UDC 519:616

METRICAL EVALUATION OF SPATIAL CONTENT FOR SEGMENTATION-BASED IMAGE RETRIEVAL

ALEXANDER KAGRAMANYAN, VLADIMIR MASHTALIR, VLADISLAV SHLYAKHOV

Interest in the image content analysis has been motivated by expansion of imaging in manifold scopes of activity, the availability of large image libraries, growth of multimedia applications, etc. One of the ways for partial elimination of a semantic gap between low-level visual features and high-level human concept is to analyze spatial properties of image parts specifically induced by segmentation. To provide a region-based image retrieval with a query 'ad exemplum' (to wide extent), a new metric to compare arbitrary nested partitions is proposed. Studied metrical properties of partially ordered quotient sets provide e.g. the objects search independent on the background.

Keywords: image segmentation, metric, nested partitions.

INTRODUCTION

Tremendous growth of automated image processing applications including content-based image retrieval (CBIR) with queries 'ad exemplum' often aligns semantic matching with foreground scene of the research issues [1, 2]. Transition of a percept into the concept strongly depends on an approach to image understanding. Spatial content of an image may be extracted from a collective structure of homogeneous in appropriate feature space regions. Therefore, there is a great need for automatic tools which should to classify and retrieve image content on the base of segmentation.

There exists a demand to bridge a semantic gap between low-level features and human concepts [3]. Consideration of spatial image content in a metric quantic of collective structure of region families is an advance (though sufficiently small) over simple features analysis. Though for last years researches have actively explored this area, the fundamental problem of similarity measuring of two complex objects described by its partitions still remains unsolved. There arises a need for search metrics which will not be sensitive to varying acquisition, partial objects occlusions, colors transformations.

Thus, there are many reasons for the study of metrical properties of set partitions since they are models of arbitrary crisp clustering. Valid metric on quotient sets (clusters, segmentations) is a crucial issue of CBIR if segmentation should be used to organize image content according to categories that are meaningful to humans. It should be emphasized that metrics on partial ordered quotient sets are underlying tools of image content analysis as they comply with construction of a hierarchy either the top-down or the bottom-up approaches and one can explain wholes by decomposing them into smaller and smaller parts or alternatively one can construct wholes from smaller parts.

One more requirement for new developed similarity measures (to be a metric) is explained by possibilities to speedup a search. The information about distances between objects in the database is utilized to discard entire sets of images at the search stage by applying of triangular inequality [4].

Our contribution consists in theoretical ground of nested partitions metrics to get novel features (spatial content of images) for content-based image retrieval. The rest of the paper is organized as follows: Section 2 gives the groundwork of the metrics on partitions, Section 3 presents metrics on nested partitions, Section 4 is devoted to metrical properties of partial ordered quotient sets, Section 5 includes discussion of experiments and results.

1. GROUNDWORK

Among the most promising metrics which particularly have desirable properties we can indicate the Earth Mover's Distance (EMD) [5], variation of information [6], Mirkin [7], van Dongen [8] metrics, and quite a few of related measures [9]. However, all of them are valid only for finite-dimensional sets and either have considerable computational complexity or have low-sensitivity under meaning changes of partitions. The partition metric introduced in [10] and extended in [11] to arbitrary measurable set has not these disadvantages. It includes similarity and dissimilarities measures of equivalence classes simultaneously with simplest computability and is most suitable to analyze partial ordered (relatively to inclusion) partitions. Consider necessary preliminaries to represent spatial content metrical evaluation.

Let Ω be an arbitrary measurable set with a measure $\mu(\Omega)<\infty$, i.e. for any $A\subset\Omega$ exists some number $\mu(A)$ which is the measure (length, area, volume, mass distribution, probability distribution, cardinality, etc.). Let $\mathcal{P}_\Omega$ be a power set in which all subsets are measurable also. Introduce the set $\Pi_\Omega\subset\mathcal{P}_\Omega$ of finite (regarding the number of cosets) partitions of set Ω s.t. $\alpha\in\Pi_\Omega$, $\alpha=\{A_i\}_{i=1}^n$, $A_i\in\mathcal{P}_\Omega$, $\Omega=\bigcup_{i=1}^n A_i$, $\forall i,j\in\{1,2,\&,n\}: i\neq j\Rightarrow A_i\cap A_j=\varnothing$.

With the key assumption of image retrieval on basis of the spatial content produced by any segmentation, the set Ω is none other than a field of view. Quotient sets viz the sets A_i are ipso facto generated either by different algorithms or by the same algorithm with varied parameters. Figure 1 illustrates the appearance of nested partitions corresponding to spatial refinement levels which this way or another go with human image understanding. Thus, there arises a necessity to compare partitions and what is more nested quotient sets. In other words, we need mathematical tools to evaluate spatial content.

The metric on Cartesian square $\Pi_\Omega \times \Pi_\Omega$ is [11]

$$\rho(\alpha,\beta)=\sum_{i=1}^{n}\sum_{j=1}^{m}\mu(A_i \Delta B_j)\mu(A_i \cap B_j) \qquad (1)$$

where $\beta=\{B_j\}_{j=1}^{m}$ and $A_i \Delta B_j=(A_i \setminus B_j)\cup(B_j \setminus A_i)$ is a symmetrical difference.

A tantamount form generalizing Mirkin metric was found for functional (1) [11]

$$\begin{aligned}\rho(\alpha,\beta)=\sum\nolimits_{i=1}^{n}[\mu(A_i)]^2+\sum\nolimits_{j=1}^{m}[\mu(B_j)]^2-\\ -2\sum\nolimits_{i=1}^{n}\sum\nolimits_{j=1}^{m}[\mu(A_i \cap B_j)]^2.\end{aligned} \qquad (2)$$

To use (1), (2) efficiently for nested partition we have at first to find formulae realized one quotient set inclusion in another and then to get relevant criterion of nesting.

2. METRICS ON NESTED PARTITIONS

Consider two nested partitions when one is splitting other. For definiteness we assume $\alpha \subseteq \beta$ i.e.: for any $A_i \in \alpha$ can be found some $B_j \in \beta$ for which $A_i \subseteq B_j$. In other words, partition β breaks up into m 'subpartitions' i.e. $\alpha=\{\alpha_1^*,\ldots,\alpha_m^*\}$ and

$$\begin{cases}\alpha_1^*=\{A_1,\ldots,A_{k_1}\};\\ \alpha_2^*=\{A_{k_1+1},\ldots,A_{k_1+k_2}\};\\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\ \alpha_j^*=\{A_{k_1+k_2+\ldots+k_{j-1}+1},\ldots,A_{k_1+k_2+\ldots+k_j}\};\\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\ \alpha_m^*=\{A_{k_1+k_2+\ldots+k_{m-1}+1},\ldots,A_{k_1+k_2+\ldots+k_m}\}\end{cases}$$

where α_j ($j\in\overline{1,m}$) contains k_j elements of partition α (see Figure 2) and

$$B_j=\bigcup\nolimits_{i=k_1+k_2+\ldots+k_{j-1}+1}^{k_1+k_2+\ldots+k_j}A_i\ ,$$

where $k_1+k_2+\ldots+k_m=n$.

Next propositions provide the computational complexity curtailment with respect to (1) and (2) formulae.

Proposition 1. For any two finite (relative to number of cosets) partitions $\alpha,\beta\in\Pi_\Omega$ of arbitrary measurable set Ω if $\alpha\subseteq\beta$ then

$$\rho(\alpha,\beta)=\sum\nolimits_{j=1}^{m}[\mu(B_j)]^2-\sum\nolimits_{i=1}^{n}[\mu(A_i)]^2\ .$$

Proposition 2. For any two partitions from Proposition 1 metric (1) can be expressed via elements of embedded partition by two means

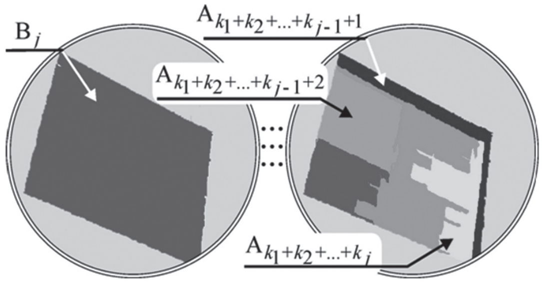

Fig. 2. Explanation of partitions indexing

$$\rho(\alpha,\beta)=\sum_{j=1}^{m}\sum_{i=1}^{k_j}\mu(A_{ji})[\sum_{i'=1\,(i'\neq i)}^{k_j}\mu(A_{ji'})]=$$

$$=2\sum_{j=1}^{m}\sum_{i=1}^{k_j-1}\sum_{i'=i+1}^{k_j}\mu(A_{ji})\mu(A_{ji'}).$$

Hereinafter we shall use the symbol $\mathbb{O}$ to denote conventional partitions origin consisting of the source set Ω i.e. $\mathbb{O}=\{\Omega\}$. Also for simplicity of notation, we write $G(\alpha)$ instead of $\rho(\alpha,\mathbb{O})$ and $S(\alpha)$ instead of $\sum_{i=1}^{n}[\mu(A_i)]^2$. It should be noted that namely these functionals $G(\alpha)$ and $S(\alpha)$ allow us to describe all properties of nested partitions.

If partitions are produced by different ways, there arises a question: are they nested? Let $f(x)$, $x\in\mathbb{R}^2$ be an image and segmentation measure is defined as

$$\mu(A_{jk})=\int_{A_{jk}}f(x)dx\ ,\ \mu(B_j)=\int_{B_j}f(x)dx\ .$$

Since partitions α,β are embedded into $\mathbb{O}$, we get triangle inequality $\rho(\alpha,\beta)\le\rho(\alpha,\mathbb{O})+\rho(\beta,\mathbb{O})$. Taking into account that

$$\rho(\alpha,\beta)=\sum\nolimits_{i=1}^{n}\sum\nolimits_{j=1}^{m}\int_{A_i\cap B_j}f(x)dx\int_{A_i\triangle B_j}f(x)dx,$$

$$\rho(\alpha,O)=\left(\int_\Omega f(x)dx\right)^2-\sum\nolimits_{i=1}^{n}\left(\int_{A_i}f(x)dx\right)^2$$

we arrive at

$$2\left(\int_\Omega f(x)dx\right)^2\ge\sum_{i=1}^{n}\left(\int_{A_i}f(x)dx\right)^2+\sum_{j=1}^{m}\left(\int_{B_j}f(x)dx\right)^2+$$

$$+\sum_{i=1}^{n}\sum_{j=1}^{m}\int_{A_i\cap B_j}f(x)dx\int_{A_i\triangle B_j}f(x)dx.$$

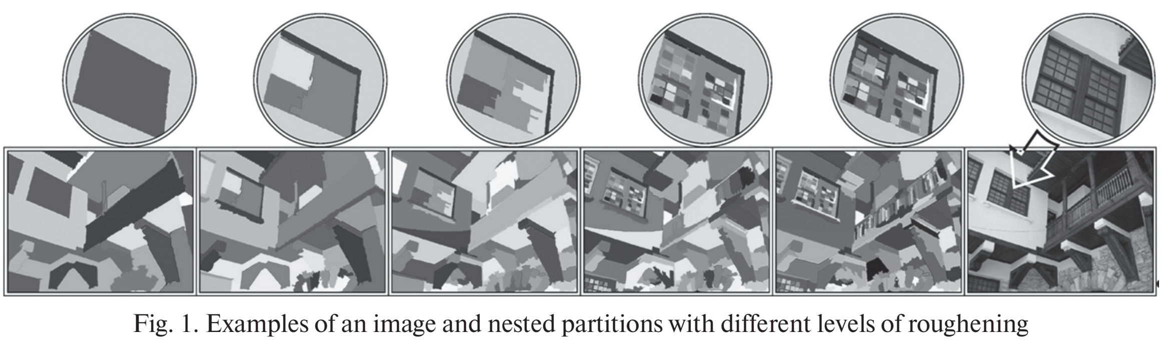

Fig. 1. Examples of an image and nested partitions with different levels of roughening

This inequality can be used as validation criterion for disclosing of nested quotient sets.

Suppose now that that β is a query in CBIR. If β is answering the semantic purpose i.e. the region of interest is detected and the desired segmented image α is embedded into it, there can be organized metric background independent search of objects. Indeed, since $\alpha,\beta\in\mathbb{O}$ and $\rho(\alpha,\beta)\le\rho(\alpha,\mathbb{O})+\rho(\mathbb{O},\beta)$ from Proposition 1 we obtain

$$\sum\nolimits_{i=1}^{n}[\mu(A_i)]^2-\sum\nolimits_{j=1}^{m}[\mu(B_j)]^2-$$
$$-\sum\nolimits_{i=1}^{n}\sum\nolimits_{j=1}^{m}\mu(A_i\Delta B_j)\mu(A_i\cap B_j)\ge 0.$$

The equality holds only if $\alpha\subseteq\beta$ so search images with minimal values of this criterion is the same that background independent objects retrieval.

3. METRICAL PROPERTIES OF NESTED PARTITIONS

Consider metrical properties of nested partitions. We shall use besides $\alpha,\beta\in\Pi_\Omega$ one more partition $\gamma=\{C_1,C_2,...,C_l\}$. Denote by $\alpha\beta$ an intersection of any quotient sets α and β. At first consider properties of $\rho(\alpha,\beta)$ $S(\alpha)$ and $G(\alpha)$.

Property 1.

$\forall\alpha,\beta\in\Pi_\Omega\Rightarrow\rho(\alpha,\beta)=S(\alpha)+S(\beta)-2S(\alpha\beta)$.

If $\alpha\subset\beta$ then $\rho(\alpha,\beta)=S(\beta)-S(\alpha)$.

Property 2.

$\forall\alpha,\beta\in\Pi_\Omega\Rightarrow\rho(\alpha,\beta)=2G(\alpha\beta)-G(\alpha)-G(\beta)$.

If $\alpha\subset\beta$ then $\rho(\alpha,\beta)=G(\alpha)-G(\beta)$.

These properties follow from (2), Proposition 1 and functionals definitions. Next interrelationships between $S(\alpha)$, $G(\alpha)$ and measure of set to be partitioned are the corollary of the same definitions.

Property 3. $S(\mathbb{O})=\mu^2(\Omega)$, $G(\mathbb{O})=0$.

Property 4.

i) $G(\alpha)=S(\mathbb{O})-S(\alpha)=\mu^2(\Omega)-S(\alpha)$;

ii) $S(\alpha)=S(\mathbb{O})-G(\alpha)=\mu^2(\Omega)-G(\alpha)$;

iii) $S(\alpha)+G(\alpha)=S(\mathbb{O})=\mu^2(\Omega)$.

For brevity of notation we shall write out properties for $S(\alpha)$. Properties of $G(\alpha)$ can be easily obtained from equalities mentioned in Property 4.

Property 5. If $\alpha\subset\beta$ then $S(\alpha\beta)=S(\alpha)$.

This property is the straightforward corollary of Proposition 1.

Property 6. The set Π_Ω is open bounded set viz $\rho(\alpha,\beta)<\mu^2(\Omega)$ $\forall\alpha,\beta\in\Pi_\Omega$.

The statement means that the set is limited but does not contain its bounds. For arbitrary $\alpha,\beta\in\Pi_\Omega$ from triangle inequality $\rho(\alpha,\beta)\le\rho(\alpha,\mathbb{O})+\rho(\beta,\mathbb{O})$ it follows

$$S(\alpha)+S(\beta)-2S(\alpha\beta)\le S(\mathbb{O})-S(\alpha)+S(\mathbb{O})-S(\beta)$$

whence it appears the inequality $\rho(\alpha,\beta)+S(\alpha\beta)\le S(\mathbb{O})$. Since by definition $S(\alpha\beta)\ge 0$ we have $\rho(\alpha,\beta)\le\mu^2(\Omega)$ from property 3. At the same time we can reach the bounds of Π_Ω, i.e. get the equality $\rho(\alpha,\beta)=\mu^2(\Omega)$ in result of infinite comminuting of partition $\alpha\beta$ tending to an unattainable in some sense partition, which consists of zero measure cosets, what is impossible. Thus $\rho(\alpha,\beta)<\mu^2(\Omega)$ what was required.

Property 7.

$2S(\alpha\beta)\le S(\alpha)+S(\beta)\le S(\alpha\beta)+\mu^2(\Omega)$.

Correctness of the inequalities results from a metric non-negativity and inequality $\rho(\alpha,\beta)+S(\alpha\beta)\le S(\mathbb{O})$.

As the implication of two above properties we can indicate for arbitrary partitions

Property 8. $S(\alpha\beta)<\mu^2(\Omega)$, $S(\alpha)<\mu^2(\Omega)$.

Property 9. If $\alpha\subset\beta$ then elements $\mathbb{O},\alpha,\beta\in\Pi_\Omega$ lay on one 'line' in the sense that

$$\rho(\alpha,\mathbb{O})=\rho(\mathbb{O},\beta)+\rho(\alpha,\beta),$$

i.e. β is located 'between' $\mathbb{O}$ and α.

Indeed, from inequality $\rho(\alpha,\beta)+\rho(\beta,\mathbb{O})\ge\rho(\alpha,\mathbb{O})$ considering $\alpha,\beta\in\mathbb{O}$ from Property 1 we find $S(\beta)-$ $-S(\alpha)+S(\mathbb{O})-S(\beta)\ge S(\mathbb{O})-S(\alpha)$ or $0\ge 0$ what proves required equality.

Property 10.

If $\alpha\subset\beta$ then $S(\alpha)\le S(\beta), G(\alpha)\ge G(\beta)$.

The explanation of the property consists in follows: under partition splitting $S(\alpha)$ decreases and $G(\beta)$ increases. The property validation is established by Property 1 and metric non-negativity.

Property 11. If partition splitting leads to the fulfillment $\lim\limits_{n\to\infty}\max\limits_{i\in\{1,...,n\}}\mu(A_i)=0$ then

$$\lim_{n\to\infty}G(\alpha)=\mu^2(\Omega),\ \lim_{n\to\infty}S(\alpha)=0.$$

Property 13.

If $\alpha,\beta\subset\gamma$ then $\rho(\alpha,\beta)+S(\alpha\beta)\le S(\gamma)$.

Property 14.

If $\alpha,\beta\subset\gamma$ then $S(\alpha\beta)<S(\gamma)$, $\rho(\alpha,\beta)<S(\gamma)$.

Property 15.

If $\alpha\subset\beta\subset\gamma$, then $\rho(\alpha,\beta)+S(\alpha\beta)\le S(\gamma)$.

Property 16.

If $\alpha\subset\beta\subset\gamma$, then $\rho(\alpha,\beta)-\rho(\beta,\gamma)=\rho(\alpha,\gamma)$, i.e. 'point' β lays on the 'line', which pass through points α and γ, and is situated 'between' them.

Summarize induced 'geometry' of set Π_Ω.

1. Π_Ω belongs to a 'circle' with centre in $\mathbb{O}$ and radius $\mu^2(\Omega)$.

2. Π_Ω has 'diameter' $\mu^2(\Omega)$ as $\rho(\alpha,\beta)<\mu^2(\Omega)$.

3. Π_Ω is a sheaf of 'lines', which pass through $\mathbb{O}$ but do not tend to infinity due to contingencies.

4. Π_Ω contains 'lines' which intersect infinite times.

5. Infinite partition *o* (as limits of uncountably infinite refinement) may be indicated which is nested into anyone other. Hence all 'lines' from Π_Ω may begin from any point α and *o* belongs to all of them.

OUTLOOK

Three expressions to evaluate similarities of nested partitions, relevant criterion of quotient sets nesting, search criterion of spatial content retrieval have been proposed. Multiple experiments with ground truth and algorithmic segmentations allow to affirm that obtained results provide the search of the images families of the cosets corresponding to the searched objects not depending on the background components, and in addition they make the retrieval not depending on segmentation technique. Both a query and image in database can be segmented with different levels of roughening or refinement (see Figure 3).

Fig. 2. Image, its partition and queries

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Поступила в редколлегию 8.12.2014

Каграманян Александр Георгиевич, кандидат технических наук, доцент кафедры естественных наук Харьковского национального университета им. В.Н. Каразина. Научные интересы: мультиалгебраические системы, мультигруппы, обработка и интерпретация видеоинформации.

Машталир Владимир Петрович, доктор технических наук, профессор, исполняющий обязанности ректора Харьковского национального университета радиоэлектроники. Научные интересы: обработка и распознавание изображений, модели и методы грануляции информации.

Шляхов Владислав Викторович, доктор технических наук, доцент, профессор кафедры высшей математики Харьковского национального университета радиоэлектроники. Научные интересы: модели и методы грануляции информации, мультиалгебраические системы.

УДК 519:616

Метрическое оценивание пространственного содержания для поиска, основанного на сегментации изображений / А.Г. Каграманян, В.П. Машталир, В.В. Шляхов // Прикладная радиоэлектроника: науч.-техн. журнал. — 2014. — Том 13. — № 4. — С. 436–439.

Интерес к анализу содержания изображений мотивируется расширением использования визуализации в различных предметных областях, наличием больших библиотек изображений, ростом мультимедиа приложений и т.д. Один из путей частичного устранения семантического разрыва между визуальными признаками низкого уровня и высоким уровнем человеческого восприятия является анализ пространственных свойств индуцированных сегментацией частей изображений. Для обеспечения поиска с запросом 'ad exemplum' на базе областей (в широком смысле) предложена новая метрика для сравнения произвольных вложенных разбиений. Изученные метрические свойства частично упорядоченных фактор-множеств обеспечивают, например, поиск объектов независимо от фона.

Ключевые слова: сегментация изображений, метрика, вложенные разбиения.

Ил.: 03. Библиогр.: 11 назв.

УДК 519:616

Метричне оцінювання просторового змісту для пошуку, який базується на сегментації зображень / О.Г. Каграманян, В.П. Машталір, В.В. Шляхов // Прикладна радіоелектроніка: наук.-техн. журнал. — 2014. — Том 13. — № 4. — С. 436–439.

Інтерес до аналізу змісту зображень мотивується розширенням використання візуалізації в різних предметних областях, наявністю великих бібліотек зображень, зростанням мультимедіа застосувань та ін. Одним зі шляхів часткового усунення семантичного розриву між візуальними ознаками низького рівня і високим рівнем людського сприйняття є аналіз просторових властивостей індукованих сегментацією частин зображень. Для забезпечення пошуку із запитом 'ad exemplum' на базі областей (у широкому сенсі) запропонована нова метрика для порівняння довільних вкладених розбиттів. Вивчені метричні властивості частково впорядкованих фактор-множин забезпечують, наприклад, пошук об'єктів незалежно від фону.

Ключові слова: сегментація зображень, метрика, вкладені розбиття.

Іл.: 03. Бібліогр.: 11 найм.