**UDC 621.9.048.6**

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# MATHEMATICAL MODEL OF NANOSTRUCTURED PROCESSING BASED ON PARTS OF NONLINEAR DISSIPATIVE OSCILLATOR

*The article presents a mathematical model of nanostructured finishing and improving surface treatment of machine parts based on nonlinear dissipative oscillator, allowing to define the bifurcation parameters of frictional force and speed of loading of the material of the surface layer and to conduct parametric synthesis tool to meet the challenge of the dynamic stability of the process. It is shown that the construction of such model is possible in advanced view taking into account restrictive conditions on the mechanical strength of the reaction tool in contact with the treated surface, expressed by equations of nonlinear dissipative oscillator dissipation nanostructured smoothing and energy analysis of the transition pressing process and emersion of the indenter .*

***Key words:*** *model, contact, system, indenter, surface layer, implantation, emersion, dynamic stability*

## Introduction

High efficiency in use of machine manufacturing of various kinds of finishing and hardening surface treatment of machine parts can be achieved due to reasonable modes selecting, selecting the tool design and the creation of conditions of dynamic stability of process. The solving of this problems is closely related with evolution of dynamical system of nanostructural finishing and hardening treatment (MRB) details, which takes into account rubbing in contact zone of instrument with treated material. That is why there is a need to review relation between energy supply and loss in dissipative system of nanostructural MRB details, which is possible only with a help of appropriate mathematical model of process dynamics. At once such mathematical model will help to determine the boundary conditions of frictional, power and speed capacity of material and MRB details and provide with dynamical stability of the process, what will positively affect on formation of qualitative results of surface layer.

## The main content and results of work

It is known that [1], generalization of mathematical model of dynamical system with one degree of freedom is equation of nonlinear dissipative oscillator kind:

$$\ddot{y} + \gamma\dot{y} + f(y) = 0, \tag{1}$$

where $y$ – dynamical variable, which represents a kind of generalized coordinate; $\gamma$ – dissipation parameter; $f(y)$ – nonlinear function.

In general, analysis of nonlinear dissipative oscillator is in determination stable conditions, which are considered as limiting set of plane phase, invariants relevant to time evolution of considered system. Then time evolution of dynamical nanostructural system of surface of MRB machine details represents by itself differential equation of indenter movement mass $m_1$, equilibrium equation of forces:

$$m_1 \frac{d^2 y_1}{dt^2} + B_1 \frac{dy_1}{dt} + P_k(y_1) - P_e \pm P_{tn} = 0. \tag{2}$$

Dissipation of energy in dynamical system of nanostructural MRB surface of details caused in location of area of used tool with the material being treated as viscous friction($B_1$), and friction without lubricant ($P_{тн}$), which depends from axial velocity of displacement of the indenter $y_1$, nonlinear relation type:

$$P_{tn} = P_{тtn0}\mathrm{sign}\left(\frac{\mathrm{d}y_1}{\mathrm{d}t}\right), \tag{3}$$

where $P_{tn0}$ – module of frictional force.

Nonlinear hysteresis dependence $P_k(t) = f(y_1(t))$ represents dynamic link with contact reaction force with generalized coordinate of considered dynamical system, in which variation force corresponds to variations of indenter position regarding treated work piece surface of detail and can be considered as dynamical characterization of this model of process nanostructural MRB.

Ambiguity of dynamical characteristics of MRB processes of machine`s detail surfaces is caused by forming roller pushed back metal in conditions of frictional force of material on the surface layer, which leads to mismatch of the curves pressing and emersion of the indenter. Coordinate of apex of indenter at the end point of pressing area corresponds to the maximum penetration depth: $y_1 = h_{bnmax}$.

At the end point coordinate emersion section of the indenter apex coincides with the vertex coordinates of the roller, that is $|y_1| = h_b$. In strengthening frictional contact properties and the height of the roller contact area of the indenter with the treated surface increases, resulting of increased inflow of energy into vibrational system.

To identify the strength of reaction forces in contact $P_k$ was offered mathematical model as a product of dynamical contact area $A_d(y_1)$ on contact pressure $P_{sk}(y_1)$, which includes changing of physical and mechanical properties of the material surface layer of work piece the multiplicity of loading $N_c$:

$$P_k(y_1(t)) = A_d(y_1)P_{sk}(y_1, N_c) = \pi R h_d(y_1(t))P_{sk}(y_1, N_c). \tag{4}$$

The change of contact area is considered by the depths of dynamical implementing indenter in the treated layer $h_d(y_1(t))$, which during pressing and emersion of indenter can be described in following way:

$$h_d(y_1) = \left[ y_1(t) + y_1(t)\frac{K_b}{1-K_b} \right] \text{ if } \dot{y}(t) > 0;$$

$$h_d(y_1) = \left[ y_1(t) + y_{\max}(t)\frac{K_b}{1-K_b} \right] \text{ if } \dot{y}(t) < 0. \tag{5}$$

Dependence of the contact pressure from the depth of the dynamic implementation of the indenter is largely determined by the physical and mechanical properties nanostructural MRB material surface layer parts [2]. To describe relation of $P_{sk}(y_1(t))$ represented in (4) was used in two zone linear model of the following type:

$$P_{sk}(y_1(t)) = a(N_c) + b(N_c)y_1(t) \quad \text{if } 0 < y_1(t) < h_{nc};$$
$$P_{sk}(y_1(t)) = c(N_c) + d(N_c)y_1(t) \quad \text{if } h_{nc} < y_1(t) < h_{\max} \tag{6}$$

First zone is determined by the thickness of nanostructured layer $h_{nc}$, second zone - thickness of surface layer $h_{max}$, corresponding to the maximum penetration depth of the indenter into the material by rehardening. In this regard, the dependence of the contact pressure $P_{sk}(y_1(t))$ can be based on the physical modeling of the process at different multiplicity of loading material $N_c$ and kinetic micro indentation of surface detail layer after

MRB. Sudden change of the parameter $P_{tn}$ hysteretic and dynamic characteristics of the work piece of smoothing detail surface $P_k = f(t)$ during pressing and emersing of indenter tool [2] makes the mathematical description of the dynamics of nanostructured MRB components in the form of piecewise continuous differential equations, the terms of which usage are determined direction of the vector of the axial velocity of the indenter $\dot{y}_1(t)$:

$$m_1\ddot{y}_1(t) + B_1\dot{y}(t) + P_b - P_{tn}\,, \quad \text{if } \dot{y}_1(t) > 0;$$

$$m_1\ddot{y}_1(t) + B_1\dot{y}(t) + P_b + P_{tn}\,, \quad \text{if } \dot{y}_1(t) < 0. \tag{7}$$

The condition of dynamical system of nanostructured MRB machine parts at any time of moment characterized vertex coordinate indenter $y_1$ relative to the origin given to the midline profile of the treated surface, and its rate of axial displacement $\dot{y}_1(t)$. The terms of the evolution of dynamic system to a stationary or oscillatory mode can be determined by analyzing the energy of the transition process of pressing and emersion of the indenter tool used. Then, considering the possible options for the development of the oscillatory process in dissipative dynamical system of nanostructured MRB details after the dynamic pressing and emersion of indenter, we can distinguish the following two special cases:

1. Energy loss in a dissipative system of nanostructured MRB details on an interval of time and dynamic pressing and emersion of indenter (oscillation cycle) exceeds the level of income. In this case total energy of system through the time decreases, and oscillatory process goes down.
2. Energy intake in this oscillation cycle exceeds its dissipation. It increases the amplitude of the oscillations until the occurrence of the energy balance in which the value of the incoming energy per unit of time equal to its losses over the same period. In this case, the indenter tool performs undamped oscillations.

Important for the dynamic stability of nanostructured components of MRB details has been used construction of the tool. In modern constructions of strengthening and smoothing tools [3], which can be taken as a basis for nanostructured surfaces of MRB details with the usage of NC machines and machining centers, a predetermined force of processing $P_b$, acting on the indenter, moving into direction with the opposing force of friction $P_{tn}$, provided by means of adjustable compression spring. Dynamical properties of the instrument can be considered as mass of indenter $m_1$ and the equivalent viscous damping coefficient $B_1$, which in types of structures of reinforcing and smoothing tool is determined by the friction force $P_{tn}$. In the zone of contact interaction of the indenter with material surface layer of the work piece machining force $P_b$, opposes the force of reaction in contact $P_k$. Frictional properties of the surface of indenter contact with the material being processed defined deformation friction force $P_{tr}$. In the general case, the total flow of energy in the area of ascent of indenter $W_+$, determined by the energy of the compressed spring $\Delta W_p$ and the kinetic energy of the moving $\Delta W_k$:

$$W_+ = \Delta W_p + \Delta W_k = k\frac{\Delta y_1^2}{2} + m_1\frac{\dot{y}_1(t)^2}{2}, \tag{8}$$

where $k$ - spring stiffness, $\Delta y_1$ - displacement of indenter increment.

The nature of the stored energy dissipation $W_-$. Is determined by the type of generalized dissipative friction forces [4]:

$$W_- = \Delta W_{bt} + \Delta W_{kt}. \tag{9}$$

Energy losses $\Delta W_{bt}$ per oscillation cycle of indenter associated with friction viscous damping force proportional to the frequency of oscillations ω, square of the amplitude $Y_1^2$ and viscous damping equivalent coefficient $B_1$:

$$\Delta W_{\text{bt}} = 2B_1 Y_1^2 \omega^2 \int_0^{\pi/\omega} \sin^2 \omega t dt = \pi B_1 Y_1^2 \omega. \quad (10)$$

Energy losses determined by the frictional force in guiding indenter, proportional to the modulus of the friction force of the oscillation amplitude and rhythm $Y_1$:

$$\Delta W_{\text{kt}} = 2P_{\text{tno}} \omega \int_0^{\pi/\omega} \sin^2 \omega t dt = 4P_{\text{tno}} Y_1^2. \quad (11)$$

In case of contact with the indenter material which is being processed in the presence of lubrication condition of the balance of the internal energy is given by:

$$k\frac{\Delta y_1^2}{2} + m_1 \frac{\dot{y}_1(t)^2}{2} = \pi B_1 Y_1^2 \omega. \quad (12)$$

In the absence of lubrication of guide indenter tool balance the condition of internal energy has the form of:

$$k\frac{\Delta y_1^2}{2} + m_1 \frac{\dot{y}_1(t)^2}{2} = 4P_{\text{tno}} Y_1. \quad (13)$$

Condition of energy balance corresponds to three points. Point with the amplitude $Y_1 = a_{\text{o}}$ corresponds to the stable static equilibrium system. If the amplitude change $Y_1 > a_1$, system comes back to the condition $Y_1 = a_{\text{o}}$.

Point $Y_1 = a_2$ corresponds to the self-oscillating mode. Point $Y_1 = a_1$ corresponds to an unstable dynamic equilibrium in which any change in the oscillation amplitude of the indenter causes the following state of the process:

- If $Y_1 < a_1$ oscillation amplitude decreases to $Y_1 = a_{\text{o}} = 0$, which corresponds to the steady state of static equilibrium;
- If $Y_1 > a_1$ oscillation amplitude increases to $Y_1 \approx a_2$, the system goes into self-oscillatory mode.

In the last case for developing self-oscillatory mode in system, which is in static equilibrium, requires external action with amplitude $Y_1 > a_1$; such type of excitation of self-oscillations are classified as hard excitation [5]. From the theory of regular and chaotic self-oscillations it is known that any self-oscillating dynamic system is nonlinear and dissipative and only the simultaneous implementation of these two conditions promotes self-oscillating mode of dynamical system in the sense of independence from the initial conditions, that is the initial energy. These conditions make necessary the analysis of dynamic systems MRB nanostructured surfaces of details as an essentially nonlinear system with mainly "soft" excitation.

To predict the effect on the dynamic stability of nanostructured components of MRB details speed indentation into the material $v_{\text{в}}$ it is necessary to establish its relationship with the speed of the axial movement of the indenter $\dot{y}_1(t)$ during its emersion after finishing of dynamical indentation. Taking into account smallness of the quantities $h_{\text{b}}$ and $h_{\text{bn}}$ in compare with radius of indenter R, it can be written:

$$\frac{v_{\text{b}}}{\dot{y}_1(t)} \approx \frac{\sqrt{2R(h_{\text{b}} + h_{\text{bn}})}}{h_{\text{b}} + h_{\text{bn}}} = \sqrt{\frac{2R}{h_{\text{b}}}}, \quad (14)$$

where $h_{\text{d}}$ - depth of dynamic implementation of the indenter into the material of the surface layer in the details of MRB.

Since the height of the roller $h_b$ and depth of dynamic implementation of the indenter $h_d$ are functionally connected with a scale coefficient of the plastic structure formation $K_b$, final velocity interrelation $v_в$ and $\dot{y}_1$ can be represented in such dependence:

$$v_b = \dot{y}_1(t)\sqrt{\frac{2RK_b}{h_b}}. \quad (15)$$

The effect of velocity of nanostructured MRB details $v_b$ on the character of development of the oscillatory process of indenter can be set based on the classification of dynamic characteristics of nanostructured surface layer of smoothing depending on the relation of defined speed $v_{bi}$ with critical smoothing speed $v_{bkr}$, corresponding to the output of the indenter in contact with the surface layer and the excitation of self-oscillations that can match:

- stationary regime of smoothing with small oscillatory process "indentation-emersion";

-relation $v_b \le v_{bkr}$ transitional process of damped oscillations in the surface layer of the indenter;

- relation $v_b > v_{bkr}$ and developing of the self-oscillatory mode nanostructural smoothing with the output of the indenter tool from the surface layer.

Mathematical model of excitation and development of self-oscillations of the indenter tool, reflecting the extreme case of the evolution of dynamical system of nanostructured smoothing (and any other type of surface MRB details), can be represented by the following differentiating equations of second order with discontinuous right side:

$$\begin{aligned}
&m_1\ddot{y}_1(t) + B_1\dot{y}_1(t) + P_k = P_b - P_{tn} \quad \text{if } y_1 > 0;\ \dot{y}_1(t) > 0;\\
&m_1\ddot{y}_1(t) + B_1\dot{y}_1(t) + P_k = P_b + P_{tn} \quad \text{if } y_1 > y_b;\ \dot{y}_1(t) < 0;\\
&m_1\ddot{y}_1(t) + B_1\dot{y}_1(t) = P_b + P_{tn} \quad \text{if } y_1 < y_b;\ \dot{y}_1(t) < 0; \qquad (16)\\
&m_1\ddot{y}_1(t) + B_1\dot{y}_1(t) = P_b - P_{tn} \quad \text{if } y_1 < 0;\ \dot{y}_1(t) > 0.
\end{aligned}$$

The main tasks of the study of dynamics of nanostructured components of MRB details are definition of bifurcation parameter values of frictional and force and velocity loading of material from the surface layer and parametric synthesis tool to do the task of the dynamic stability of the process. The task of parametrical synthesis of node of dynamic stability is to determine the parameters of the mass of the indenter tool $m_1$ and equivalent damping coefficient $B_1$, providing vibration resistance of the process at different levels of frictional and force loading of the material from the surface layer, that is for different values of the coefficient of the height of the roller $K_b$ and processing speed $v_в$. As the roller is formed during processing of the metal push back is a source of energy supply in the oscillation process, determining nonlinear dynamic characteristics of MRB details components and causing the excitation of oscillations in the indenter tool, may be assumed that the nonlinear dependence of the friction and force in the guiding of the indenter tool a typical model that determines the energy dissipation and thereby prevents the excitation of oscillations.

**The general algorithm and recommendations**

Mathematical model of nanostructured MRB details, which determines stable condition of dynamical system, it is advisable to create it on the base of nonlinear dissipative oscillator equation (1), keeping the next sequence:

1. Prepare the evolution operator of dynamical system of MRB details of nanostructured surfaces in machine details in the form of balance of power equation (2).

2. Determine the type of the contact with the used tool with treated material and compose nonlinear hysteretic relation of the type $P_k(t) = f(y_1(t))$.

3. Identify the depths of dynamic implementation of indenter $h_d(y_1(t))$ in treated surface layer of the detail, depending on the conditions defined by formulas (5).

4. Establish the relationship between the value of the contact pressure $P_{sk}(y_1(t))$ and thickness of nanostructured layer $h_{max}$ at different multiplicity of loading material $N_c$ and the results of kinetic micro indentation layer after MRB details.

5. Make the description of dynamics of nanostructured MRB details in the form of a continuous piecewise differential equations (7).

6. Energetic losses and incomes are analyzed in dissipative of nanostuctured MRB details and defined frictional properties of the interaction of the indenter with the material of the surface layer in view of the viscous damping lubrication and energy balance.

7. Determine the degree of influence of speed indentation axial movement of the indenter on the dynamic stability of nanostructured MRB components in accordance with the dependence (15).

8. Using second order differential equations with discontinuous right - side of part (16), the bifurcation parameter values are determined with frictional force of speed loading of the material of the surface layer parametric synthesis is carried out of the tool which provides dynamic stability of MRB details.

**Numerical models**

In general, the numerical solution of the problem of determining the boundary conditions of frictional and force of speed loading of the material of the surface layer in with MRB details which contains mainly in setting allowable values of indentation force $P_b$ and velocity of processing $v_в$ in the space of parameter values $K_b$ at various initial states of treated material surface hardness. Taking into account essential nonlinearity of mathematical model of a dynamical system of MRB details nanostructured surfaces which are expressed by differential equations (16), analysis of the stability of the system in the space of technological parameters and initial conditions can be carried out by the phase portrait, described in work [6]. In this case change of the margins of technological regimes of nanostructured MRB details components can be determined on the basis of the simulation software package for VisSim bifurcation changes of the multi sheet portrait of nonlinear dissipative oscillator process.

**Conclusion**

So the study made it possible to implement the following:

1. To develop generalized mathematical model of nanostructured surfaces of MRB details on the basis of a nonlinear dissipative oscillator.
2. To determine the effect speed indentation into the processed material which is on dynamic stability of nanostructured MRB details .
3. To identify the bifurcation parameter of values of frictional and force of speed loading of material surface layer details if MRB and conduct parametric synthesis tool for solving the problem of dynamic stability of the process.

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Надійшла до редакції 30.01.2014

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**МАТЕМАТИЧНА МОДЕЛЬ ДИНАМІКИ НАНОСТРУКТУРУЮЧОЇ ОБРОБКИ ДЕТАЛЕЙ НА ҐРУНТІ НЕЛІНІЙНОГО ОСЦИЛЯТОРА**

*В статті показана математична модель наноструктуруючої обробно-зміцнюючої обробки деталей машин на ґрунті нелінійного дисипативного осцилятора, що дозволяє визначити біфуркаційні параметри фрикційно-силового та швидкісного навантаження матеріалу поверхневого шару і проводити параметричний синтез інструменту для рішення задачі забезпечення динамічної стійкості процесу. Вказано, що побудування такої моделі можливе в розверненому вигляді з обліком обміжних умов на механічну силу реакції в контакті інструменту з оброблюваною поверхнею, виражених рівняннями нелінійної дисипації дисипативного осцилятору наноструктуруючого вигладжування і аналізу енергетики перехідного процесу втискування та виплування індентору.*

***Ключові слова**: модель, контакт, система, індентор, поверхневий шар, втискування, виплування, динамічна усталеність.*

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**МАТЕМАТИЧЕСКАЯ МОДЕЛЬ ДИНАМИКИ НАНОСТРУКТУРИРУЮЩЕЙ ОБРАБОТКИ ДЕТАЛЕЙ НА ОСНОВЕ НЕЛИНЕЙНОГО ДИССИПАТИВНОГО ОСЦИЛЛЯТОРА**

*В статье представлена математическая модель наноструктурирующей отделочно-упрочняющей обработки поверхностей деталей машин на основе нелинейного диссипативного осциллятора, позволяющая определять бифуркационные параметры фрикционно-силового и скоростного нагружения материала поверхностного слоя и проводить параметрический синтез инструмента для решения задачи обеспечения динамической устойчивости процесса. Показано, что построение такой модели возможно в развернутом виде с учетом ограничительных условий на механическую силу реакции в контакте инструмента с обрабатываемой поверхностью, выраженных уравнениями нелинейной диссипации диссипативного осциллятора наноструктурирующего выглаживания, и анализа энергетики переходного процесса вдавливания и всплытия индентора.*

***Ключевые слова:** модель, контакт, система, индентор, поверхностный слой, внедрение, всплытие, динамическая устойчивость.*