison with delay time one can neglect the derivatives in (4) and consider the variables in discrete moments of time  $t_n = 2n\tau$ . As a result, after some mathematical transformations DDEs (4) are reduced to the 2-D iterative map

$$F_{n+1} e^{i\varphi_{n+1}} \approx r J_2(2F_n) e^{i(\Delta - 2\varphi_n)}, \quad (5)$$

where  $r = 4\alpha_1\alpha_2\rho_1\rho_2\delta^{-2}J_1(F_0)$ ,  $\Delta = \psi_2 - \psi_1 + \theta_0$  [10,11]. From (5) one can see that the dynamics of the phase obeys the Bernoulli map  $\varphi_{n+1} = \Delta - 2\varphi_n$  that demonstrates the hyperbolic chaotic dynamics with a positive Lyapunov exponent of  $\Lambda = \ln 2$ .

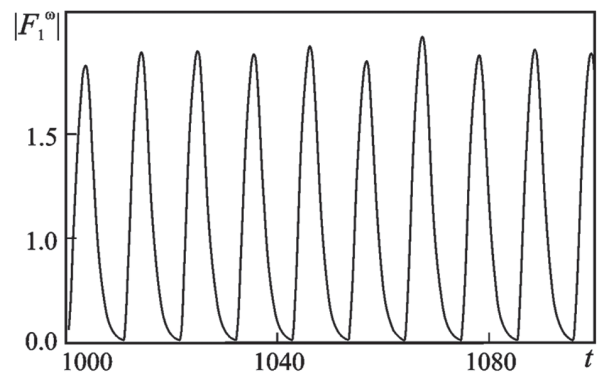
Numerical simulation of the DDEs (4) confirms that the generator is capable to produce robust

hyperbolic chaotic oscillation. The values of parameters approximately correspond to the parameters of the millimeter band oscillators described in [12]. In Fig. 4, the plot of largest Lyapunov exponent of the hyperbolic attractor vs. the excitation parameter  $\alpha_1$  is shown. For comparison with the results for the map (5), we calculated the Lyapunov exponent for the stroboscopic Poincare map ( $t = 2n\tau$ ). The largest Lyapunov exponent is almost independent from the parameter and approximately equal to  $\ln 2$  that indicates structural stability of the chaotic attractor.



**Fig. 4.** Largest Lyapunov exponent of the hyperbolic attractor vs.  $\alpha_1$  at  $\alpha_2 = 15.0$

Fig. 5 shows the typical waveform of the amplitude in the input cavity of the first klystron in the regime of hyperbolic chaos. One can see that the signal has the form of pulse sequence with nearly constant amplitude. However, the phase of the signal varies irregularly from pulse to pulse [9] providing robust chaotic signal. This is confirmed by Fig. 6 where typical examples of iterative diagram for the phase of subsequent pulses and projection of the attractor onto the  $\text{Re } F_1^\omega - \text{Im } F_1^\omega$  plane are presented. For this plots we take the values of the variables at the moments of time when the amplitude  $|F_1^\omega|$  reaches its local maximum. The attractor has a topology of the Smale–Williams solenoid, which is typical for the systems with hyperbolic chaos. The angular coordinate of the attractor obeys the Bernoulli map.



**Fig. 5.** Waveform of the field amplitude in the input cavity of the first klystron in the regime of hyperbolic chaos

Despite of the hypothesized hyperbolic nature of the chaotic attractor, the considered scheme of the generator is of interest itself, since it reveals an opportunity to obtain robust structurally stable chaos at

microwave frequencies. This property is very important for possible applications in chaos-based communication and radar systems.

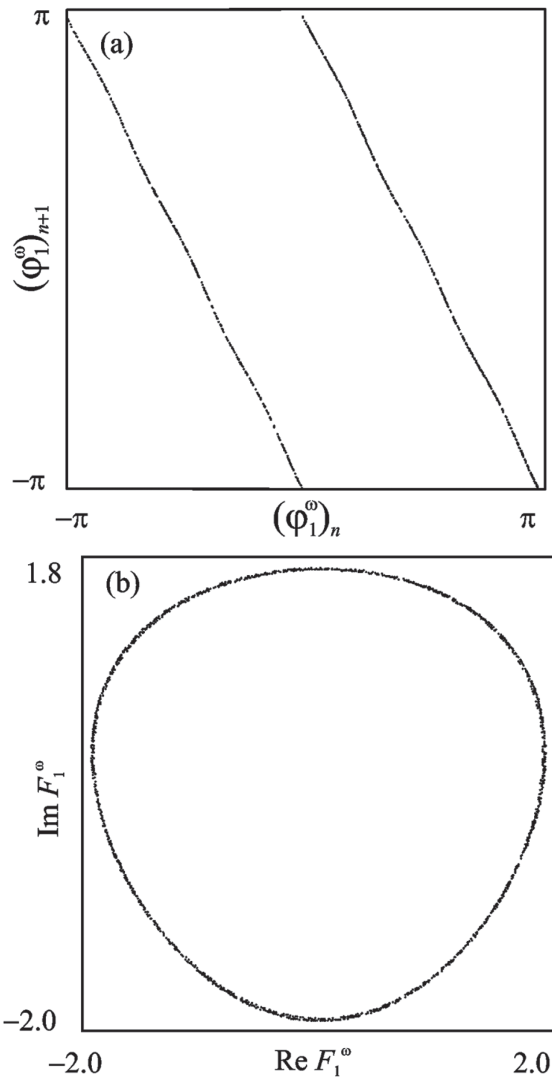


Fig. 6. The iterative diagram for the phase of the oscillations (a) and the projection of the phase portrait on  $\text{Re } F_1^\omega - \text{Im } F_1^\omega$  plane (b)

#### 4. SUPPRESSING SELF-MODULATION AND CONTROLLING CHAOS IN DELAYED FEEDBACK OSCILLATORS

As is mentioned in Sec. 2, self-modulation instability is typical for the oscillators with time delayed feedback. This instability results in generation of multiple frequencies or even spread spectrum chaotic signal. Thus the instability restricts the maximal output power and efficiency of an oscillator in the single-frequency regime.

In [13-15] a method for suppression of the self-modulation was proposed. This method expands the well-known idea of time-delayed feedback chaos control [16] on delayed feedback oscillators. It is based on adding an additional control feedback with the parameters chosen so that after passing through the two feedback legs the fundamental waves appear in the same phase, while the self-modulation sidebands appear in anti-phase, and thus, suppress each other.

Consider the general scheme of a ring-loop oscillator which consists of an amplifier and a delayed feedback path. To stabilize single frequency generation regimes we split the feedback leg into two paths as shown in Fig. 7. Let parameter  $k$  define relative power level of the signals passing through the two feedback legs. Assume that we are able to adjust the delay times  $\tau_{1,2}$  and phase shifts  $\psi_{1,2}$ . Considering propagation of a modulated signal with fundamental frequency  $\omega$  and modulation frequency  $\Omega$  one can show [13-15] that for suppression of the sidebands the parameters should satisfy the following relations:

$$\psi_1 - \psi_2 - \omega(\tau_1 - \tau_2) = 2\pi n, \quad (6)$$

$$\Omega(\tau_1 - \tau_2) = 2\pi m + \pi. \quad (7)$$

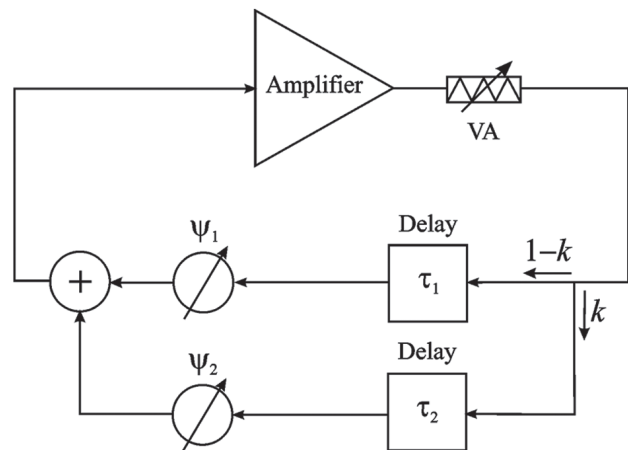


Fig. 7. Scheme of an oscillator with double delayed feedback loop

We demonstrate successful application of the method on two examples. We performed numerical simulations for a generalized model of a delayed-feedback oscillator (1) [13], as well as for klystron [15] and TWT [14] oscillators. For example, the Eq. (1) for the case of two feedback loops should be modified as follows

$$\begin{aligned} \frac{dA}{dt} + \gamma A = \\ = \alpha \left[ (1-k)e^{i\psi_1} F(A_{\tau_1}) + ke^{i\psi_2} F(A_{\tau_2}) \right]. \end{aligned} \quad (8)$$

Numerical results show that the application of the additional control feedback allows suppression of self modulation including chaotic spread spectrum oscillation. Fig. 8 shows a typical example of steady-state output signal amplitude  $F_{out}$  vs. the excitation parameter  $\alpha$  for a two-cavity klystron oscillator. The circles denote stable steady states while the squares denote unstable steady states that are stabilized by applying the control feedback. One can see that the self-modulation threshold increases from  $\alpha = 24.06$  to  $\alpha = 44.01$ . Suppose that we keep the amount of feedback constant and increase  $\alpha$  by increase of the beam current. In the steady-state regime output power  $P \sim |F_{out}|^2$ . So from Fig. 8 one can estimate that the beam current at which the single frequency regime is stable can be increased in approximately in 1.83 times, and this

result in 1.5 increase of the output power. However, the electronic efficiency  $\eta \sim F_{out} J_1(F_{out})$  decreases approximately in 1.25 times.

By adjusting the phase of the controlling feedback to  $\psi_2 \approx \psi_1 + \pi$  the power can be increased by a factor of 3 or more [15].

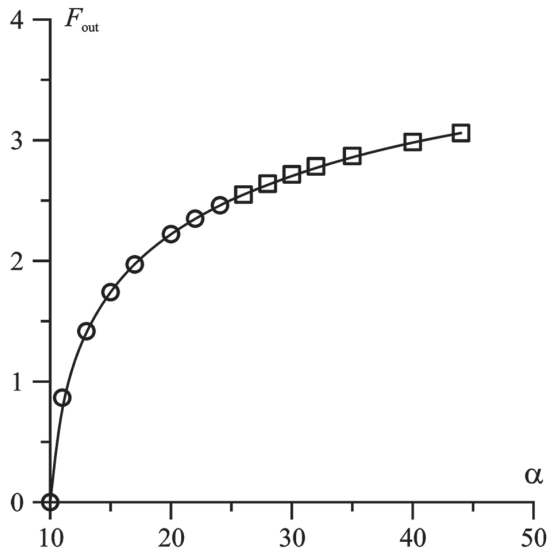


Fig. 8. Typical example of output signal amplitude vs.  $\alpha$  for the two-cavity klystron oscillator

Similar results were obtained for the TWT oscillator. Basic equations and details of nonstationary simulations are described in [14]. Fig. 9 shows a typical plot of output power and electronic efficiency vs. normalized length  $L = 2\pi CN$  where  $C$  is the Pierce gain parameter and  $N$  is the phase length of the tube [17].

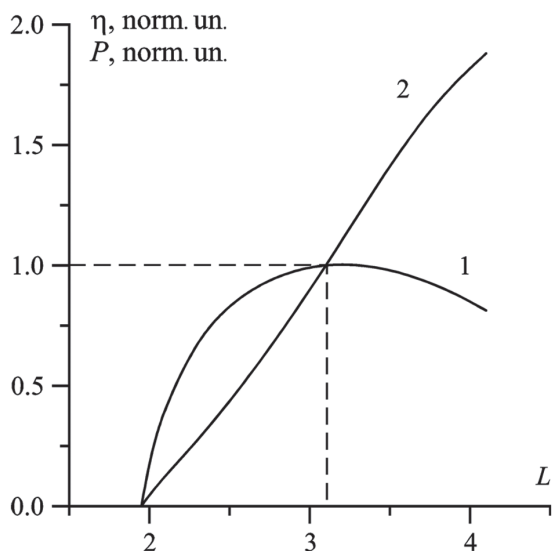


Fig. 9. Normalized TWT efficiency (1) and output power (2) vs. normalized length  $L = 2\pi CN$ . Self-modulation in a system without control arise at  $L = 3.1$

Power and efficiency are normalized to the values at  $L = 3.1$ , that is the self-modulation threshold in the oscillator without the control. Again we suppose that the amount of feedback is kept constant while the beam current, and thus,  $L$  increases. Nearly twice increase of the threshold beam current and output

power is observed, with simultaneous decrease of the efficiency. Similar to the klystron oscillator, maximal efficiency is reached near the self-modulation threshold in the oscillator without the control. However, in the TWT oscillator it is possible to stabilize not only fundamental mode, but also higher-order modes which interact with the beam more efficiently. In that case, efficiency enhancement becomes possible [14].

### 5. FORCED SYNCHRONIZATION OF A DELAYED FEEDBACK OSCILLATOR

Investigation of forced synchronization of a delayed feedback oscillator by an external harmonic driving is important in connection with recent projects of using non-autonomous klystron and TWT oscillators in chaos-based communication systems [6, 7]. Owing to multimode nature of delayed-feedback systems one can expect that processes of synchronization will have a number of special features in comparison with systems with low number of degrees of freedom. The picture of forced synchronization of a general model of the delayed-feedback oscillator (1) was thoroughly investigated in [18].

Consider oscillator (1) with cubic nonlinearity driven by an external harmonic signal

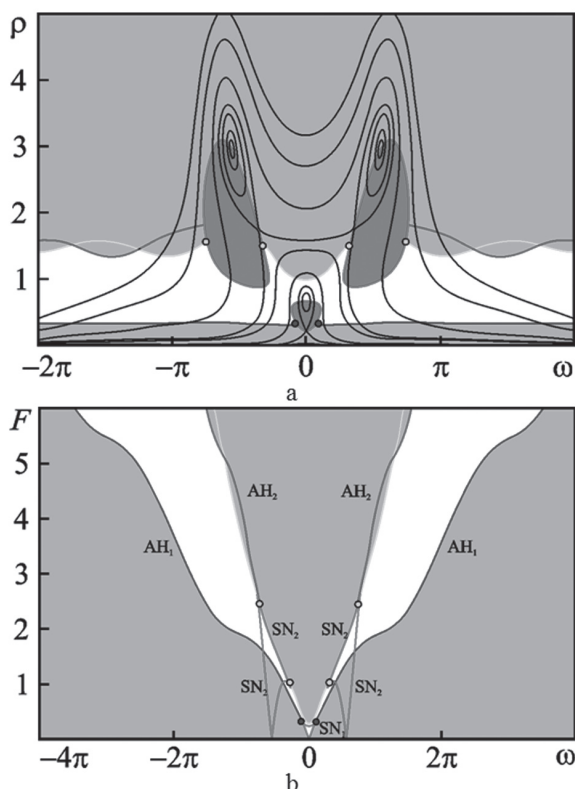
$$\frac{dA}{dt} + \gamma A = \alpha e^{i\psi} (1 - |A_c|^2) A_c + F e^{i\omega t}. \quad (9)$$

Let us start from the case  $\psi = 0$  that corresponds to the centre of a generation zone that is optimal conditions for self-excitation, and choose the other parameters  $\gamma = 0.3$  and  $\alpha = 0.9$ . Such a value of  $\alpha$  exceeds the generation threshold  $\alpha_{st} = 0.3$  but lies below the self-modulation threshold  $\alpha_{sm} = 1.33$ , so the free-running system exhibits stable single-frequency oscillations.

Solutions of (9)  $A_0 \exp(i\omega t)$  correspond to the forced synchronization mode. We studied the stability of these solutions analytically and numerically. In Fig. 10a the picture of frequency responses (resonant curves)  $\rho(\omega)$  where  $\rho = |A_0|^2$  is presented. The bottom part of the Fig. 10a is similar to a classical picture of resonance curves for synchronization of a system with one degree of freedom [19]. However in the domain of high amplitudes there exist significant differences. First of all, one should notice the domains of phase locking at frequencies of self-modulation modes. Second, the resonance curves become unstable in the domain of high amplitudes. This instability is caused by excitation of sidebands with frequencies of self-modulation modes. Despite there is no self-modulation in the free-running oscillator for the chosen values of parameters, excitation of self-modulation modes is possible when intensive external driving is applied. We name this regime as *drive-induced self-modulation*. Fig. 10b shows a corresponding synchronization tongue on the  $\omega - F$  plane. Lines of saddle-node (SN) and Andronov–Hopf (AH) bifurcations are shown. These lines contact with each other in Bogdanov–Takens (BT) points marked by circles. Domain of stable synchronous operation is shown in white color, unstable domains are shaded.

Notice that in the free-running oscillator self-modulation is caused by excitation of two sidebands which are equidistant from the fundamental frequency (modes with numbers  $n = \pm 1$  in Fig. 2a). Accordingly, the boundary of drive-induced self-modulation in Fig. 10 consists of two intertwined curves which correspond to excitation of mode with either  $n = 1$  or  $n = -1$ .

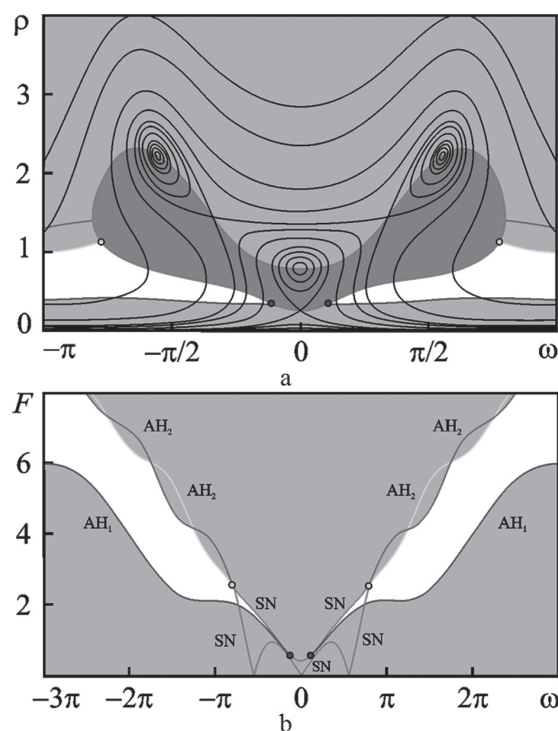
It is well known that SN bifurcation corresponds to phase-locking of the free-running oscillator [19]. Note that there exist two kinds of SN bifurcations, one corresponding to phase-locking of the fundamental mode near  $\omega = 0$  ( $SN_1$ ) and the others near  $\omega \approx \pm 0.55\pi$  ( $SN_2$ ) corresponding to the self-modulation modes (see Sec. 1) which are always unstable. There also exist domains of phase-locking of higher-order self-modulation modes not shown in Fig. 10. Similarly, there are two kinds of AH bifurcations: one ( $AH_1$ ) corresponds to synchronization via suppression of natural dynamics, while the other ( $AH_2$ ) one corresponds to drive-induced self-modulation. Therefore, there are two kinds of BT points marked by light and dark circles respectively.



**Fig. 10.** (a) The picture of resonance curves in the center of generation zone,  $\psi = 0$ ,  $\gamma = 0.3$ ,  $\alpha = 0.9$ ; (b) Synchronizations tongue on  $\omega - F$  plane. SN — saddle-node bifurcations; AH — Andronov–Hopf bifurcations. Bogdanov–Takens points are shown by circles

We also investigated transformation of resonant curves and synchronization tongues with the approaching to the self-modulation threshold. With the increase of  $\alpha$ , the phase-locking domains formed at the fundamental and two self-modulation modes merge, breaking the domain of stability into three separated parts. Accordingly, the synchronization

tongue also breaks into three partly overlapping parts that corresponds to bistability. While parameter  $\alpha$  approaches to the self-modulation threshold the central part of synchronization tongue decreases in size. Finally, when  $\alpha > \alpha_{sm} = 1.33$ , the central domain of synchronization vanishes (Fig. 11). Now the tongue consists of two separate parts and the synchronization tongue does not touch the  $F = 0$  axis, i.e. synchronization threshold appears. This occurs since the free-running oscillator generates quasiperiodic self-modulated oscillation with two independent frequencies.



**Fig. 11.** The picture of resonance curves (a) and synchronizations tongue (b) above the self-modulation threshold:  $\psi = 0$ ,  $\gamma = 0.3$ ,  $\alpha = 1.45$

The analytical results presented above were verified by direct numerical simulation of the Eq. (9). Simulation of the process of transition to synchronous regime with increasing of the driving force detected the very complicated picture of dynamical regimes. When the driving frequency is close to the natural frequency of the oscillator, synchronization via phase locking occurs similar to system with one degree of freedom. However, the mode-locking process is accompanied by periodic excitation and decay of self-modulation modes [18].

When the driving frequency shifts off the natural frequency, synchronization occurs via suppression of the natural frequency. In that case, period-doubling cascade and transition to chaos precedes transition to the synchronous mode. Such a behavior can be explained as follows. The free-running system operates close to the self-modulation threshold, i.e. with the increase of the excitation parameter  $\alpha$  the sequence of period doublings is observed. External driving increases the amplitude of oscillation thus stimulating the sequence of bifurcations observed in the free-running system.

When the driving frequency is close to the frequency of a secondary eigenmode hard transition to the synchronous regime is observed. The hard transition is accompanied by hysteresis and a narrow band of bistability appears near the boundary of the domain of synchronization.

The mechanisms described above are illustrated in Fig. 12 where the phase diagram on the  $\omega - F$  plane is presented. The lines of different bifurcations which form the boundary of a synchronization tongue (similar to Figs. 10,11) are also shown. Domain of synchronization is shown by white, domain of bistability is hatched. Domains of period doublings ( $T_2$ ) and chaotic dynamics (C) are also shown. One can see that the domain of chaotic dynamics is located close to the self-modulation frequency ( $\omega \approx 0.6\pi$ ). Inside this domain there exist a lot of narrow windows of periodic motion which are not shown in Fig. 12.

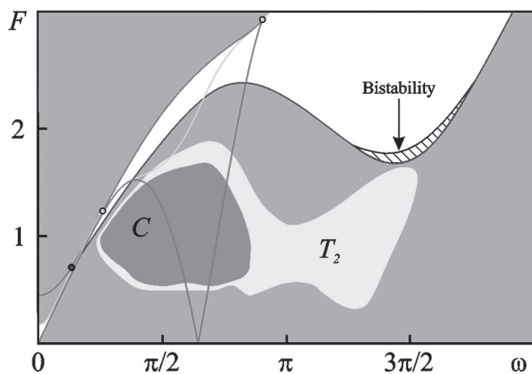


Fig. 12. Phase diagram on  $\omega - F$  plane at  $\psi = 0$ ,  $\gamma = 1.0$ ,  $\alpha = 2.5$

## 6. CONCLUSION

In this paper, we presented an overview of research activity on nonlinear dynamics of delayed feedback microwave oscillators in Saratov State University. The delayed feedback oscillators are distributed systems with infinite number of degrees of freedom and demonstrate a very complicated behavior including multiple transitions between regular and chaotic regimes. By that reason, they have good prospects as sources of chaotic microwave radiation for communication and radar systems.

In particular, the ring-loop oscillator consisting of two coupled klystrons with resonators tuned to fundamental and second harmonic frequencies is proposed. This oscillator is capable to generate structurally stable hyperbolic chaos at microwave frequencies. Structural stability means insensitivity of the system dynamics and the attractor structure to variations of parameters of the system.

The method to suppress self-modulation instability based on an additional delayed feedback circuit is proposed. It allows substantial increase in the beam current that corresponds to the stability of the stationary single-frequency generation and nearly threefold increase in the generation power.

The picture of forced synchronization in delayed feedback systems is investigated. In the domain of the small amplitudes the picture of synchronization is

similar to the classical one of the oscillator with one degree of freedom. However, with growth of amplitude there are a lot of differences caused by excitation of different self-modulation modes.

## ACKNOWLEDGMENTS

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**Нелинейная динамика микроволновых генераторов с запаздывающей обратной связью** / Н.М. Рыскин, В.В. Емельянов, О.С. Хаврошин, С.А. Усачева, А.В. Яковлев // Прикладная радиоэлектроника: науч.-техн. журнал. – 2013. – Том 12. – № 1. – С. 37–44.

Представлен обзор исследований в области нелинейной динамики микроволновых автогенераторов с запаздыванием, выполненных в Саратовском государственном университете в течение последних десяти лет. В статье затрагивается широкий круг вопросов. Описана общая картина нелинейной динамики генераторов с запаздыванием. Обсуждаются последние достижения в области разработки микроволновых генераторов грубого гиперболического хаоса клистронного типа. Рассматриваются проблемы управления хаосом в генераторах с запаздыванием, а также синхронизации таких генераторов внешним гармоническим сигналом.

**Ключевые слова:** генератор с запаздывающей обратной связью, микроволны, клистрон, лампа бегущей волны, гиперболический хаос, управление хаосом, синхронизация.

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**Нелінійна динаміка мікрохвильових генераторів із запізненим зворотним зв'язком** / Н.М. Рискін, В.В. Ємельянов, О.С. Хаврошин, С.А. Усачова, А.В. Яковлев // Прикладна радіоелектроніка: наук.-техн. журнал. – 2013. – Том 12. – № 1. – С. 37–44.

Представлено огляд досліджень в галузі нелінійної динаміки мікрохвильових автогенераторів із запізненням, виконаних у Саратовському державному університеті протягом останніх десяти років. У статті порушується широке коло питань. Описана загальна картина нелінійної динаміки генераторів з запізненням. Обговорюються останні досягнення в галузі розробки мікрохвильових генераторів грубого гіперболического хаосу клістронного типу. Розглядаються проблеми управління хаосом в генераторах з запізненням, а також синхронізації таких генераторів зовнішнім гармонічним сигналом.

**Ключові слова:** генератор із запізненим зворотним зв'язком, мікрохвилі, клістрон, лампа біжучої хвилі, гіперболический хаос, управління хаосом, синхронізація.

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